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Logic morphisms as a framework for backward transfer of lemmas and strategies in some modal and epistemic logics

Ricardo Caferra, Stéphane Demri, Michel Herment

LIFIA-IMAG,
46, Av. Félix Viallet,
38031 Grenoble Cedex, FRANCE
{caferra | stephane | herment}@lifia.imag.fr (uucp)

Abstract

There exist methods in automated theorem proving for non-classical logics based on translation of logics from a (non-classical) source logic (abbreviated henceforth SL) into a (classical) target logic (abbreviated henceforth TL). These valuable methods do not address the important practical problem of *presenting proofs in SL*. We propose a framework applicable at least to S4(p), K, T, K4 for presenting proofs of theorems of these logics found in a familiar TL: Order-Sorted Predicate Logic (abbreviated henceforth OSPL). The method backward translates lemmas in a deduction (in TL) either (a) into lemmas in a corresponding deduction in SL (in the best case), or (b) into formulas semantically related to lemmas in a corresponding deduction (in the worst case). As a natural consequence we bring to the fore the fact that this framework can also be used to help in solving another important and very difficult problem: *the transfer of strategies from one logic to another*. One conjecture -with corresponding theorem which is a particular case of it- is stated. When (b) above holds we give sufficient (and in general satisfactory) conditions in order to obtain the lemmas in SL. Two examples are treated in full detail : the well known problem of the "wise man puzzle" and another one which shows how our method can be used to transfer strategies. No additional theoretical result is given in this direction, but it is clear from the example how the proposed framework can help to transfer strategies.

1. Introduction

Relating logics is a technique that has been used in automated deduction for non classical logics:

. By E. Orlowska, who introduced the notion of *resolution-interpretability of a logic in another logic* and applied it in order to construct theorem proving systems for algorithmic and m-valued Post logics (Orlowska 79, 80).

. More recently by A. Hertzig and H-J. Ohlbach who emphasize the idea of *logic morphism* which is implicitly used in the work of E. Orlowska (Herzig 89, Ohlbach 89).

The works of Ohlbach and Hertzig, in which unification plays a central role, were applied to several classes of modal and temporal logics. In these works the notions of *source logic (SL)* and *target logic (TL)* can be identified. SL is a logic to which we want to transfer results, in which we want to prove theorems and for which we do not have a good theorem prover or a theorem prover at all,...TL is a logic for which we know a lot of results, for which we have good theorem provers with good complete strategies,...

In automated deduction for non-classical logics we can clearly, either use existing methods for these logics: tableau-based (see for ex. (Fitting 83)) or resolution-based (see for ex. (Enjalbert & Fariñas del Cerro 1989)), or use translation methods (i.e. based on logic morphism). Our work is set in the latter context.

A common feature of all works centered on logic morphisms is that they do not worry about a very important practical problem: *presenting proofs in the source logic*. In a fully general approach this amounts to translating proofs between (arbitrary) logics and, obviously, we shall not attack this problem. Instead, we shall deal with the problem of translating (for certain logics) some formulas of a proof found in the TL into formulas in a corresponding proof in the SL.

We shall identify conditions allowing backward translation of lemmas of proofs from a particular TL: the Order-Sorted Predicate Logic (OSPL) into some differents (non classical) SL: S4(p), K, T, K4. OSPL is a "good" TL because very good theorem provers with good complete strategies are available for it. In order to expound our work we shall limit ourselves in this paper to the multi-modal propositional logic S4(p) but it will distinctly appear that similar results can be straightforwardly obtained for K, T and K4.

A nice "by-product" of our results about transfer of lemmas is their usefulness as a *framework for studying strategies transfer*. It is well known that finding good strategies (and if it is possible to prove their completeness) is one of the central problems in automated deduction. If a strategy has been used to obtain a proof in TL, the steps of the proof can be considered as "keeping trace" of the strategy. Therefore a framework for studying the transfer of lemmas is also a framework for studying the transfer of strategies from TL to SL.

The structure of the paper is the following: in section 2 we present the basic definitions concerning logic

morphisms, a recall of S4(p) (S4 with several agents) and the definition of a partial inverse morphism. Section 3 contains the definition of a particular partial inverse morphism between OSPL and S4(p). Section 4 contains the main results of this paper: we state one conjecture and prove one related theorem. A well kown example ("the wise man puzzle") is treated in detail. Another example shows the possibilities of using our approach as a framework for translating strategies. We have chosen the latter example in order to compare our method with a completely different one. Recently (Auffray, Enjalbert, & Hebrard 1990) proved that the unit strategy is complete also for some modal logics. In applying our backward lemma translation we obtain a similar result as the latter one. Presentation of the examples in section 4 is done with our proof-editor system (Caferra, Demri, & Herment 91). Section 5 gives some ideas of future work.

2. Basic definitions

We use the notion of logic morphism from (Ohlbach 89) and we add the notion of *partial inverse morphism* corresponding to the backward lemma transfer from TL to SL. For the sake of simplicity we shall consider, both in TL and SL only formulas in clausal form. We shall not formally define the notions of logics, specification morphisms and logic morphisms here but the main ideas of the approach in (Ohlbach 89) can be summarized as follows.This approach is based on the *proviso* that : there exist i) a (good) theorem prover for a target logic and ii) a standard translation from a *context logic* (i.e. an intermediate logic, abbreviated henceforth CL) to a target logic. The task of writing theorem provers for new logics is therefore changed in the task of finding the way of translating these new logics to CL.

"Translating" means here translating the syntax and the semantics. Ohlbach uses "morphism" to name this translation. We shall keep this name in this paper. In some standard cases the semantics of SL is syntactically captured by terms of the context logic *via* an axiomatization of the properties of semantic structures. Moreover, CL distinguishes in its syntax, the context terms which capture the semantics of SL (for instance, "$\Downarrow$" denotes the application function, "o" the composition function and "O_C" the initial world) from the domain terms. The first-order logic, CL, S4(p) and OSPL can be redefined with this formalism. Our definition of a partial inverse morphism fits in this theoretical framework. The following definition uses the notations from (Ohlbach 89).

Definition 1

A *partial inverse mophism* φ for the logic morphism Ψ between the two logics L_1 and L_2 is a mapping such that:

For $\sigma \in \Sigma_1$ (set of signatures for L_1), $\varphi(\sigma)$ is a partial function between φ_2 ($\Psi_\Sigma(\sigma)$) and φ_1 (σ) (φ_1 and φ_2 map a signature to a set of formulas). In the sequel we shall note $\varphi(F)$ the application of $\varphi(\sigma)$ to F (if it exists)♦

We illustrate the use of morphisms and partial inverse morphisms between logics by taking S4(p) and SL and OSPL as TL. The logic S4(p) is widely considered as the basic model for epistemic logics. In a world with p agents the intended interpretation of $\Box_i A$ is "the agent i knows that A is true". The syntax and semantics for S4(p) and its associated resolution calculus are those used in (Enjalbert & Fariñas del Cerro 89). In the sequel we shall use a slight modification of the logic morphism defined in (Ohlbach 89): each agent generates its own context sorts. We associate to OSPL the resolution method defined in (Schmidt-Schauß 88).

3. Morphism and partial inverse morphism

The morphism we shall use is a slight modification of the one between S4(p) and CL defined in (Ohlbach 89): the context sorts are indexed by the different agents. For example the modal part of the formula morphism is:

$$\Psi_F(\Box_\iota f) = \forall\ x\ \text{'W->}^{rt}\text{W'}_\iota\ \Psi_F(f),$$

$$\Psi_F(\Diamond_\iota f) = \exists\ x\ \text{'W->}^{rt}\text{W'}_\iota\ \Psi_F(f).$$

Let Σ be a set of clauses in S4(p) and let Σ' be the translation of those clauses into OSPL. Let Φ' a clause provable from Σ' and let Φ be the possible inverse translation of Φ' back into S4(p). We will define the conditions to backward translate Φ' and we will state, as a conjecture, that Φ is always provable from Σ. We can now define the partial inverse morphism.

As proofs in OSPL are sequences of clauses, each lemma is a clause. Therefore it suffices to define backward translation for clauses .

To translate a clause, we capture the underlying semantics of the transition from one world to another by introducing a modal operator. The principle of the backward translation consists in gathering the literals which have some context in common. Each literal L has the form sP(pw(C)) where $s \in \{\Lambda, \neg\}$ (as usual Λ denotes the empty string), P is a predicate symbol not appearing in the axioms, pw is a projection operator introduced by the morphism, and C is a context term. First of all, we algorithmically define auxiliary functions which deal with syntactic transformations.

* generate-op(t) := **if** t is a variable of sort 'W->^{rt}W'$_\iota$ **then** $\Box_\iota$ **else** $\Diamond_\iota$ %t is a context term %

* local-context ((t_1 ... t_n)) := **if** n=1 **then** generate-op(t_1) **else** generate-op(t_1) . local-context((t_2, ..., t_n)) %'.' is the concatenation operator, the t_i are context terms %

* $\beta(\Downarrow(x_1 \circ \ldots \circ x_n, O_C), m)$:= **if** m = n **then** O_C **else** $\Downarrow(x_{m+1} \circ \ldots \circ x_n, O_C)$ % $0 \le m \le n$%

This last function takes into account the fact that each context term denoting a *path-world* is equivalent to O_C or to a term of the form $\Downarrow(t_1 \circ \ldots \circ t_n, O_C)$.

* $\varphi_1(L)$:=

% L = sP(pw(C)) and C is a context term %

if C = O_C **then** P

else generate-op(t_1) . φ_1(sP(pw(β(C,1))))

Definition of φ

Let $\mathcal{C}$ be a clause $L_1 \vee \ldots \vee L_n$. We define a set of classes of literals $\{C_i, 1 \le i \le k\}$ ($k \le n$) which is a partition of the set of literals of $\mathcal{C}$. Moreover the classes have the following properties :

For $1 \le j \le k$, $\exists\, \alpha_j$ such that

% $L_i = s_i.P_{Li}(pw(\Downarrow(t^{Li}_1 \circ ... \circ t^{Li}_{ni}, O_C)))$ or $s_i.P_{Li}(pw(O_C))$ %

$\forall\, P,Q \in Cj$, $\forall\, i, 1 \le i \le \alpha_j\ t^P_i = t^Q_i$, and $t^P_{\alpha j+1} \ne t^Q_{\alpha j+1}$ if both $t^P_{\alpha j+1}$ and $t^Q_{\alpha j+1}$ exist

$\forall\, P \in Cj\ \forall Q \notin Cj$, $\forall\, i, 1 \le i \le \alpha_j\ t^P_i \ne t^Q_i$

If $\alpha_j = -1$ then $\forall\, L \in Cj$ L has the following form $sP(pw(O_C))$.

It is easy to verify that the decomposition in classes is unique. The definition of $\varphi(\mathcal{C})$ will use the function α which takes as argument a set of literals and which returns a modal formula.

Calculus of $\alpha(C)$ for the set of literals $\{l_1,...,l_p\}$

$\alpha(C)$:= Case : % $l_i = s_iP_i(pw(C_i))$ %

$p = 1$; $\varphi_1(L_1)$

$\alpha_C = -1$; $s_1P_1 \vee ... \vee s_pP_p$ % α_C is the α of the class C%

Otherwise : $l = (t^{l1}_1, ..., t^{l1}_{\alpha C})$

$L_1 = \{lit \mid \exists\, s.p(pw(c)) \in C$ and lit =s. $p(pw(\beta(c,\alpha_C)))$ and $\alpha_C - n_p \ne 0\}$;

$L_2 = \{lit \mid \exists\, s.p(pw(c)) \in C$ and lit =s. $p(pw(O_C))$ and $\alpha_C - n_p = 0\}$;

local-context(l). $\varphi(\vee\, lit)$

$lit \in L_1 \cup L_2$

We can now define $\varphi(\mathcal{C})$: $\varphi(\mathcal{C}) = \alpha(C_1) \vee ... \vee \alpha(C_k)$.

Example of a backward translation

$L = P(pw(\Downarrow(x \circ z, O_C))) \vee Q(pw(\Downarrow(x \circ y, O_C)))$

$\varphi(L) = \Box_n\, \varphi(P(pw(\Downarrow(z, O_C))) \vee Q(pw(\Downarrow(y, O_C)))) = \Box_n(\Box_n P \vee \Box_n Q)$ where the sort of x,y,z is '$W \to^{rt} W'_n$ .We shall abbreviate $pw(\Downarrow(t_1 \circ ... \circ t_n, O_C))$ by $[t_1,...,t_n]$.

4. Properties of the backward translation

Given a formula in SL and a proof of its translation in TL it seems natural to expect from a backward translated lemma to belong to the proof in SL, or at least to be in some relation with formulas in the SL proof. This relation precisely stated by the following conjecture. We did not find any counterexample for it, but we can establish as a theorem only a weaker result (see the Theorem 1 below).

Conjecture

i) Let f be a S4(p)-formula and $C_1,..., C_N$ be the S4(p)-clauses such that $\models_{S4(p)} C_1 \wedge ... \wedge C_N \Leftrightarrow f$.

ii) Let Ψ be the logic morphism between S4(p) and OSPL (the one describe in section 2). $\{A_1,...,A_k\}$ is the set of axioms introduced by the specification morphism.

iii) We note for $1 \le i \le N$ $\Psi_F(C_i) = K_i$. Let $L^1_i,...,L^{ni}_i$ be OSPL-clauses such that $L^1_i \wedge ... \wedge L^{ni}_i$ is satisfiable iff K_i is satisfiable.

iv) Let $P=\{s_1,...,s_n\}$ be a deduction using the resolution and paramodulation rules from the set of OSPL-clauses $\{L^j_i \mid 1 \le i \le N$ and $1 \le j \le n_i\} \cup \{A_1,...,A_k\}$.

Then there exists a proof $P' = \{s'_1,...,s'_m\}$ in S4(p) (using the resolution system defined in (Enjalbert & Fariñas del Cerro 89)) from $\{C_1,..., C_N\}$ such that:

(*) if each predicate symbol that occurs in s_j $(1 \le j \le n)$ corresponds to one symbol in f

then there exists k $(1 \le k \le m)$ such that

$s'_k \models_{S4(p)} \varphi(s_j)$ and i f $s_j \ne \Box$OSPL (OSPL empty clause) then $s'_k \ne \Box_{S4(p)}$ (S4(p) empty clause) ■

The theorem 2 below gives conditions allowing to obtain s'_k from $\varphi(s_j)$. Initially we propose a theorem which particularizes the conjecture to a special kind of S4(p)-clauses. We need the following definition.

Definition 2

Let O_i $(1 \le i \le n)$ be multi-modal operators.

A *simple formula* F is a propositional multi-modal formula iff $F = O_1 ... O_n L$ $n \ge 0$, where L is a propositional litteral (i.e. of the form sP, $s \in \{\wedge, \neg\}$) ♦

Theorem 1

- i) and ii) as in the conjecture, moreover we assume that each C_i has the following form : $S_1 \vee ... \vee S_n$ where each S_i is a simple formula.
- For $1 \le i \le N$ $\Psi_F(C_i) = C'_i$ and in this case C'_i is an OSPL-clause.
- Let $P=\{s_1,...,s_n\}$ be a deduction using the resolution rule from the following set of OSPL-clauses $\{C'_1,..., C'_N\} \cup \{A_1,...,A_k\}$.
- Then there exists a proof $P'=\{s'_1,...,s'_m\}$ in S4(p) from $\{C_1,...,C_N\}$ such that (*) (in the Conjecture) holds.

Proof (*sketch*)

To build P' verifying the conditions stated in the theorem, we can show that for this particular case n=m and for $1 \le i \le n$ $\varphi(s_i) = s'_i$. The proof is by induction on the number of steps of P.

• *Base case*

If L is an initial clause (in TL) and if $\varphi(L)$ exists then $\varphi(L)$ is also an initial clause (in SL)

• *Induction step*

If L is obtained by resolution rule from S_u and S_v then $\varphi(L)$ can be obtained from $\varphi(S_u)$ and $\varphi(S_v)$ with the modal resolution. To do so, we distinguish three cases according to the nature of the unifier ■

The corollary below gives a simple syntactic criterion to test whether $s'_k \models_{S4(p)} \varphi(s_j)$ is verified. It offers a satisfactory solution in a lot of cases. We first define a set of syntactic transformation rules .

Definition 3

According to the definition of clauses found in (Enjalbert & Fariñas del Cerro 89) (and usually adopted in the modal logic literature) let $\{A_1,..., A_n\}$ be a set of disjunctive S4(p)-formulas, $\{B_1,...,B_m\}$ be a set of conjunctive S4(p)-formulas, and $\{F_0,...,F_s\}$ be a set of S4(p)-formulas. We note $u \to v$ the substitution of u by v (see the simplification rules in (Abadi & Manna 86)). Here is a *set of rules :*

(R_1) $\Diamond(A_1 \wedge ... \wedge A_n) \to \Diamond A_1 \wedge ... \wedge \Diamond A_n$

(R_2) $\Box(B_1 \wedge ... \wedge B_m) \to \Box B_1 \wedge ... \wedge \Box B_m$

(R_3) $\Diamond(F_1 \vee ... \vee F_s) \to \Diamond F_1 \vee ... \vee \Diamond F_s$

(D_1) $F_0 \vee (F_1 \wedge ... \wedge F_s) \to (F_0 \vee F_1) \wedge ... \wedge (F_0 \vee F_s)$ ♦

• Transformation S (applied to a clause C):
Step 1: To modify C by applying R_1 and R_2 as long as possible with a higher priority to the deepest subformulae of C.
Step 2: Apply D_1 to C with the same restrictions. If the resulting formula is equal to C then go to the step 3) else go to the step 1).
Step 3: The result of S is C, and C has the following form $C = C_1 \wedge \ldots \wedge C_n$ where each C_i is a S4(p)-clauses. We note $S(C) = C_1 \wedge \ldots \wedge C_n$ or $C \text{ ->> } \{C_1, \ldots, C_n\}$.
• Transformation T (applied to $\varphi(L)$):
Apply R_3 to $\varphi(L)$ as long as possible with a higher priority for the deepest subformulae of $\varphi(L)$. Return the resulting formula.
To prove the theorem 2 the *Monotonicity of Entailment Lemma* from (Abadi & Manna 86), can be used.

Theorem 2

(1) S terminates for every clause C and $\models_{S4(p)} C \Rightarrow S(C)$ and (2) T terminates for every backward lemma $\varphi(L)$ and $\models_{S4(p)} \varphi(L) \Leftrightarrow T(\varphi(L))$ ■

Corollary

If the conditions i), ii) and iii) below hold
i)L is a lemma verifying the conditions of the Conjecture.
ii)C is a clause which has been deduced from $\{C_1,\ldots,C_N\}$. Let $\{f_1,\ldots,f_k\}$ be the set such that $C \text{ ->> } \{f_1,\ldots,f_k\}$.
iii) There exists α ($1 \le \alpha \le k$) such that every disjunct of f_α is a disjunct of $T(\varphi(L))$, **then** $C \models_{S4(p)} \varphi(L)$ ■

The backward translation we built between OSPL and S4(p), and its properties considering the modal resolution system for S4(p) (Enjalbert & Fariñas del Cerro 89) can be straightforwardly adapted to some other normal modal logics. By keeping the same backward translation between OSPL and K, K4 or T the logic morphism between OSPL and these logics is mainly contained in the one defined for the multi-modal logic (Ohlbach 89). We get some similar properties considering the modal resolution system (Enjalbert & Fariñas del Cerro 89) for them. For instance the theorem 1 (and the theorem 2) can be proved in the same way for K, K4 or T. The corresponding conjecture still remains a plausible theorem. The next example shows how the backward translation guide the proof in SL.

Example 1 ("Wise Man Puzzle")
We illustrate our method with a famous example from Mc Carthy. Its traditional form is:

> *"A certain king wishes to determine which of his three wise men is the wisest. He arranges them in a circle so that they can see and hear each other and tells them that he will put a white or black spot on each of their foreheads but at least one spot will be white. In fact all three spots are white. He then offers his favor to the one who first tells him the color of his spot. After a while, the wisest announces that his spot is white. How does he know?"*

To axiomatize this puzzle in S4(p), we assume that the three wise men are A, B, C and C is the wisest. At least one of them has a white spot and everyone knows that everybody else knows that his colleagues know this :

A_1 $\Box_i \Box_j \Box_k (P_A \vee P_B \vee P_C)$ for $i, j, k \in \{A, B, C\}$ and $\{i, j, k\} = \{A, B, C\}$ (P_i for $i \in \{A, B, C\}$ means that the wise i has a white spot ; A_1 denotes 6 axioms according to i, j and k).

The three men can see each other and they know this. Whenever one of them has a white or black spots he knows that his colleagues know this and he knows also that his colleagues knows this from each other :

A_2 $\Box_i(\neg P_i \Rightarrow \Box_j \neg P_i)$ for $i,j \in \{A, B, C\}$ and $i \neq j$.

A_3 $\Box_i \Box_j (\neg P_i \Rightarrow \Box_k \neg P_i)$ for $i, j, k \in \{A, B, C\}$ and $\{i, j, k\} = \{A, B, C\}$.

A_4 $\Box_i \Box_j (\neg P_j \Rightarrow \Box_k \neg P_j)$ for $i, j, k \in \{A, B, C\}$ and $\{i, j, k\} = \{A, B, C\}$.

C knows that B does not know that the colour of his spot and C knows that B knows that A does not know the colour of his spot :

A_5 $\Box_C \neg \Box_B P_B$ A_6 $\Box_C \Box_B \neg \Box_A P_A$.

We have considered 26 axioms from which we would like to deduce $\Box_C P_C$ (A_7). We translate these formulas into OSPL.

A'_1 $\forall x^i_1 : \text{'W->}^{rt}\text{W'}_i, \forall x^j_2 : \text{'W->}^{rt}\text{W'}_j, \forall x^k_3 : \text{'W->}^{rt}\text{W'}_k \; P_A([x^i_1, x^j_2, x^k_3]) \vee P_B([x^i_1, x^j_2, x^k_3]) \vee P_C([x^i_1, x^j_2, x^k_3])$ for $i, j, k \in \{A, B, C\}$ and $\{i, j, k\} = \{A, B, C\}$

A'_2 $\forall x^i_1 : \text{'W->}^{rt}\text{W'}_i, \forall x^j_2 : \text{'W->}^{rt}\text{W'}_j \; \neg P_i([x^i_1]) \Rightarrow \neg P_i([x^i_1, x^j_2])$ for $i,j \in \{A, B, C\}$ and $i \neq j$.

A'_3 $\forall x^i_1 : \text{'W->}^{rt}\text{W'}_i, \forall x^j_2 : \text{'W->}^{rt}\text{W'}_j, \forall x^k_3 : \text{'W->}^{rt}\text{W'}_k \; \neg P_i([x^i_1, x^j_2]) \Rightarrow \neg P_i([x^i_1, x^j_2, x^k_3])$ for $i, j, k \in \{A, B, C\}$ and $\{i, j, k\} = \{A, B, C\}$.

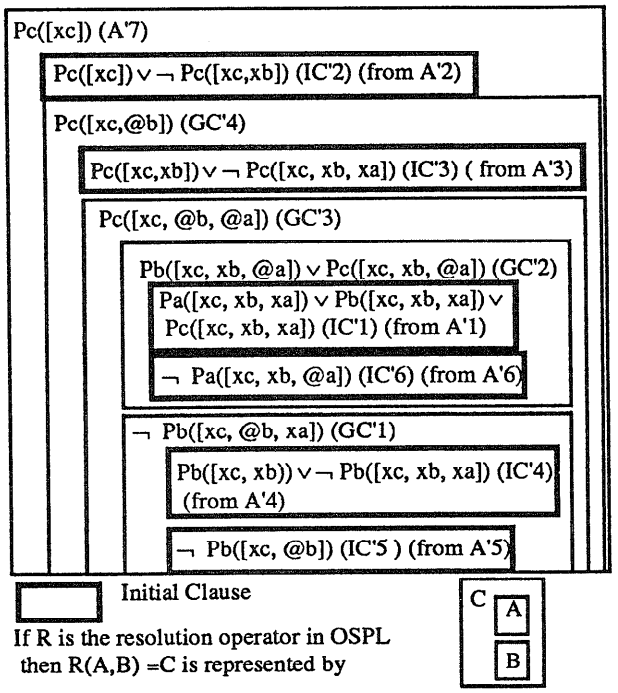
A'_4 $\forall x^i_1 : \text{'W->}^{rt}\text{W'}_i, \forall x^j_2 : \text{'W->}^{rt}\text{W'}_j, \forall x^k_3 : \text{'W->}^{rt}\text{W'}_k \; \neg P_j([x^i_1, x^j_2]) \Rightarrow \neg P_j([x^i_1, x^j_2, x^k_3])$ for $i, j, k \in \{A, B, C\}$ and $\{i, j, k\} = \{A, B, C\}$.

A'_5 $\forall x^C_1 : \text{'W->}^{rt}\text{W'}_C, \neg P_B([x^C_1 @_B])$, $@_B$ is a constant of sort $\text{'W->}^{rt}\text{W'}_B$

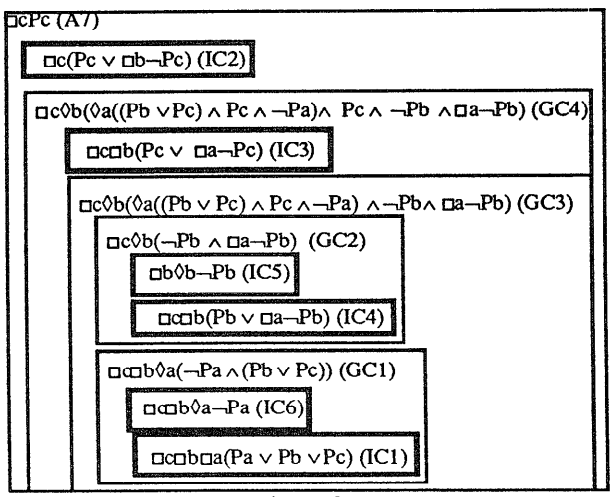
A'_6 $\forall x^C_1 : \text{'W->}^{rt}\text{W'}_C, \forall x^B_1 : \text{'W->}^{rt}\text{W'}_B, \neg P_B[x^C_1, x^B_2, @_A]$ where $@_A$ is a constant of sort $\text{'W->}^{rt}\text{W'}_A$

The translation of A_7 is (A'_7) $\forall x^C_1 : \text{'W->}^{rt}\text{W'}_C \; P_C([x^C_1])$. The deduction of the particular lemma A'_7 is presented in the Figure 1 (with our proof-editor).
Let L be the lemma $P_C([x_C])$. The backward translation of L is A_7. To deduce A_7 we can only consider the source S4(p)-axioms which have generated through the specification morphism the initial clause IC'_i useful to deduce A'_7 (cf the OSPL-proof). The only initial axioms (in clausal form) we consider, are $IC_1, \ldots, IC_6$ (cf the S4(p)-proof below). It should be noticed that the space search has been reduced with respect to the initial set of axioms (we have now 6 axioms instead of 26). We present the backward translation of every lemma of the OSPL-proof.
Backward translation of the clauses from the OSPL-proof:
For $1 \le i \le 6$ $\varphi(IC'_i) = IC_i$ (initial clauses),

$\varphi(CG'_1) = \Box_C \Diamond_B \Box_A \neg P_B$;
$\varphi(CG'_2) = \Box_C \Box_B \Diamond_A (P_B \vee P_C)$; $\varphi(A'_7) = A_7$
$\varphi(CG'_3) = \Box_C \Diamond_B \Diamond_A P_C$; $\varphi(CG'_4) = \Box_C \Diamond_B P_C$
We first present the proof of $\varphi(L)$ in S4(p) (Figure 2) and then we show how the conjecture holds and how the corollary has been a guide to build the S4(p)-proof.



(Figure 1)



(Figure 2)

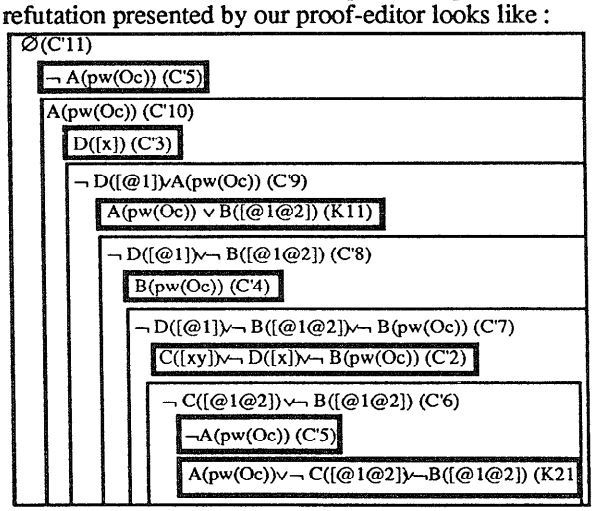
We note that , for $1 \leq i \leq 6$ $IC_i \models_{S4(p)} \varphi(IC'_i)$ and, $GC_2 \models_{S4(p)} \varphi(GC'_1)$, $GC_1 \models_{S4(p)} \varphi(GC'_2)$, $GC_i \models_{S4(p)} \varphi(GC'_i)$, i = 3,4 and $A_7 \models_{S4(p)} \varphi(A'_7)$.
We prove $GC_3 \models_{S4(p)} \varphi(GC'_3)$.

GC_3 ->> $\{\Box_C \Diamond_B \Diamond_A (P_B \vee P_C), \Box_C \Diamond_B \Diamond_A P_C, \Box_C \Diamond_B \Diamond_A \neg P_A, \Box_C \Diamond_B \neg P_B, \Box_C \Diamond_B \Box_A \neg P\}$ and $T(\varphi(GC'_3)) = \varphi(GC'_3)$. Since every disjunct of $\Box_C \Diamond_B \Diamond_A P_C$ is a disjunct of $\varphi(GC'_3)$ by the application of the corollary $GC_3 \models_{S4(p)} \varphi(GC'_3)$ holds. We get similar results for the other backward translated generated clauses.

In SL, in order to find a proof of A_7 we can consider the lemma CG'_3 of the proof of TL. In the modal system we first generate clauses until some clauses (GC_3) which verify the above conditions (a simple criterion is presented in the corollary) are deduced. The sequel of the proof will favour the use of these clauses. If we consider that every deduced clause is a lemma, we can "almost" build the proof in SL (*in this particular case it is possible to build the whole proof*). The next example shows how our method can be used as *a framework for the transfer of strategies.*

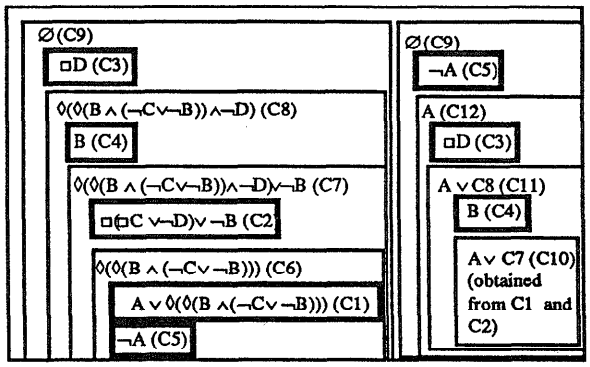
Example 2

The following formula (from (Auffray, Enjalbert, & Hebrard 90)) is a S4-theorem for which the proof in OPSL verifies the hypothesis of the Conjecture .
We consider the following S4-clauses:
(C_1): $A \vee \Diamond(\Diamond(B \wedge \neg C \vee \neg B))$ (C_2): $\Box(\Box C \vee \neg D) \vee \neg B$
(C_3): $\Box D$ (C_4): B (C_5): $\neg A$
We translate these formulas into OSPL:
(C'_1): $A(pw(O_C)) \vee (B([@_1 @_2]) \wedge (\neg C([@_1 @_2]) \vee \neg B([@_1 @_2])$ where $@_1$ and $@_2$ are constants of sort '$W \text{->}^{rt} W$'. Let K^1_1 , K^2_1 be some OSPL-clauses such that $\models_{OSPL} K^1_1 \wedge K^2_1 \Leftrightarrow C'_1$ (see Figure 3).
(C'_2): $\forall x,y$:'$W \text{->}^{rt} W$' $C([xy]) \vee \neg D([x]) \vee \neg B(pw(O_C))$
(C'_3): $\forall x$:'$W \text{->}^{rt} W$' $D([x])$ (C'_4): $B(pw(O_C))$
(C'_5): $\neg A(pw(O_C))$
A refutation has been obtained using a linear-input strategy (i.e. a linear strategy in which one at least of the clauses used in a resolution step is an input clause). The refutation presented by our proof-editor looks like :



(Figure 3)

Using results of the preceding sections we get the proof in S4 presented in the Figure 4.



(Figure 4)

It is easy to verify that by backward translating the proof found in OSPL, *the proof is essentially the same that the one in* (Auffray, Enjalbert, & Hebrard 90): the S4-proof implicitly uses the input strategy but the linear strategy is only partially used in the global proof. Clearly *we did not prove* the completeness of the unit strategy but, from a practical point of view the result is the same. On the other hand our approach is much more general because it allows us to study *different strategies* (or *heuristics*) and *different logics*.

5. Conclusion and future work

We have presented a framework for the backward transfer of lemmas and strategies and we have shown how it works on two detailed examples. It is natural to ask about the possibilities and limits of our approach. We succinctly analyse now both of them. Theoretically, if SL admits a logic morphism to TL and if there exists a proof calculus in SL then our method could be applied. But as the proof calculus considered for TL is resolution, a first step to extend the class of source logics to which our method could be applied, consists in considering logics with a resolution proof calculus. Among the existing resolution proof calculi for non-standard logics we shall study the extension of our method to the following ones :

- the first-order modal logic S5 (Fariñas del Cerro 84)
- the propositional temporal linear logic (Cavalli & Fariñas del Cerro 84)
- some epistemic logics (Konolige 86).

For the two latter logics, the problem of defining a logic morphism to OSPL and an inverse partial morphism is open. In principle there is no theoretical impossibility to answer to it. Obviously our approach could be applied with some other proof calculi and we are investigating in this way.

We are presently studying a backward morphism for linear temporal logic, for propositional S5 and for first-order modal logics. The main lines of future work are:

- To try to prove the conjecture stated in section 4.
- To characterize theoretically the SL, TL and logic morphism adapted to the transfer of strategies and, simultaneously
- To use the ideas presented in the second example in order to experimentally help in the study of the possibilities for transferring strategies.

Finally, it is worth pointing out that our method has similarities with the so called "reversed skolemisation" (see for ex. (Cox and Pietrzykowski 80)) - though our approach seems more general - . We shall deepen the study of their respective merits and limitations.

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