Identifying Linear Causal Effects

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Abstract

This paper concerns the assessment of linear cause-effect relationships from a combination of observational data and qualitative causal structures. The paper shows how techniques developed for identifying causal effects in causal Bayesian networks can be used to identify linear causal effects, and thus provides a new approach for assessing linear causal effects in structural equation models. Using this approach the paper develops a systematic procedure for recognizing identifiable direct causal effects.

Introduction

Structural equation models (SEMs) have dominated causal reasoning in the social sciences and economics, in which interactions among variables are usually assumed to be linear (Duncan 1975; Bollen 1989). This paper deals with one fundamental problem in SEMs, accessing the strength of linear cause-effect relationships from a combination of observational data and model structures. This problem has been under study for half a century, primarily by econometricians and social scientists, under the name "The Identification Problem". Traditional approaches are based on algebraic manipulation of the linear equations, for example, the rank and order conditions (Fisher 1966), the well-known instrumental variable method (Bowden & Turkington 1984). The applications of these methods are limited in scope and the identification problem is still far from being solved.

In recent years, causal reasoning with graphical models has been an active research area in the artificial intelligence community (Heckerman & Shachter 1995; Lauritzen 2000; Pearl 2000; Spirtes, Glymour, & Scheines 2001; Dawid 2002). The most common representation of graphical causal models involves a directed acyclic graph (DAG) with causal interpretation, called a *causal Bayesian network (BN)*. The relation between linear SEMs and causal BNs, in which no assumptions were made about the functional forms of how the variables interact with each other, is analyzed in (Pearl 1998; 2000; Spirtes, Glymour, & Scheines 2001). Graphical approaches for identifying linear causal effects in SEMs have been developed in which the structure of a SEM is represented by a DAG with bidirected edges, and some sufficient graphical conditions were established (McDonald 1997; Pearl 1998; Spirtes *et al.* 1998). Recently, (Brito & Pearl 2002b) developed a sufficient graphical criterion for the identification of the entire model. (Brito & Pearl 2002a) gave a generalization of the instrumental variable method that allows its application to models with few conditional independences.

The problem of identifying (nonlinear) causal effects in causal BNs concerns the assessment of the effects of actions or interventions from a combination of observational data and causal structures. This problem has been under extensive study over the last ten years (Pearl 1993; 1995; Galles & Pearl 1995; Glymour & Cooper 1999; Pearl 2000; Spirtes, Glymour, & Scheines 2001; Tian & Pearl 2002). In this paper we show that all the results in identifying nonlinear causal effects in causal BNs can be directly used to identify linear causal effects in SEMs, and thus provide a new tool for the identification problem in SEMs.

We consider three types of linear causal effects in SEMs – direct effects, partial effects, and total effects, and show how they can be computed through identifying nonlinear causal effects. Identifiable causal effects are expressed in terms of observed convariances. One feature of this approach is that it may directly identify total effects and partial effects even though some individual parameters involved are not identifiable. We also give a procedure that systematically looks for identifiable direct causal effects. The approach adds another tool to our repertoire of identification methods. However, its application is limited and it may well fail to identify some causal effects that are identifiable by other approaches.

Linear SEMs and Identification

A linear SEM over a set of variables $V = \{V_1, \ldots, V_n\}$ is given by a set of equations of the form

$$V_j = \sum_{i<j} c_{V_j V_i} V_i + \epsilon_j, \quad j = 1, \ldots, n, \tag{1}$$

where $c_{V_j V_i}$'s are called *path coefficients*, and ϵ_j's represent "error" terms and are assumed to have normal distribution with zero mean and covariance matrix Ψ with $\Psi_{ij} = Cov(\epsilon_i, \epsilon_j)$. In this paper we consider recursive models and assume that the summation is for $i < j$.

The equations and the non-zero entries in the covariance matrix Ψ define the structure of the model. The model struc-

$$
\begin{aligned}
X &= \epsilon_x \\
Z &= aX + \epsilon_z \\
W &= bZ + \epsilon_w \\
Y &= cX + dZ + eW + \epsilon_y \\
Cov&(\epsilon_x, \epsilon_w) \neq 0 \\
Cov&(\epsilon_z, \epsilon_y) \neq 0
\end{aligned}
$$

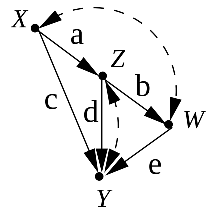


Figure 1: A linear SEM.

ture can be represented by a DAG G with (dashed) bidirected edges, called a *causal diagram* (or *path diagram*), as follows: the nodes of G are the variables $V_1, \dots, V_n$; there is a directed edge from V_i to V_j in G if the coefficient of V_i in the equation for V_j is not zero ($c_{V_j V_i} \neq 0$); there is a bidirected edge between V_i and V_j if the error terms ϵ_i and ϵ_j have non-zero correlation. Figure 1 shows a simple SEM and the corresponding causal diagram in which each directed edge is annotated by the corresponding path coefficient.

In linear SEMs, the observed distribution $P(v)$ is fully specified by a covariance matrix Σ over V. Let C be the matrix of path coefficients $c_{V_j V_i}$. Given the model structure and parameters C and Ψ, the covariance matrix Σ is given by (see, for example, (Bollen 1989))

$$\Sigma = (I - C)^{-1} \Psi (I - C)^{'-1}. \tag{2}$$

Conversely, in the identification problem, given the structure of a model, one attempts to solve for C in terms of the (observed) covariance Σ. If the equation (2) gives a unique solution to some path coefficient $c_{V_j V_i}$, independent of the (unobserved) error correlations Ψ, that coefficient is said to be *identifiable*. In other words, the *identification problem* is that whether a path coefficient is determined uniquely from the covariance matrix Σ given the causal diagram. Note that the identifiability conditions we seek involve the structure of the model alone, not particular numerical values of parameters, allowing for pathological exceptions.

Causal BNs and Identification

A causal Bayesian network consists of a DAG G over a set $V = \{V_1, \dots, V_n\}$ of vertices, called a *causal diagram*. The interpretation of such a diagram has two components, probabilistic and causal. The probabilistic interpretation views G as representing conditional independence assertions: Each variable is independent of all its non-descendants given its direct parents in the graph.[1] These assertions imply that the joint probability function $P(v) = P(v_1, \dots, v_n)$ factorizes according to the product (Pearl 1988)

$$P(v) = \prod_i P(v_i | pa_i) \tag{3}$$

where pa_i are (values of) the parents of variable V_i in the graph.[2]

The causal interpretation views the directed edges in G as representing causal influences between the corresponding variables. This additional assumption enables us to predict the effects of interventions, whenever interventions are described as specific modifications of some factors in the product of (3). The simplest such intervention involves fixing a set T of variables to some constants $T = t$, denoted by the action $do(T = t)$ or simply $do(t)$, which yields the post-intervention distribution[3]

$$P_t(v) = \begin{cases} \prod_{\{i | V_i \notin T\}} P(v_i | pa_i) & v \text{ consistent with } t. \\ 0 & v \text{ inconsistent with } t. \end{cases} \tag{4}$$

Eq. (4) represents a truncated factorization of (3), with factors corresponding to the manipulated variables removed. If T stands for a set of treatment variables and Y for an outcome variable in $V \setminus T$, then Eq. (4) permits us to calculate the probability $P_t(y)$ that event $Y = y$ would occur if treatment condition $T = t$ were enforced uniformly over the population. This quantity, often called the *causal effect* of T on Y, is what we normally assess in a controlled experiment with T randomized, in which the distribution of Y is estimated for each level t of T.

We see that, whenever all variables in V are observed, given the causal diagram G, all causal effects can be computed from the observed distribution $P(v)$. Our ability to estimate $P_t(v)$ from observed data is severely curtailed when some variables are unobserved, or, equivalently, if two or more variables in V are affected by unobserved confounders; the presence of such confounders would not permit the decomposition of the observed distribution $P(v)$ in (3). The question of *identifiability* arises, i.e., whether it is possible to express a given causal effect $P_t(s)$ as a function of the observed distribution $P(v)$ in a given causal diagram.

It is convenient to represent a causal BN with unobserved confounders in the form of a DAG G that does not show the elements of unobserved variables explicitly but, instead, represents the confounding effects of unobserved variables using (dashed) bidirected edges. A bidirected edge between nodes V_i and V_j represents the presence of unobserved factors (or confounders) that may influence both V_i and V_j. For example, the diagram in Figure 1 can be used to represent a causal BN with unobserved confounders.

In general, identifiability can be decided using *do*-calculus derivations (Pearl 1995). Sufficient graphical conditions for ensuring identifiability were established by several authors and are summarized in (Pearl 2000, Chapters 3 and 4), and see (Tian & Pearl 2002) for latest development. These graphical conditions often use a graphical relation called d-separation defined as follows (Pearl 1988).

[1] We use family relationships such as "parents," "children," and "ancestors" to describe the obvious graphical relationships.

[2] We use uppercase letters to represent variables or sets of variables, and use corresponding lowercase letters to represent their values (instantiations).

[3] (Pearl 2000) used the notation $P(v|set(t))$, $P(v|do(t))$, or $P(v|\hat{t})$ for the post-intervention distribution, while (Lauritzen 2000) used $P(v||t)$.

Definition 1 (d-separation) *A path*[4] p *is said to be* d-separated *by a set of nodes* Z *if and only if*

1. p *contains a chain* $V_i \rightarrow V_j \rightarrow V_k$ *or a fork* $V_i \leftarrow V_j \rightarrow V_k$ *such that the node* V_j *is in* Z, *or*
2. p *contains an inverted fork* $V_i \rightarrow V_j \leftarrow V_k$ *such that* V_j *is not in* Z *and no descendant of* V_j *is in* Z.

For example, a criterion called "back-door" (Pearl 1993) says that the causal effect $P_x(y)$ of variable X on Y is identifiable if there exists a set of variables Z such that Z satisfies the following *back-door condition* relative to (X, Y): (i) no member of Z is a descendant of X; and (ii) Z d-separates all the back-door paths from X to Y, where a path from X to Y is said to be a *back-door path* if it contains an arrow into X. Moreover, $P_x(y)$ is given by

$$P_x(y) = \sum_z P(y|x, z)P(z). \tag{5}$$

Linear SEMs can be thought of as a special case of causal BNs in which all interactions among variables are assumed to be linear. If a causal effect $P_t(y)$ is identifiable given a causal diagram, it will provide information about the identifiability of certain path coefficients in the SEM with the same causal diagram. In the next section, we show how causal effects $P_t(y)$ are related to path coefficients in SEMs.

Identifying Linear Causal Effects: from Causal BNs to SEMs

In SEMs, we define three types of linear causal effects as follows. The path coefficient $c_{V_jV_i}$ quantifies the direct causal influence of V_i on V_j, and is called a *direct effect*. Assume that there is a directed path p from V_k to V_i in the causal diagram G, then the product of path coefficients along the path p is called the *partial effect* of V_k on V_i along the path p, and will be denoted by $PE(p)$. For example, in the model shown in Figure 1, the partial effect of X on Y along the path (X, Z, Y) is given by ad. Let $\Gamma(V_k, V_i)$ denote the set of all directed paths from V_k to V_i, and let $\gamma \subseteq \Gamma(V_k, V_i)$. Then $\sum_{p \in \gamma} PE(p)$ is called the partial effect of V_k on V_i along the set of paths γ and will be denoted by $PE(\gamma)$. In particular, $PE(\Gamma(V_k, V_i))$, which represents the summation of products of path coefficients along the directed paths from V_k to V_i, is called the *total effect* of V_k on V_i and will be denoted by $TE(V_k, V_i)$. For example, in the model shown in Figure 1, the total effect of X on Y is given by $TE(X, Y) = c + ad + abe$.

The three types of linear causal effects – direct effects, partial effects, and total effects, can be computed from the nonlinear causal effects $P_t(y)$. An identifiable causal effect $P_t(y)$ would mean certain linear causal effects are identifiable in the SEM with the same causal diagram G. Let $E[.|do(t)]$ denote the expectations in the post-intervention distribution $P_t(.)$:

$$E[X|do(t)] = \int x P_t(x) dx \tag{6}$$

The following proposition is intuitive and not hard to prove.

[4] A path is a sequence of consecutive edges (of any directionality).

Proposition 1 (Total Effects) *The total effect of* V_k *on* V_i *is identifiable if* $P_{v_k}(v_i)$ *is identifiable, and* $TE(V_k, V_i)$ *can be computed as*

$$TE(V_k, V_i) = E[V_i|do(v_k)]/v_k. \tag{7}$$

For example, in the model shown in Figure 1, if $P_x(y)$ is identifiable, then $TE(X, Y) = c + ad + abe = E[Y|do(x)]/x$.

Let $Pa_j = \{V_{j_1}, \ldots, V_{j_l}\}$ be the set of parents of V_j in the causal diagram (Pa_j is the set of variables that appear in the right hand side of the equation for V_j in Eq. (1)), then it is easy to show that

$$E[V_j|do(pa_j)] = \sum_i c_{V_jV_{j_i}} v_{j_i}, \tag{8}$$

from which we have the following proposition.

Proposition 2 (Direct Effects) *The direct effect of* V_k *on* V_j *is identifiable if* $P_{pa_j}(v_j)$ *is identifiable, and the path coefficient* $c_{V_jV_k}$ *can be computed as*

$$c_{V_jV_k} = \frac{\partial}{\partial v_k} E[V_j|do(pa_j)], \quad V_k \in Pa_j. \tag{9}$$

For example, in the model shown in Figure 1, if $P_{xzw}(y)$ were identifiable, then $E[y|do(xzw)] = cx + dz + ew$ and path coefficients c, d, e would be identifiable.

Let $S = \{V_{i_1}, \ldots, V_{i_m}\}$ be a set of variables that does not contain V_i. Let γ_j be the set of directed paths from V_{i_j} to V_i that does not pass any other variables in S. Then it is easy to show that

$$E[V_i|do(s)] = \sum_j PE(\gamma_j) v_{i_j}, \tag{10}$$

where we define $PE(\emptyset) = 0$. Eq. (10) leads to the following proposition.

Proposition 3 *Let* $S = \{V_{i_1}, \ldots, V_{i_m}\}$, *and let* γ_j *be the set of directed paths from* V_{i_j} *to* V_i *that does not pass any variables in* $S \setminus \{V_{i_j}\}$. *If* $P_s(v_i)$ *is identifiable, then each partial effect* $PE(\gamma_j)$, $j = 1, \ldots, m$, *is identifiable and is given by*

$$PE(\gamma_j) = \frac{\partial}{\partial v_{i_j}} E[V_i|do(s)]. \tag{11}$$

If we seek to identify some particular partial effects, then Proposition 3 can be modified to obtain the following proposition.

Proposition 4 (Partial Effects) *Given a set* $\gamma \subseteq \Gamma(V_k, V_i)$ *of directed paths from* V_k *to* V_i, *if there exists a set of variables* S *such that none of the variables in* S *lies in the paths in* γ *and each path in* $\Gamma(V_k, V_i) \setminus \gamma$ *passes a variable in* S,[5] *and if* $P_{s,v_k}(v_i)$ *is identifiable, then the partial effect* $PE(\gamma)$ *is identifiable and can be computed as*

$$PE(\gamma) = \frac{\partial}{\partial v_k} E[V_i|do(s), do(v_k)]. \tag{12}$$

[5] Note that such a set S may not exist for some γ.

For example, in the model shown in Figure 1, the partial effect $c+ad$ of X on Y along the paths (X,Y) and (X,Z,Y) is equal to $\frac{\partial}{\partial x}E[Y|do(xw)]$ if $P_{xw}(y)$ is identifiable.

Propositions 1-4 provide a method for identifying linear causal effects in SEMs by identifying certain causal effects $P_t(y)$ and then computing expectations. Identified causal effects $P_t(y)$ are expressed in terms of the observed joint $P(v)$ which is a normal distribution in SEMs. To compute expectations with respect to the distribution $P_t(y)$, we will often need to compute conditional expectations. Let $S = \{V_{i_1},\ldots,V_{i_m}\}$ and $S_j = S \setminus \{V_{i_j}\}$. We have the following formula for conditional expectations in normal distributions (Cramer 1946)

$$E[V_i|s] = \sum_j \beta_{V_iV_{i_j}.S_j}\ v_{i_j}, \tag{13}$$

where $\beta_{V_iV_{i_j}.S_j}$ denotes the *partial regression coefficient* and represents the coefficient of V_{i_j} in the linear regression of V_i on S. (Note that the order of the subscripts in $\beta_{V_iV_j.S}$ is essential.) Partial regression coefficients can be expressed in terms of covariance matrices as follows (Cramer 1946):

$$\beta_{V_iV_j.S} = \frac{\Sigma_{V_iV_j} - \Sigma^T_{V_iS}\Sigma^{-1}_{SS}\Sigma_{V_jS}}{\Sigma_{V_jV_j} - \Sigma^T_{V_jS}\Sigma^{-1}_{SS}\Sigma_{V_jS}}, \tag{14}$$

where Σ_{SS} etc. represent covariance matrices over corresponding variables.

Whenever a causal effect $P_t(y)$ is determined as identifiable, we can use Eqs. (7)–(13) to identify certain linear causal effects. We illustrate the computation process by an example. The "back-door" criterion (Pearl 1993) says that if a set of variables Z satisfies the back-door condition relative to an ordered pair of variables (X,Y), then $P_x(y)$ is identifiable and is given by Eq. (5). Then by Proposition 1, the total effect of X on Y is identifiable. Let $Z = \{Z_1,\ldots,Z_k\}$ and $Z^i = Z \setminus \{Z_i\}$. $TE(X,Y)$ can be computed as

$$\begin{aligned} &TE(X,Y) = E[Y|do(x)]/x \\ &= \sum_z E[Y|x,z]P(z)/x \ \text{ (by Eq. (5))} \\ &= \sum_z (\beta_{YX.Z}\ x + \sum_i \beta_{YZ_i.XZ^i}\ z_i)P(z)/x \ \text{ (by Eq. (13))} \\ &= \beta_{YX.Z} \quad (E[Z_i]=0) \end{aligned} \tag{15}$$

Therefore, we conclude that if a set of variables Z satisfies the back-door condition relative to (X,Y), then the total effect of X on Y is identifiable and is given by $\beta_{YX.Z}$. This result is given as Theorem 5.3.2 in (Pearl 2000, p. 152).

Using this approach, all the results for identifying causal effects in causal BNs can be used for identifying linear causal effects. Some of these results provide new graphical criteria for the identification of linear causal effects in SEMs. Consider the "front-door" criterion (Pearl 1995), which says that the causal effect $P_x(y)$ of variable X on Y is identifiable if there exists a set of variables Z satisfying the following *front-door condition* relative to (X,Y): (i) Z intercepts all directed paths from X to Y; (ii) all back-door paths from X to Z are d-separated by the empty set; and (iii) all back-door paths from Z to Y are d-separated by X. Moreover, $P_x(y)$ is given by

$$P_x(y) = \sum_z P(z|x)\sum_{x'} P(y|x',z)P(x') \tag{16}$$

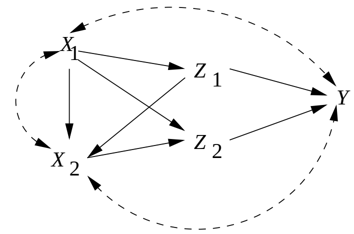


Figure 2: $P_{x_1x_2}(y)$ is identifiable.

Proposition 1 then leads to the following criterion for identifying total effects in SEMs.

Theorem 1 *If a set of variables $Z = \{Z_1,\ldots,Z_k\}$ satisfies the front-door condition relative to (X,Y), then the total effect of X on Y is identifiable and is given by $\sum_i \beta_{YZ_i.XS_i}\beta_{Z_iX}$ where $S_i = Z \setminus \{Z_i\}$.*

For X being a singleton a general criterion for identifying $P_x(y)$ is given by Theorem 4 in (Tian & Pearl 2002). Let a path composed entirely of bidirected edges be called a *bidirected path*. Let $An(Y)$ denote the union of $\{Y\}$ and the set of ancestors of Y, and let $G_{An(Y)}$ denote the subgraph of G composed only of variables in $An(Y)$. The theorem says that $P_x(y)$ is identifiable if there is no bidirected path connecting X to any of its children in $G_{An(Y)}$. Proposition 1 then leads to the following theorem.

Theorem 2 *(Tian & Pearl 2002) The total effect of X on Y is identifiable if there is no bidirected path connecting X to any of its children in $G_{An(Y)}$.*

Partial effects can be obtained by Proposition 4 from identifying the causal effects $P_t(y)$ of a set of variables T on Y. Some graphical criteria for identifying $P_t(y)$ are given in (Pearl & Robins 1995; Kuroki & Miyakawa 1999). Those criteria are complicated and here we show the use of those criteria by an example. It was shown in (Kuroki & Miyakawa 1999) that $P_{x_1x_2}(y)$ in Figure 2 is identifiable. Then by Proposition 3, the partial effect $c_{Z_1X_1}c_{YZ_1} + c_{Z_2X_1}c_{YZ_2}$ of X_1 on Y is identifiable, and the total effect $c_{Z_2X_2}c_{YZ_2}$ of X_2 on Y is identifiable. The computation of actual expressions for these partial effects are lengthy and will not be shown here.

Direct effects can be thought of as a special case of partial effects, and by Proposition 2, direct effects on a variable V_j can be obtained by identifying $P_{pa_j}(v_j)$, the causal effects of V_j's parents on V_j. The criteria in (Pearl & Robins 1995; Kuroki & Miyakawa 1999) can also be used for identifying direct effects. For example, it is shown in (Pearl & Robins 1995) that the causal effect $P_{x_1x_2}(y)$ in Figure 3 is identifiable and is given by (Pearl 2000, page 122)

$$P_{x_1x_2}(y) = \sum_z P(y|x_1,x_2,z)P(z|x_1), \tag{17}$$

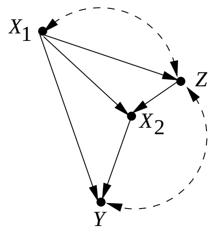


Figure 3: $P_{x_1x_2}(y)$ is identifiable.

from which we compute

$$E[Y|do(x_1, x_2)] = (\beta_{YX_1.X_2Z} + \beta_{YZ.X_1X_2}\beta_{ZX_1})x_1 + \beta_{YX_2.X_1Z}\ x_2. \quad (18)$$

By Proposition 2, we conclude that the direct effects of X_1 and X_2 on Y are both identifiable and are given by

$$c_{YX_1} = \beta_{YX_1.X_2Z} + \beta_{YZ.X_1X_2}\beta_{ZX_1}, \quad (19)$$
$$c_{YX_2} = \beta_{YX_2.X_1Z}. \quad (20)$$

This method of translating identifiability results in causal BNs to linear models provides another tool for the identification problem in SEMs. Traditional SEM methods (Bollen 1989; Bowden & Turkington 1984) and recent graphical methods (Brito & Pearl 2002a) have focused on the identification of individual path coefficients, i.e. direct effects (with the exception of (Pearl 1998)). The method presented in this paper not only can be used to identify individual path coefficients, but may also directly identify some total effects and partial effects even though some individual path coefficients involved are not identifiable.

Some individual path coefficients that can not be directly identified as direct effects may be identified indirectly by the identification of some other causal effects of which they are a part. For example, in Figure 4(a), the path coefficient c_{WY} can not be identified directly (nonlinear causal effect $P_y(w)$ is not identifiable), yet it can be shown that the total effect of Z_2 on Y, c_{YZ_2}, and the total effect of Z_2 on W, $c_{YZ_2}c_{WY}$, are both identifiable, and therefore c_{WY} can be computed as $TE(Z_2, W)/TE(Z_2, Y)$. In general, a set of identifiable causal effects means that a set of algebraic expressions (sum of products) of path coefficients are identifiable, which can be treated as a set of equations over path coefficients. Individual path coefficients may be identified by solving those equations. For the purpose of identifying individual path coefficients, in the next section, we develop a procedure that systematically determines identifiable causal effects and solves for path coefficients.

Identifying Path Coefficients Systematically

Let a topological order over V be $V_1 < \ldots < V_n$ such that if $V_i < V_j$ then there is no directed path from V_j to V_i. For j from 2 to n, at each step, we will try to identify the path coefficients associated with edges pointing at V_j (i.e., the direct effects on V_j, $c_{V_jV_i}$'s). For this purpose we consider the subgraph composed of those variables ordered before V_j. Let $V^{(j)} = \{V_1, \ldots, V_j\}$, $j = 1, \ldots, n$, and let $G_{V^{(j)}}$ denote the subgraph composed of the nodes in $V^{(j)}$. At step j, we seek identifiable causal effects imposed by other variables on V_j. Ideally, if $P_{pa_j}(v_j)$ is identifiable, then all those path coefficients $c_{V_jV_i}$'s are identifiable. In general, this is usually not the case. One general type of causal effects on V_j that is guaranteed to be identifiable is given in (Tian & Pearl 2002), as shown in the following lemma.

Lemma 1 *(Tian & Pearl 2002) Let a topological order over V be $V_1 < \ldots < V_n$, and let $V^{(j)} = \{V_1, \ldots, V_j\}$, $j = 1, \ldots, n$, and $V^{(0)} = \emptyset$. Let S_j be the set of variables (including V_j) that are connected with V_j by a bidirected path (i.e., a path composed entirely of bidirected edges) in $G_{V^{(j)}}$ and let $\bar{S}_j = V^{(j)} \setminus S_j$. Then $P_{\bar{s}_j}(s_j)$ is identifiable and is given by*

$$P_{\bar{s}_j}(s_j) = \prod_{\{i|V_i \in S_j\}} P(v_i|v^{(i-1)}). \quad (21)$$

Using Lemma 1 and Proposition 3, at step j, we can compute $E[V_j|do(\bar{s}_j))]$ and identify some partial effects on V_j. Next, we study in detail what information we could obtain from these partial effects. Let $Z = \{Z_1, \ldots, Z_k\}$ be the set of variables in $\bar{S}_j$ such that for each Z_i there exists a directed path from Z_i to V_j that does not pass any other variables in $\bar{S}_j$. Let γ_i be the set of directed paths from Z_i to V_j that do not pass any other variables in $\bar{S}_j$. By Proposition 3, each partial effect $PE(\gamma_i)$ is identifiable and is given by

$$PE(\gamma_i) = \frac{\partial}{\partial z_i} E[V_j|do(\bar{s}_j)]. \quad (22)$$

Let the set of parents of V_j that are connected with V_j by a bidirected path be $Pa_j \cap S_j = \{Y_1, \ldots, Y_l\}$. Then the partial effect $PE(\gamma_i)$ as a summation of products of path coefficients along directed paths from Z_i to V_j can be decomposed into

$$PE(\gamma_i) = \sum_{m=1}^{l} PE(\delta_{im})c_{V_jY_m}, \text{ for } Z_i \notin Pa_j, \quad (23)$$

or when Z_i is a parent of V_j,

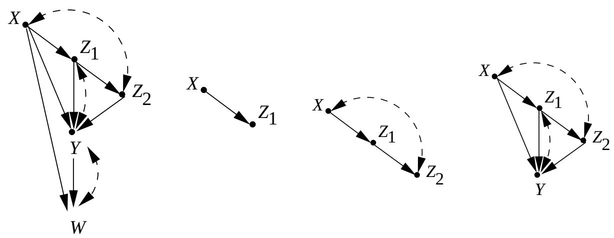
$$PE(\gamma_i) = c_{V_jZ_i} + \sum_{m=1}^{l} PE(\delta_{im})c_{V_jY_m}, \text{ for } Z_i \in Pa_j, \quad (24)$$

where δ_{im} is the set of directed paths from Z_i to Y_m that do not pass any other variables in $\bar{S}_j$, and the summation is over the parents of V_j that are in S_j because γ_i only contains paths that do not pass variables in $\bar{S}_j$.

Since $P_{\bar{s}_j}(s_j)$ is identifiable and $Y_m \in S_j$, by Proposition 3, each partial effect $PE(\delta_{im})$ is identifiable and is given by

$$PE(\delta_{im}) = \frac{\partial}{\partial z_i} E[Y_m|do(\bar{s}_j)], \quad (25)$$

which would have been identified before the step j. From Eqs. (22)–(25), we conclude that, at step j, we will obtain a set of equations which are linear in path coefficients $c_{V_jV_i}$'s



(a) G (b) (c) (d)

Figure 4: Subgraphs for identifying path coefficients in G.

associated with edges pointing at V_j and in which $c_{V_jV_i}$'s are the only unknowns. Eq. (24) will be linearly independent with other equations because the path coefficient $c_{V_jZ_i}$ will not appear in any other equations. However, in general, we can not guarantee that Eq. (23) is independent with each other.

In summary, we propose the following procedure for systematically identifying path coefficients.

- Let a topological order over V be $V_1 < \ldots < V_n$ and let $V^{(j)} = \{V_1, \ldots, V_j\}$, $j = 1, \ldots, n$,
- At step j, for j from 2 to n, do the following
  1. Let S_j be the set of variables (including V_j) that are connected with V_j by a bidirected path in $G_{V^{(j)}}$ and let $\bar{S}_j = V^{(j)} \setminus S_j$.
  2. Find the expression for $P_{\bar{s}_j}(s_j)$ by Lemma 1.
  3. Compute $E[V_j|do(\bar{s}_j)]$ and use Proposition 3 to get a set of equations linear in path coefficients $c_{V_jV_i}$'s associated with edges pointing at V_j.
  4. Try to solve the set of linear equations for $c_{V_jV_i}$'s.

We demonstrate this procedure by systematically identifying path coefficients in a SEM with causal diagram shown in Figure 4(a). The only admissible order of variables is $X < Z_1 < Z_2 < Y < W$. At step 1, we consider the subgraph in Figure 4(b). It is obvious that $P_x(z_1) = P(z_1|x)$, and we obtain

$$c_{Z_1X} = E[Z_1|do(x)]/x = \beta_{Z_1X}. \tag{26}$$

At step 2, we consider the subgraph in Figure 4(c). Z_2 is connected with X by a bidirected path and Lemma 1 gives

$$P_{z_1}(x, z_2) = P(z_2|z_1, x)P(x), \tag{27}$$

which leads to

$$P_{z_1}(z_2) = \sum_x P(z_2|z_1, x)P(x). \tag{28}$$

Therefore, we obtain

$$c_{Z_2Z_1} = E[Z_2|do(z_1)]/z_1 = \beta_{Z_2Z_1.X}. \tag{29}$$

At step 3, we consider the subgraph in Figure 4(d). Y is connected with Z_1 by a bidirected path and Lemma 1 gives

$$P_{xz_2}(y, z_1) = P(y|z_2, z_1, x)P(z_1|x). \tag{30}$$

We then compute the expectation

$$\begin{aligned} &E[Y|do(x, z_2)] \\ &= \beta_{YZ_2.Z_1X}\, z_2 + (\beta_{YZ_1.Z_2X}\beta_{Z_1X} + \beta_{YX.Z_2Z_1})x \end{aligned} \tag{31}$$

By Proposition 3 we obtain the path coefficient

$$c_{YZ_2} = \beta_{YZ_2.Z_1X}, \tag{32}$$

and the following partial effect

$$c_{YX} + c_{Z_1X}c_{YZ_1} = \beta_{YX.Z_2Z_1} + \beta_{Z_1X}\beta_{YZ_1.Z_2X}, \tag{33}$$

where c_{Z_1X} is identified in Eq. (26) and c_{YX} and c_{YZ_1} are unknown.

Finally, at the last step, we consider the graph in Figure 4(a). W is is connected with Y and Z_1 by a bidirected path and Lemma 1 gives

$$P_{xz_2}(w, y, z_1) = P(w|y, z_2, z_1, x)P(y|z_2, z_1, x)P(z_1|x). \tag{34}$$

We then compute the expectation

$$\begin{aligned} &E[W|do(x, z_2)] \\ &= (\beta_{WY.Z_2Z_1X}\beta_{YZ_2.Z_1X} + \beta_{WZ_2.YZ_1X})z_2 \\ &\quad + [\beta_{WY.Z_2Z_1X}(\beta_{YZ_1.Z_2X}\beta_{Z_1X} + \beta_{YX.Z_2Z_1}) \\ &\quad + \beta_{WZ_1.YZ_2X}\beta_{Z_1X} + \beta_{WX.YZ_2Z_1}]\, x, \end{aligned} \tag{35}$$

and by Proposition 3, the total effect of Z_2 on W is identifiable:

$$c_{YZ_2}c_{WY} = \beta_{WY.Z_2Z_1X}\beta_{YZ_2.Z_1X} + \beta_{WZ_2.YZ_1X}, \tag{36}$$

and the following partial effect of X on W is identifiable:

$$\begin{aligned} &c_{WX} + (c_{YX} + c_{Z_1X}c_{YZ_1})c_{WY} \\ &= \beta_{WY.Z_2Z_1X}(\beta_{YZ_1.Z_2X}\beta_{Z_1X} + \beta_{YX.Z_2Z_1}) \\ &\quad + \beta_{WZ_1.YZ_2X}\beta_{Z_1X} + \beta_{WX.YZ_2Z_1}. \end{aligned} \tag{37}$$

The path coefficient c_{YZ_2} is identified in (32), and from Eq. (36), the path coefficient c_{WY} is identifiable and is given by

$$c_{WY} = \beta_{WY.Z_2Z_1X} + \frac{\beta_{WZ_2.YZ_1X}}{\beta_{YZ_2.Z_1X}}. \tag{38}$$

Then from Eqs. (37), (33), and (38), the path coefficient c_{WX} is identifiable and is given by

$$\begin{aligned} c_{WX} &= \beta_{WX.YZ_2Z_1} + \beta_{WZ_1.YZ_2X}\beta_{Z_1X} \\ &\quad - \frac{\beta_{WZ_2.YZ_1X}}{\beta_{YZ_2.Z_1X}}(\beta_{YZ_1.Z_2X}\beta_{Z_1X} + \beta_{YX.Z_2Z_1}) \end{aligned} \tag{39}$$

Conclusion

We show how the three types of linear causal effects in SEMs – direct effects, partial effects, and total effects, can be identified through identifying nonlinear causal effects of the form $P_t(y)$ using techniques developed in causal BNs. The approach provides a new tool for the identification problem in SEMs. Every progress in causal effects identification in causal BNs can be directly translated into progress in SEMs. One feature of this approach is that it may directly identify total effects and partial effects even though some individual path coefficients involved are not identifiable. The approach does not mean to be a complete identification rule, and it may fail to identify some path coefficients that are identifiable (possibly by other approaches).

Acknowledgments

This research was partly supported by NSF grant IIS-0347846.

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