

Formal Basis for Commonsense Abstraction of Dynamic Systems¹

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Abstract

Abstraction is an essential technique in reasoning about complex systems involving a large number of variables and interconnections. Aggregation of dynamic systems is an abstraction technique whose application is easily observable in everyday life. The basic intuition behind aggregation of variables may be summarized as follows: if variables in a large dynamic system can be partitioned into subsets such that variables in each subset are more strongly connected to each other than to variables in other subsets, one can describe the short-run behavior of each subsystem independently of other subsystems. Furthermore, one can describe the long-run behavior of the entire system in terms of these subsets instead of individual variables, treating each subset as a black box. This paper provides a formal justification for commonsense abstraction based on aggregation of a dynamic system and presents a procedure for doing so.

1. Introduction

Abstracting a detailed description to produce a simpler description is essential in reasoning about a complex system. Aggregation is one such abstraction mechanism whose application is easily observable in everyday life. The general problem is to compute the values of some variables of a complex, dynamic system. People confront such situations on a daily basis and are successful in quickly approximating the desired values with sufficient precision for their tasks. One such technique is to deal with aggregated variables rather than the original variables. An aggregated variable is one whose value depends on the values of some collection of variables. For example, it could be the sum of some variables. If the number of aggregated variables is less than the original number of variables, then it is clearly a computational advantage to re-cast the problem in terms of the aggregated variables. Let us take some concrete examples.

1. A well-known example by Simon of aggregation of variables is about heat flow within a building [Simon 81]. Consider a building divided into a large number of rooms, which are in turn divided into a number of offices by partitions. The building walls provide perfect thermal insulation from the environment. The wall between rooms are good but not perfect insulators while the partitions are poor insulators. In this situation, the temperature

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The building walls provide perfect thermal insulation from the environment. The wall between rooms are good but not perfect insulators while the partitions are poor insulators. In this situation, the temperature equilibrium among offices within one room will be reached very rapidly while equilibrium among rooms will be reached only slowly. Therefore, as long as one is not interested in modeling rapid temperature fluctuations within one room, a useful aggregation will be to have one temperature variable for each room and to assume equilibrium within a room is reached instantaneously.

2. 'Common sense' aggregation is also useful to experts in making quick but reasonably accurate computations. Consider the domain of electrical circuits. Aggregation of variables is used in the simplest of circuit calculations. An example would be to determine the behavior of charge flow between two banks of capacitors connected in parallel. If the banks are connected via a large resistor while the capacitors within a bank are connected in parallel using low resistance lines, a useful approximation would be to determine the expressions of the aggregated charge for a bank and assume instantaneous sharing of this charge within a bank. Hence having calculated the aggregated charge for a bank the approximate expressions for individual charges is easily obtained. Aggregation of variables can also be used in really complex systems. For example, an engineer in charge of a sub-station that supplies electrical power to a city block is more likely to be using an aggregated load in his calculations rather than the average individual loads of the houses in the block.

Aggregation has been discussed by many researchers in Artificial Intelligence. Weld proposes an aggregation procedure by discovering a cycle in the simulated behavior of a model [Weld 86]. Fishwick also detects cyclic behavior for the purpose of aggregation, but he does so not by simulation but by static analysis of data-flow graph of a process [Fishwick 87].

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the behavior description of components into a behavior description of the whole [Bylander 87]. Their approach is to combine behavior descriptions of more components to generate a description of the behavior of the device as a whole by recognizing certain patterns, called *causal patterns*, in combinations of causal steps in component behavior descriptions and aggregating the steps into one abstract step.

All these techniques are concerned with aggregating behavior consisting of a sequence of discrete steps. In this paper, we discuss a rather different aggregation technique for aggregating a behavior model represented, not in terms of explicit causal steps, but in terms of functional relations among variables.

Simon and Ando provided a formal basis for aggregation, namely aggregation of variables [Simon and Ando 61]. They proved that the above intuition was indeed true for the case of a nearly decomposable dynamic matrix with one significant characteristic root for each subsystem. Courtois [Courtois 77] specifies an aggregation procedure along with an error-of-approximation analysis for the special case of stochastic matrices that satisfy the Simon-Ando requirements. In this paper we extend the work by Simon, Ando, and Courtois by presenting an aggregation procedure for more general, non-stochastic dynamic systems that satisfy the Simon-Ando requirements. Though the procedure and example presented in this paper are numerical, the concepts of aggregation applies to qualitative models, also. The aggregation technique presented here provides justifications and suggests procedures for qualitative abstractions.

This paper is organized as follows. Section 2 presents the concept of near decomposability of dynamic systems and its implication for their behaviors. Substantial background is presented to make this paper self-contained. Section 3, then, discusses construction of an aggregate system from a nearly decomposable system such that the behavior of the aggregate system is a reasonable approximation to the long-term behavior of the original system. Finally we discuss the implications of this work for common sense aggregation in Artificial Intelligence.

2. Decomposable and nearly decomposable systems

This section formally introduces the concept of a nearly decomposable system and describes the theorems proven by Simon and Ando about the behavior of such a system [Simon and Ando 61].

Let M^* be a self-contained dynamic system of n equations and P^* be the matrix of coefficients in M^* . Thus, M^* consists of equations of the form;

$$x'_i = a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n,$$

and P^* is a matrix of the form;

$$P^* = \begin{array}{cccc|c} a_{11} & a_{12} & \dots & a_{1n} & | \\ | & \dots & & & | \\ | & \dots & & & | \\ | & \dots & & & | \\ a_{n1} & a_{n2} & \dots & a_{nn} & | \end{array}$$

where a_{ij} is the coefficient of x_j in the i th equation of M .

P^* is called *completely decomposable* if, by simple rearrangement of rows and columns, P^* can be put in a block diagonal form as

$$P^* = \begin{array}{ccc|ccc} | & P_1^* & & & & | \\ | & & P_2^* & & & | \\ | & & & & & | \\ | & & & & & | \\ | & & & & & | \\ & & & & & P_N^* | \end{array}$$

where P_j^* s are square submatrices, N is the number of such submatrices in P^* , and all the elements of P^* not in any of the submatrices are zero.

If the matrix of coefficients is completely decomposable, the dynamic system consists of independent components which do not interact at all and which behave independently of each other. The submatrices represent the components.

Now, consider a slightly different, self-contained dynamic system M and its matrix P of the same size as M^* and P^* , such that P has the same diagonal submatrices as P^* but the elements outside of the submatrices are either zero or very small, the magnitudes being less than ϵ for some given ϵ . P looks like,

$$P = \begin{array}{ccc|ccc} | & P_1^* & & & & | \\ | & & P_2^* & & & | \\ | & & & & & | \\ | & & & & & | \\ | & & & & & | \\ & & & & & P_N^* | \end{array}$$

where the elements of P outside of the submatrices are either ϵ or zero. Then P can be expressed as

$$P = P^* + \epsilon C,$$

where C is an arbitrary $n \times n$ matrix. A matrix such as P that can be put in this form is called a *nearly completely decomposable matrix* or a *nearly decomposable matrix*.

The system M whose matrix P is nearly completely decomposable consists of components such that variables within each component interact strongly, but variables from different component interact relatively weakly. The submatrices represent such components and the ϵ elements outside the submatrices represent weak links among components.

2.1. Behavior of a Nearly Decomposable System

For a dynamic system to be dynamically stable all its eigenvalues must be negative. We will assume that this is in fact the case for all systems we will discuss in this chapter. Furthermore, we assume that all the roots of the system are distinct. When all the roots are distinct, the time paths of the variables can be expressed as

$$x_i(t) = \sum_{j=1}^n z_{ij} c_j \exp(\lambda_j t), \quad (1)$$

where $\lambda_1, \dots, \lambda_n$ are the characteristic roots, and (z_{1j}, \dots, z_{nj}) is the eigenvector corresponding to the j th root.

If all the roots $\lambda_1, \dots, \lambda_n$ are arranged in ascending order of the absolute values of their real parts, the contribution of the roots toward the end of the list to the dynamic behavior of the system will be damped rapidly, and will be of importance only to the short-term behavior of the system. For the long-term behavior, we can ignore these roots and treat the system as having a smaller number of degrees of freedom, corresponding to the number of roots we retain.

Let M be a dynamic structure with the corresponding matrix P that is nearly decomposable with N square submatrices. Let m_I be the size of the I th submatrix, and x_{1_I}, \dots, x_{m_I} be the variables belonging to the I th submatrix. Let $\lambda_{1_I}, \dots, \lambda_{m_I}$ be the characteristic roots belonging to the I th submatrix. Without loss of generality, we will assume that the roots in each subsystem are arranged in ascending ordering of the absolute values of their real parts. Therefore,

$$|Re(\lambda_{1_I})| < |Re(\lambda_{2_I})| < \dots < |Re(\lambda_{m_I})|$$

for $I = 1$ to N .

The time path of each variable in the system can be expressed as follows:

$$x_{h_K}(t) = \sum_{I=1}^N \sum_{j=1}^{m_I} z_{jI} h_K \exp(\lambda_{jI} t) \quad (2)$$

Alternatively, the time path of the entire system can be expressed as

$$x(t) = Z \times \Lambda(t),$$

where $x(t)$ is the the column vector of the variables,

$$x(t)^T = \{x(t)_{1_1}, x(t)_{2_1}, \dots, x(t)_{m_1}, x(t)_{1_2}, \dots, x(t)_{m_2}, \dots, x(t)_{1_N}, \dots, x(t)_{m_N}\}$$

and Z is the matrix whose columns are eigenvectors corresponding to the eigenvalues, $\lambda_{1_1}, \dots, \lambda_{m_N}$ and Λ is the column vector of the exponential terms as

$$\Lambda(t)^T = \{ \exp(\lambda_{1_1} t), \exp(\lambda_{2_1} t), \dots, \exp(\lambda_{m_1} t), \exp(\lambda_{1_2} t), \dots, \exp(\lambda_{m_2} t), \dots, \exp(\lambda_{1_N} t), \dots, \exp(\lambda_{m_N} t) \}$$

Since P is nearly decomposable, the eigenvector z_{jI} corresponding to the eigenvalue λ_{jI} ($I = 1$ to N and $j = 1$ to m_I) is such that its elements z_{kIjI} are very small for $L \neq I$. In the above expression for $x(t)_{h_K}$, these small $z_{jI} h_K$'s represent the effects of the variables outside the K th subsystem on $x(t)_{h_K}$.

Simon and Ando show that the behavior of such a system may be approximately described in the following four stages [Simon and Ando 61]:

1. short-run dynamics

Variables in each subsystem are moving towards their relative equilibrium independently of other

subsystems.

2. short-run equilibrium

The most significant root of each subsystem dominates the behavior of the subsystem.

3. long-run dynamics

The variables in each subsystem move together towards over-all equilibrium while maintaining relative equilibrium in each subsystem.

4. long-run equilibrium

Finally, the most significant root of the entire system dominates.

When the behavior of a large system is approximately described in four stages as above, the goodness of the approximation naturally depends on how small the ϵ 's are and also how dominant the most significant root of each subsystem is compared to the rest of the roots.

3. Aggregation of Variables

We describe the procedure for producing an aggregated matrix from a nearly decomposable matrix. In the description below we often drop the argument (t) of variables that are functions of time to improve readability.

3.1. Procedure for aggregation

M is the nearly decomposable system defined in 2.1 with N submatrices and M^* is the corresponding completely decomposable system. M and M^* consist of n variables and equations. Each subsystem M_i and M^*_i ($i = 1$ to N) consists of m_i variables x_{1_i}, \dots, x_{m_i} . M_i also consists of equations of the following form;

$$x'_{j_i} = f_{j_i}(x_{1_i}, x_{2_i}, \dots, x_{m_i}, x_{1_2}, \dots, x_{m_2}, \dots, x_{1_N}, \dots, x_{m_N})$$

for $j = 1$ to m_i , (3)

and M^*_i of the following form;

$$x'_{j_i} = f^*_{j_i}(x_{1_i}, x_{2_i}, \dots, x_{m_i})$$

for $j = 1$ to m_i , (4)

where f_{j_i} 's and $f^*_{j_i}$'s are linear functions of their arguments.

Let λ_i be the most significant eigenvalue of M_i and let λ_i^* be the corresponding eigenvalue of M^*_i . Let $z^*_i = \{z^*_{1_i}, z^*_{2_i}, \dots, z^*_{m_i}\}$ be the eigenvector corresponding to λ_i^* . Simon and Ando show that

$$x_{j_i}/x_{k_i} \approx z^*_{j_i}/z^*_{k_i} \quad \text{for } j = 1 \text{ to } m_i. \quad (5)$$

We define the aggregate variable y_i and another useful variable Z_i for each subsystem M_i as follows:

$$y_i = \sum_{j=1}^{m_i} x_{j_i} \quad (6)$$

$$Z_i = \sum_{j=1}^{m_i} z^*_{j_i} \quad (7)$$

It follows from equation (5) that

$$x_j/y_j \approx z^*_{j_i} / Z^*_{j_i} \text{ for } j = 1 \text{ to } m_i. \quad (8)$$

To prove this, it is sufficient to cross multiply and then approximately equate terms on both sides of the equation using the relations in (5).

To derive the aggregate matrix we need expressions for all y'_j in terms of the aggregate variables. Observe that it follows from the definition of the aggregate variables in (6) that

$$y'_i = \sum_{j=1}^{m_i} x'_j. \quad (9)$$

It is therefore possible to compute the aggregate matrix by doing the following for every subsystem M_i .

1. Add the m_i equations in (3). Note that the left-hand-side of the resultant equation is nothing but y'_i while the right-hand-side looks like the right-hand-side of (3).
2. Using the relations in (8) replace all x_j in the right-hand-side by $z^*_{j_k} / Z^*_{j_k}$ for $j=1$ to m_k and for $k=1$ to N .

The aggregate system consists of the variables y_1 to y_N and the N differential equations thus generated.

3.2. Example of aggregation

We will give an example of model aggregation. Consider an environment where four species, a , b , c , and d , of organisms live. Assume that available resources and living space are fixed and limited, that the environment is isolated, and that there is no new resources added to the environment. The life of the four species of organisms are coupled in the following manner: a mainly preys on b but occasionally preys on c ; c mainly preys on d but also preys on b occasionally; also b preys on d and d preys on a very infrequently. Let M be a nearly decomposable dynamic system with four variables x_1 , x_2 , x_3 , and x_4 representing the populations of the four species, a , b , c , and d respectively. Suppose that the following relations hold among the variables;

$$x'_1 = -50.000x_1 + 23.000x_2 + (1.0000e-3)x_3 \quad (10)$$

$$x'_2 = -1.0000x_1 - 0.10000x_2 + (2.0000e-3)x_3 \quad (11)$$

$$x'_3 = (1.0000e-3)x_2 - 47.000x_3 + 17.000x_4 \quad (12)$$

$$x'_4 = (3.0000e-3)x_1 - 3.0000x_3 - 0.90000x_4 \quad (13)$$

Let P be the following matrix of coefficients for the structure M .

-50.000	23.000	1.0000e-03	0.0000
-1.0000	-0.10000	0.0000	2.0000e-03
0.0000	1.0000e-03	-47.000	17.000
3.0000e-03	0.0000	-3.0000	-0.90000

P is nearly decomposable and has the following two 2×2 submatrices, P^*_1 and P^*_2 .

P^*_1	=	-50.000	23.000
		-1.0000	-0.10000
P^*_2	=	-47.000	17.000
		-3.0000	-0.90000

The eigenvalues of P associated with each submatrix are as follows:

subsystem	eigenvalues
P_1	$\lambda_{1_1} = -0.56526$ $\lambda_{2_1} = -49.535$
P_2	$\lambda_{1_2} = -2.6170$ $\lambda_{2_2} = -47.383$

The eigenvectors of P^*_1 and P^*_2 corresponding to the eigenvalues λ_{1_1} and λ_{1_2} are as follows:

$$z_{1_1}^* = (0.46526, 1)$$

$$z_{1_2}^* = (0.38303, 1)$$

We will let variables $y_1(t)$ and $y_2(t)$ to be the aggregate variables for submatrices M_1 and M_2 defined as follows:

$$y_1 = x_{1_1} + x_{2_1}$$

$$y_2 = x_{1_2} + x_{2_2}$$

Differentiating both sides of the above two equations with respect to time yields;

$$y'_1 = x'_{1_1} + x'_{2_1} \quad (14)$$

$$y'_2 = x'_{1_2} + x'_{2_2} \quad (15)$$

y_1 and y_2 have the following approximate, linear relations to the variables of the submatrices:

$$x_1 \approx \frac{0.46526}{1+0.46526} y_1 = 0.317527 y_1$$

$$x_2 \approx \frac{1}{1+0.46526} y_1 = 0.682473 y_1$$

$$x_{1_2} \approx \frac{0.38303}{1+0.38303} y_2 = 0.2769494 y_2$$

$$x_{2_2} \approx \frac{1}{1+0.38303} y_2 = 0.723051 y_2$$

Substituting the derivatives on the right-hand-side of (14) and (15) by the expressions on the right-hand-side of equations (10) through (13), and further substituting the occurrences of the original variables by their approximations in terms of y_i 's given above yields the following aggregate systems;

$$y'_1 = -0.565278 y_1 + 1.723047 e-3 y_2 \quad (16)$$

$$y'_2 = 1.63505-3 y_1 - 2.616978 e-3 y_2. \quad (17)$$

The variables y_1 and y_2 and equations (16) and (17) constitute the aggregate structure M' . The timepaths of the two eigenvectors are as follows:

$$y_1 = c_1 \exp(-0.565277t) + c_2 0.839814e-03 \exp(-2.61698t) \quad (18)$$

$$y_2 = c_1(-0.79692e-03) \exp(-0.565277t) + c_2 \exp(-2.61698t) \quad (19)$$

c_1 and c_2 are such constants that the relations (6) are satisfied at $t = 0$. Given the values the aggregate variables, the values of the original variables can be estimated by equation (8).

3.3. Conditions for Aggregation

When the behavior of a nearly decomposable structure is approximated by an aggregate structure, the goodness of approximation will depend on the smallness of the matrix elements outside the submatrices and also on the degree of the dominance of the most significant root of each submatrix over other roots. This second condition can be stated more precisely as follows: For a nearly decomposable dynamic matrix to be aggregated as described in this section, the matrix must satisfy the following conditions;

1. There is one root in each submatrix such that the absolute value of its real part is smaller than any others in the submatrix.
2. The absolute value of the real part of the primary root of any subsystem must be smaller than that of any non-primary root of any subsystem. In other words,

$$\begin{aligned} |Re(\lambda_{1I})| &< |Re(\lambda_{jJ})| \\ \text{for } I &= 1 \text{ to } N, j = 2 \text{ to } m, \\ \text{and } J &= 1 \text{ to } N. \end{aligned}$$

The necessity of the condition (1) above is obvious if the movement of each subsystem is to be represented by one aggregate variable. The second condition is given by Courtois [Courtois 77]. Courtois showed that the second condition is necessary for the case of stochastic matrices, but that it is also necessary for more general cases can be easily seen as follows: Given a dynamic structure, equation (2) expresses the time path of each variable. Without loss of generality, assume that all the characteristic roots λ_i 's ($i = 1$ to m_j) associated with each submatrix P_j ($j = 1$ to N) are arranged in increasing order of the magnitude of their real parts. Therefore, for each submatrix P_j , λ_{1j} is the most significant root.

Then, equation (2) can be expanded as follows:

$$x_{h_k}(t) = S_1 + S_2 \quad (20)$$

where

$$S_1 = \sum_{l=1}^N z_{jlh_k} \exp(\lambda_{1l}t)$$

and

$$S_2 = \sum_{l=1}^N \sum_{k=2}^{m_l} z_{jlh_k} \exp(\lambda_{jl}t)$$

In aggregating a matrix, one discards S_2 in (20). Therefore, if aggregation is to produce a reasonable approximation, the exponential terms in the discarded term, S_2 , must diminish before the exponential terms in S_1 . In other words, the magnitudes of the real parts of the eigenvalues involved in S_1 must be smaller than those in S_2 .

3.4. Subsystems with multiple significant roots

The two conditions discussed above for aggregation makes the applicability of the aggregation procedure somewhat limited. However, the concept of nearly decomposable systems and the discussion of behavior of such systems in Section 2 are more generally applicable, and the aggregation procedure can be generalized to cases where the conditions are not necessarily satisfied; in particular to cases where a submatrix have any number of non-negligible roots.

Let $\lambda_{1l}, \dots, \lambda_{m_l}$ be the eigenvalues associated with the l th submatrix. Without loss of generality, we will assume that these eigenvalues are arranged in ascending order of the absolute values of their real parts. For some given threshold value $\lambda_0 > 0$, we partition this set of eigenvalues for the l th submatrix into two subsets, one containing those the absolute values of whose real parts are less than or equal to λ_0 and the other containing the rest. Let s_l be the number of the eigenvalues belonging to the first subset. We will refer to the eigenvalues in the first subset as *significant roots* and those in the second subset as *insignificant roots*. Therefore, the set of the significant roots is

$$\{\lambda_{1l}, \lambda_{2l}, \dots, \lambda_{s_l}\}, \quad \text{where } |Re(\lambda_{il})| \leq \lambda_0,$$

and the set of insignificant roots is

$$\{\lambda_{1+s_l}, \lambda_{2+s_l}, \dots, \lambda_{m_l}\},$$

where $|Re(\lambda_{il})| > \lambda_0$.

After the contributions of the insignificant roots vanish, the long-run behavior of each subsystem M_l can be described with s_l ($s_l \leq m_l$) significant roots. Therefore, one can define for each subsystem s_l aggregate variables such that the values of the original variables $x_{1l}, x_{2l}, \dots, x_{m_l}$ after some time can be approximated as linear combinations of the aggregate variables. Then, M can be rewritten in terms of these aggregate variables to produce an aggregate system. The aggregate system will describe the long-term behavior and will have fewer degrees of freedom than the original one.

4. Discussion

This paper focuses on devising a formal model, namely aggregation of variables, for common sense abstraction. This work builds on existing work on nearly decomposable dynamic systems. An aggregation procedure aggregates a dynamic structure by defining one aggregate variable for each subsystem based on its most significant roots and rewriting the entire structure in terms of these aggregate variables. For

this procedure to be applicable, the most significant root of each subsystem must be such that the magnitude of its real part is smaller than any insignificant root of any subsystem. When a dynamic system is aggregated in this manner, each subsystem can be represented by one aggregate variables. We generalized this procedure to make it applicable to cases where each subsystem has more than one non-negligible roots. In such cases, as many aggregate variables as there are significant roots (with respect to some threshold value) in each subsystem are defined for a subsystem. Courtois performed in-depth analysis of approximation error in aggregation for stochastic systems [Courtois 77]. Similar analysis or approximation error for the general aggregation procedure presented in this paper will be useful and represents immediate future work for this project.

Note that the aggregate variables are defined as sums of the variables in a subsystem. As the formal model treated here covers quite a large class of systems we think that sums or simple linear functions of strongly connected variables will comprise a significant percentage of common sense aggregation examples. The examples in Section 1 seem to support this point.

Though the procedure and example presented in this paper are numerical, the relevance of this work is not limited to cases where numerical information of functional relations among variables is available. Even when only a qualitative model exists, model aggregation is possible and is often performed based on such qualitative knowledge as relative strengths of interactions among variables and groups of variables and relative speeds at which groups of variables reach equilibrium through workings of causal mechanisms in the system. The work presented here provides justifications and suggests procedures for performing such qualitative aggregation. One of the authors discusses related qualitative aggregation techniques and their relations to the notion of causality in a separate document [Iwasaki 88].

Kuipers uses abstraction by time-scale in order to control the exponential growth of the number of possible courses of behavior in qualitative simulation [Kuipers 87]. Kuipers has a hierarchy of constraint networks of very fast to very slow mechanisms. When simulating a fast mechanism, variables controlled by slower mechanisms are considered constant, and when simulating a slow mechanism, equilibrium among variables controlled by faster mechanisms is considered to be reached instantaneously. This idea of abstraction by time-scale is very similar to the notion of abstraction discussed in this thesis. However, Kuipers does not explore the issue of generating such a hierarchy of models from one original model. The aggregation technique discussed in this paper can be used to generate a hierarchy of models of different time-scales.

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