

## Troubleshooting : when modeling is the trouble

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### Abstract

This paper shows how order of magnitude reasoning has been successfully used for troubleshooting complex analog circuits. The originality of this approach was to be able to remove the gap between the information required to apply a general theory of diagnosis and the limited information actually available. The expert's ability to detect a defect by reasoning about the significant changes in behavior it induces is extensively exploited here: as a kind of reasoning that justifies the qualitative modeling, as a heuristic that defines a strategy and as a working hypothesis that makes clear the scope of this approach.

### I. Introduction

The challenge of troubleshooting is to localize, in a malfunctioning device, those faulty components (elementary physical elements having a well defined function) which can be replaced or modified

A classical approach would be to provide a set of dependency relations between failures and faults. The efficiency of such "shallow" reasoning relies on the description of all possible failures. This knowledge is strongly dependent on a particular device and is often not complete. Troubleshooting another device with the same functioning principles requires reconsidering the knowledge base.

The model-based paradigm [Davis *et al.*, 1982], [Brown *et al.*, 1982] leads to a more general approach, since only models of correct behavior for generic components have to be given. An interesting feature of this approach is that basically there is no need for either a fault model or a set of heuristically defined dependencies between failures and faults. The device specific knowledge is organized around a structural decomposition of the device. It is assumed that all correct behavior of a complex device can be predicted from its structure and the models of its components. Thus, a difference between the predicted behavior of a block (i.e. a set of connected components) which is presumed to be correct, and the observed behavior indicates that there is at least one defect in the block. The task of troubleshooting is then to identify those differences and to progressively refine their localization until a small faulty replaceable part has been located.

But determining the differences between the presumed correct behavior and the actual observations requires defining

relevant models of behavior for generic components. For analog circuits this is where problems arise, as explained in Section II. There is a lack of numerical models. Numerical models which are used in classical simulation algorithms to predict the behavior of a well functioning device are not adequate for troubleshooting purposes, once correct components work outside their normal functioning limits. In addition, basic qualitative models [De Kleer, 1984] that mainly handle signs of quantities are not powerful enough to find inconsistencies between the predicted behavior and observed behavior. Therefore in the two cases the predictive procedure may fail to detect conflicts. Modeling becomes the trouble. Section III shows how, in order to overcome this difficulty, we take advantage of the expert's ability to reason about the main changes in the behavior of a device. It allows us to make the *fundamental assumption that a defect leads to significant changes in the behavior of a device* and to exploit it by performing *order of magnitude reasoning*. Section IV demonstrates how this is used in the expert system DEDALE<sup>1</sup> to obtain a relevant qualitative modeling that distinguishes between different patterns of behavior. An example of diagnosis is given in Section V. Section VI explains that the fundamental assumption also provides a troubleshooting strategy when the circuits are more complex.

### II. Complexity of Troubleshooting in Analog Circuits

The difficulty in troubleshooting analog circuits is to characterize the correct behavior of a component in a malfunctioning circuit. The main reason is that, in a malfunctioning analog circuit, a component can behave in a way which is radically different from its designed behavior, and yet be correct. Thus knowing the designed behavior of components provides only part of the relevant modeling. In addition, the information required to numerically describe all possible correct behavior of a component is often too complex or not available for troubleshooting purposes. The following problems must be tackled:<sup>2</sup>

<sup>1</sup> DEDALE is an expert system for troubleshooting analog hybrid circuits, that has been jointly developed by Electronique Serge Dassault and IBM.

<sup>2</sup> The case of intermittent failures is not taken into account in this paper: it is assumed the faulty circuit is in a steady electric state, and observations are reproducible.

#### - Multiple Correct Behavior Patterns

The behavior of each analog electronic component depends, most of the time, on all the components which are connected to it. A defect on one component may change the functioning state of others. Thus, predicting the behavior of the different components quickly becomes very complex. For example, a transistor may behave very differently, as a current amplifier, a switch, etc depending on its electronic environment. For instance, if the value of a resistor on the emitter of a transistor is too high the transistor changes from its normal (current amplifier) functioning state to an open (no current) functioning state.

#### - Lack of Numerical Models

Numerical models of behavior for each component are often useless. Time dependence and non linearity make such models complex to use, except for some simple components (e.g. Ohm's law for resistors). For example, a model of a transistor requires the specification of a dozen parameters. Today, such models are used only to simulate correct functioning.

#### - Lack of Measurements

The observation of analog electronic behavior means providing the values of state variables, some of which cannot be measured. The inability, in an analog circuit, to measure currents is the most crucial of these limitations, because current is a state variable which is essential to distinguish between the different models.

### III. Exploiting Significant Changes in Behavior

Setting the troubleshooting of analog circuits in a model-based approach raises a certain number of basic problems. In order to explain these difficulties, and to set the debate in a well defined framework, the General Diagnostic Engine [De Kleer and Williams, 1986] is taken here as a reference.

The General Diagnostic Engine consists essentially of three parts: a *predictive procedure*, an *ATMS* (Assumption-based Truth Maintenance System) and a *measurement strategy*. The predictive procedure uses models and structure to make behavioral predictions from observations and assumptions of good functioning; it also detects *symptoms* (i.e. discrepancies between predictions or discrepancies between predictions and observations). A *symptom* is when at least one of a set of assumptions on the correct behavior of components is false. The ATMS manages these assumptions. From the *symptoms* it determines minimal *conflicts* (a *conflict* is a set of components, at least one of which is faulty) and generates a complete set of minimal *candidates*. A candidate is a set of components which, if they are faulty, explain, i.e. intersect, all the *conflicts*<sup>3</sup>. The diagnosis procedure is incremental and guided by the measurement strategy. The adequacy of a GDE to real

<sup>3</sup> Every superset of a conflict must be a conflict, and every superset of a candidate must be a candidate. Representing minimal conflicts and minimal candidates is thus sufficient.

problems is closely linked to the efficiency of the predictive procedure and of the strategy.

The trouble for analog circuits is that (see I) the models of behavior required to have a predictive procedure are not available. The solution we propose is to take advantage of the fact that the troubleshooter reasons in terms of order of magnitude. This justifies the underlying assumption made in DEDALE that a defect leads to significant changes in the behavior of the circuit. This assumption makes it possible to perform qualitative reasoning to:

- use models of behavior based on order of magnitude relations, as defined by the expert,

- search for significant *symptoms*. This is achieved by using a problem solver that checks the consistency of a set of order of magnitude equations.

- define a strategy based on the concept of *deviation*. A *deviation* is when a function behaves in a way which is significantly different from its designed behavior. These deviations are looked for in the *functional hierarchical decomposition* of the circuit.

It should be noticed that the checking process here is not a predictive procedure in the strict sense of the word. Such a procedure would lead to a qualitative "big crunch" [Brown, 1976] (a brute force approach) due to the multiplicity of correct behavior patterns. In addition, the implication *symptom* → *conflict* is replaced for high level functions by the heuristic rule *deviation* → *focusing*.

### IV. Qualitative Modeling in DEDALE

#### A. Order of Magnitude Reasoning

Reasoning about significant changes in the behavior of a circuit means performing qualitative reasoning. Models handling only signs of quantities fail, even in simple cases, to distinguish between radically different patterns of behavior. In order to take significant changes into account, the qualitative value of a quantity that must be considered is both its sign and its relative order of magnitude. To describe order of magnitude relations, three key operators  $\ll$ ,  $\cong$ ,  $\sim$  are defined, which represent the following intuitive concepts:

$A \ll B$  stands for A is negligible in comparison with B,

$A \cong B$  stands for A is close to B, i.e.  $(A - B)$  is negligible in comparison with B,

$A \sim B$  stands for A has the same order of magnitude as B. The underlying idea is that if  $A \sim B$ , then  $B \ll C$  implies  $A \ll C$ .

A formal system FOG [Raiman, 1986] defines a set of rules that can be applied to these relations (see Appendix). The basic axioms are the following:  $\cong$  and  $\sim$  are both equivalence relations and  $\cong$  is finer than  $\sim$ ,  $\ll$  is a partial ordering between the equivalence classes for  $\sim$ .

Other operators useful in DEDALE are defined in terms of the three previous ones:

$A \sim^+ B$  stands for  $A > B$ ,  $A \sim B$ , and  $\neg(A \cong B)$   
 $A \sim^- B$  stands for  $A < B$ ,  $A \sim B$ , and  $\neg(A \cong B)$ .

### B. Library of Qualitative Models

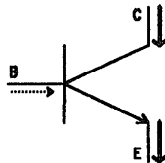
Some simple components have only one correct behavior, easily described by a unique model: Ohm's law for resistors, Kirchoff's laws for nodes (remember that even a node is a component since it can be faulty). But, in general, generic components may have several possible correct behavior patterns, and different models are needed to describe all of them.

A model  $M_i$  for a generic component consists of a set  $C_i$  of constraints linking the electrical parameters attached to this component. Voltages are linked by numerical constraints and currents by qualitative constraints. These models are based on physical laws and expertise. This expertise is required to describe all qualitatively correct behavior of complex components and to specify ranges for the numerical values of voltages corresponding to each behavior. For two different models of behavior  $M_i$  and  $M_j$  of the same component there is a significant change in terms of order of magnitude of at least one parameter. In order to reason about changes of behavior of the component, a set  $C_{ij}$  of constraints must be given for each pair  $(M_i, M_j)$  of models. These constraints express the relative order of magnitude of any given parameter in  $M_i$  and  $M_j$ . Thus, each generic component has a set of models described by all the constraints  $C_i$  and  $C_{ij}$ .

For a given circuit and given input patterns<sup>4</sup>, each component will have a particular model,  $M_N$ , (nominal) selected from its library. It is the model which corresponds to its designed behavior within the correct circuit. The values of the parameters for this particular model, called nominal values, are available by simulation or by measurement on a correct circuit. They are noted:  $V^N, I^N, \dots$  Each model  $M_i$  is now described by its variation with respect to the reference model,  $M_N$ , i.e. by the sets of constraints  $C_N, C_i$  and  $C_{iN}$ .

For instance, let us assume that the nominal model of a transistor is (this is the so called "normal" state in electronics):<sup>5</sup>

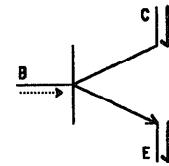
$$\begin{aligned} I_B^N &\ll I_C^N \\ I_C^N &\cong I_E^N \\ 0.6 &< V_{BE}^N < 0.9 \\ 0.2 &< V_{CE}^N \end{aligned}$$



With this nominal model, possible correct behavior patterns for the transistor are:

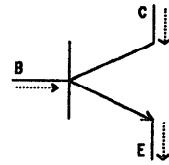
"normal" state: no significant change in currents, but some possible changes in voltage.

$$\begin{aligned} \text{no} : I_C &\cong I_C^N \\ \text{no} : I_E &\cong I_E^N \\ \text{no} : 0.6 &\leq V_{BE} < V_{BE}^N + 0.4 \\ \text{no} : V_{CE} &\geq V_{CE}^N \end{aligned}$$



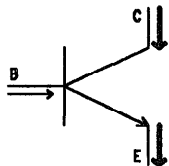
open state: great reduction in currents

$$\begin{aligned} \text{op} : I_C &\ll I_C^N \\ \text{op} : I_E &\ll I_E^N \\ \text{op} : I_B &\ll I_B^N \\ \text{op} : V_{BE} &< 0.5 \end{aligned}$$



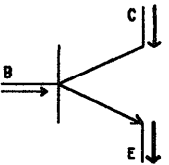
on state: limited increase in currents

$$\begin{aligned} \text{on} : I_C &\sim^+ I_C^N \\ \text{on} : I_E &\sim^+ I_E^N \\ \text{on} : I_B^N &\ll I_B \\ \text{on} : V_{CE} &< V_{CE}^N \\ \text{on} : V_{BE} &\geq V_{BE}^N \end{aligned}$$



s state: same as on state, but with a lower collector current

$$\begin{aligned} \text{s} : I_C &\sim^- I_C^N \\ \text{s} : I_E &\sim I_E^N \\ \text{s} : I_B^N &\ll I_B \\ \text{s} : V_{CE} &< V_{CE}^N \\ \text{s} : V_{BE} &\geq V_{BE}^N \end{aligned}$$



### C. Assumptions and local consistency

If we presume that a component is correct, we attach a qualitative model of correct behavior to this component. This implies selecting a model from among the several models of correct behavior. This choice can be made only if relevant observations are available. Remember that the only observations available are measurements for voltages, not for current intensity. Since there is not a one to one mapping between ranges for voltages and qualitative models of behavior, different models are generally consistent with the observations. Thus, selecting a model involves making an assumption.

For example, consider the transistor T1 for which measurements indicate the following changes with respect to the nominal values:

$$\begin{aligned} V_{BE}^N &= 0.74 & V_{BE} &= 1 \\ V_{CE}^N &= 2 & V_{CE} &= 0.25 \end{aligned}$$

Two models of correct behavior (on,s) for T1 are consistent with these observations. The two corresponding assumptions are noted T1(on) and T1(s).

If we now presume that a block B, i.e. a set of connected components, is correct we attach a model of correct behavior to each of its components. Thus, an assumption for block B is a set  $a_B$  of elementary assumptions for each of its components.  $A(B)$  stands for the set of all potential assumptions  $a_B$ .

<sup>4</sup> We refer here to input patterns where the failure has been observed.

<sup>5</sup> Base B, emitter E and collector C are the three terminals of a transistor.

for block B. The topology of a circuit makes it possible to define a set of links L(B) between the terminals of the components of block B. A link stands for a connection between two terminals of two different components, and is viewed as a constraint<sup>6</sup>. An assumption  $a_b$  is consistent if it satisfies all the constraints attached to L(B). FOG checks whether this set of constraints is satisfied or not. The set of assumptions  $a_b$  that satisfy L(B), is noted C(B). If C(B) is empty, then the set of components in B is a *conflict*. This means that there is a defect in B.

Minimal *conflicts* are searched for by focusing first on minimal blocks, which are blocks composed of a node and the components connected to it. Such conflicts are minimal in that no available observation could reduce their size: to be able to detect an inconsistency in Kirchoff's law for currents in a node, we need to know the order of magnitude of each of these currents, in other words to provide models of behavior of each component connected to that node.

## V. Example of Diagnosis

Consider a simple basic block, called a voltage follower. It is composed of: two transistors (T1 and T2), five resistors (R1-R5) and four internal nodes (N2-N5). Voltage is measured at the terminals of the different components. These values are available for the nominal behavior (see Fig. 1) and the actual behavior (see Fig. 2). With these nominal values, T1 and T2 are both in the no state. Values of resistors R4 and R5 and Ohm's law imply that there are two main currents of the same order of magnitude (see Fig. 3 nominal behavior):

$$I_{R4}^N \sim I_{R5}^N$$

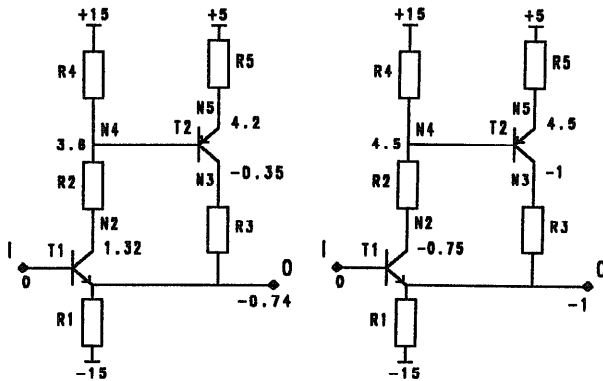


Fig. 1 Nominal behavior

Fig. 2 Malfunctioning

In this example, a *deviation* is observed for the voltage follower: the observed voltage on input I is equal to its nominal value, but the voltage on output O is appreciably less than its

<sup>6</sup> A link indicates that the same electrical signal is propagated on the two terminals. It implies the same voltage on the two terminals, with opposite currents.

nominal value. Because of *deviation*, attention is focused on the components of the follower. The assumptions of possible correct behavior for transistors, consistent with the observed measurements, are: T1(on) and T1(s), and T2(op).

The sets of possible assumptions for the minimal blocks are:<sup>7</sup>

$$\begin{aligned} B2 &= [N2, T1, R2] \\ A(B2) &= \{ \{ N2, T1(\text{on}), R2 \}, \{ N2, T1(\text{s}), R2 \} \} \\ B3 &= [N3, T2, R3] \\ A(B3) &= \{ \{ N3, T2(\text{op}), R3 \} \} \\ B4 &= [N4, T2, R2, R4] \\ A(B4) &= \{ \{ N4, T2(\text{op}), R2, R4 \} \} \\ B5 &= [N5, T2, R5] \\ A(B5) &= \{ \{ N5, T2(\text{op}), R5 \} \}. \end{aligned}$$

Let's examine the consistency of  $A(B4)$ . The nominal behavior implies:

$$\begin{aligned} I_B^N &\ll I_B^N && \text{(T2 no)} \\ I_{R5}^N &\sim I_{R4}^N && \text{(Ohm's law, see above)} \\ I_{R5}^N &\cong I_B^N && \text{(Kirchoff's law for N5)} \end{aligned}$$

Using axioms of FOG (see Appendix), we deduce from these three relations:

$$I_B^N \ll I_{R4}^N$$

This relation and Kirchoff's law for N4 give:

$$\begin{aligned} I_{R2}^N &\cong (-I_{R4}^N) && (1) \\ I_B^N &\ll I_{R2}^N && (2) \end{aligned}$$

For the actual behavior, the assumptions of correct behavior for components of B4 give:

$$\begin{aligned} T2(\text{op}) : I_B &\ll I_{R2} && (3) \\ R2 : I_{R2} &\sim^+ I_{R2}^N && (4) \text{ (Ohm's law)} \\ R4 : I_{R4} &\cong I_{R4}^N && (5) \text{ (Ohm's law)} \\ N4 : (I_{R2} + I_B) &\cong (-I_{R4}) && (6) \text{ (Kirchoff's law)} \end{aligned}$$

Using FOG once again, we obtain:

$$\begin{aligned} (2) + (4) &\rightarrow I_B^N \ll I_{R2} && (7) \\ (1) + (4) + (5) &\rightarrow I_{R2} \sim^+ (-I_{R4}) && (8) \\ (3) + (7) &\rightarrow I_B \ll I_{R2} && (9) \\ (6) + (9) &\rightarrow I_{R2} \cong (-I_{R4}) && (10) \\ (8) + (10) &\rightarrow \text{contradiction by definition of } \sim^+ \end{aligned}$$

Thus, C(B4) is empty. The same reasoning leads to:

$$\begin{aligned} C(B2) &= \{ \{ N2, T1(\text{on}), R2 \} \}. \\ C(B3) &= \{ \{ N3, T2(\text{op}), R3 \} \}. \\ C(B4) &= \{ \}. \\ C(B5) &= \{ \}. \end{aligned}$$

<sup>7</sup> Assumptions of correct behavior for resistors and nodes, that correspond to a unique model, are indicated by the name of the component.

Since C(B4) and C(B5) are empty, at least one of the components N4, T2, R2 and R4 is faulty as is at least one of the components N5, T2 and R5. Thus, two minimal *conflicts* are identified:

$$\langle N4, T2, R2, R4 \rangle \text{ and } \langle N5, T2, R5 \rangle.$$

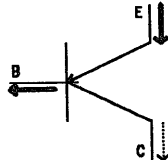
If the nodes are correct, then the three minimal *candidates* are [T2], [R2,R5] and [R4,R5]. This means that the set of defects of the circuit contains at least one of these three sets.

With the more restrictive assumption that there is a unique defect, it is certain that T2 is the faulty component, because T2 is the only component that may cause both *conflicts*. But in addition we can also discover the kind of electrical defect occurring in T2. Finding the behavior of the faulty component is obtained by suppressing its assumption of correct behavior. Here, suppressing the assumption that T2 is correct (in particular suppressing equation (3)) and applying FOG once again gives:

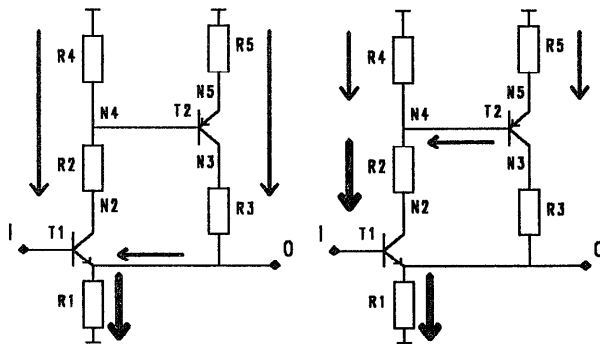
$$(6) + (8) \rightarrow -(I_B \ll I_{R2})$$

The complete reasoning for the other terminals of T2 leads to:

$$I_C \ll I_B \\ I_E \cong I_B$$



This shows that the defect on T2 is a short-circuit between the base and the emitter. Unlike the situation in shallow reasoning, where all possible faults have to be described beforehand, no models of misbehavior are needed here. Even better, such models can be discovered. Finally the qualitative reasoning describes the main changes in the behavior of the circuit due to the defect (see Fig.3). Notice that T1 is correct, although its behavior has changed: T1(n) instead of T1(normal).



nominal behavior      short-circuit of T2  
Fig.3 Main currents

## VI. Strategy

The troubleshooting example described above does not require using a strategy, since few components are involved. Troubleshooting circuits containing about a hundred components is more complex. Using once again the fundamental assumption that a defect in a component leads to significant changes in the behavior of the circuit allows us to define three basic strategies.

### A. Top-Down Strategy

According to the assumption, a defect in a component induces changes in the behavior of higher level blocks which contain this component. Most of the time, such changes occur in an observable way for at least one of these blocks, i.e. a *deviation* can be observed for this block. The top-down strategy consists then in focusing on functional hierarchical blocks where there are *deviations*, i.e. changes in order of magnitude between the actual and the nominal behavior of the block. The process repeats itself until it reaches a basic function  $B_j$ , for which the sub-functions are components. It is then possible to use models of behavior for these components<sup>8</sup>. The search for minimal *conflicts* inside  $B_j$  proceeds as for the above example by looking first for minimal blocks in  $B_j$ . If no *conflict* has been detected inside  $B_j$ , we obtain a non empty set of locally consistent assumptions  $C(B_j)$ . The fact of obtaining a non empty set  $C(B_j)$  is because the *deviation*  $\rightarrow$  *focusing* rule is just a heuristic: a *deviation* for a function does not necessarily imply a defect in one of its components. Another block,  $B_k$ , must then be considered.

### B. Horizontal Strategy

In fact, a *deviation* for a block may result from a defect in another block linked to it. Thus, the horizontal strategy means selecting block  $B_k$  which is of the same hierarchical level as  $B_j$ , i.e. both are contained in the same higher level block B, and focusing first on a block  $B_k$  linked to  $B_j$ . According to the top-down strategy, we first search for a  $B_k$  for which there is a *deviation*. If such a  $B_k$  no longer exists, we look for a block without any observable *deviation* because a defect in one block does not necessarily imply a *deviation* for this block. If for all sub-functions  $B_i$  within B, no  $C(B_i)$  is empty, we construct the set  $C(B)$ , which is the subset of  $\prod C(B_i)$  made up of assumptions that satisfy L(B), where L(B) is the set of links between the  $B_i$  sub-functions.

If  $C(B)$  is empty, then B is a *conflict*. Since it is not usually minimal, we therefore look for minimal *conflicts* in B. Such *conflicts* obviously do not respect the hierarchy. To find them, we begin by taking the  $B_i$  subfunctions two by two. For each pair  $(B_i, B_k)$ , we construct the set  $C(B_i \cup B_k)$  of assumptions which satisfy the links between  $B_i$  and  $B_k$ . If  $C(B_i \cup B_k)$  is

<sup>8</sup> For higher level functions, models of behavior describing exhaustively all good functioning states are not available. The only knowledge of the nominal behavior simply allows to observe *deviations*. In particular, no assumption is made during the top-down process.

empty, minimal *conflicts* in  $B_j \cup B_k$  are searched for by considering first, for each link  $l$  between  $B_j$  and  $B_k$ , the minimal block of components linked by  $l$ . It should be pointed out that it is rare to have  $C(B_j \cup B_k)$  empty but  $C(B_j)$  and  $C(B_k)$  not empty at the same time. Indeed, if there actually is a defect in  $B_j$ , for example, it means that the observations of measurable parameters on  $B_j$  and on all its components do not make it possible to distinguish the behavior of the faulty block  $B_j$  from a possible correct behavior of  $B_j$ .

### C. Bottom-Up Strategy

It is possible to have  $C(B)$  not empty because:

- a *deviation* for B does not imply a defect in one of its components,

- a defect in B may lead to a behavior of B consistent so far with correct patterns of behavior of its components.

This means there is a possible consistency at a higher level. The process repeats itself by searching for *conflicts* in another block at the same hierarchical level as B and, if no *conflict* has been so far detected, by considering the higher level block which contains B. It guarantees the detection of the smallest *conflicts*, with respect to the functional decomposition of the circuit.

## VII. Conclusion

The expert system DEDALE has been implemented in VM/PROLOG [VM/PROLOG, 1985]. It has 4 components: (1) An object oriented language with which to describe a circuit structurally and functionally; (2) A library of qualitative models for generic components; (3) FOG, a problem solver which performs order of magnitude reasoning; (4) strategic rules.

The expert system DEDALE is now being experimented on real size applications in a factory environment to troubleshoot complex analog circuits. According to the first results, for about 75 % of investigated failures, there are significant change in the behavior of the circuit. In these cases, DEDALE is able to find the defects.

The remaining 25% of failures are not due to faulty components, but rather to components that work to the limits of their designed behavior. In such cases there are *no significant deviations inside* the circuit. Experience has shown that such failures are identified before trying a model-based approach. Specific heuristics can be added to DEDALE to try to handle these cases as well.

These results, coupled with the highlighting of our working hypothesis, are, we hope, a step forward in knowing if and when qualitative reasoning techniques are efficient for real size applications.

## Acknowledgments

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## Appendix

Here are some rules of FOG ([A] stands for the sign of A):

$A \cong A$   
 $A \cong B \rightarrow B \cong A$   
 $A \cong B, B \cong C \rightarrow A \cong C$   
 $A \cong B, [C] = [A] \rightarrow (A + C) \cong (B + C)$   
 $A \sim B \rightarrow B \sim A$   
 $A \sim B, B \sim C \rightarrow A \sim C$   
 $A \sim B \rightarrow [A] = [B]$   
 $A \cong B \rightarrow A \sim B$   
 $A \ll B, B \ll C \rightarrow A \ll C$   
 $A \ll B, B \sim C \rightarrow A \ll C$   
 $A \cong B \rightarrow (A - B) \ll B$   
 $A \ll B \rightarrow (B + A) \cong B$   
 $A \ll B \rightarrow -A \ll B$   
 $A \sim B, [A] \neq 0 \rightarrow \neg(A \ll B)$   
 $A \sim^+ B \leftrightarrow A \sim B, \neg(A \cong B), [A - B] = +$   
 $A \sim^- B \leftrightarrow A \sim B, \neg(A \cong B), [A - B] = -$

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