Negotiation as Mutual Belief Revision

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Abstract

This paper presents a logical framework for negotiation based on belief revision theory. We consider that a negotiation process is a course or multiple courses of mutual belief revision. A set of AGM-style postulates are proposed to capture the rationality of competitive and cooperative behaviors of negotiation. We first show that the AGM revision and its iterated extension is a special case of negotiation function. Then we show that a negotiation function can be constructed by two related iterated belief revision functions under a certain coordination mechanism. This provides a qualitative method for constructing negotiation space and rational concessions. It also shows a glimpse of how to express game-theoretical concepts in logical framework.

Introduction

Negotiation has been investigated from many perspectives, including economics, applied mathematics, psychology, sociology and computer science. Significant advances have been made in both quantitative and qualitative analysis of negotiating processes (Pruitt 1981)(Rosenschein and Zlotkin 1994)(Faratin *et al.* 1998) (Kraus *et al.* 1998) (Parsons *et al.* 1998) (Sadri *et al.* 2001). Quantitative approaches, especially those which are inspired by game-theory, dominate much of the existing work. In many cases, however, numeric utility functions are either unreliable or simply unavailable. This paper is a contribution to the body of literature, such as (Sycara 1990) (Kraus *et al.* 1998)(Parsons *et al.* 1998) (Wooldridge and Parsons 2000), which, instead, views negotiation in a qualitative light. Different from most of the existing work, we provide an axiomatic analysis of negotiating processes based on belief revision theory(AGM 1985)(Zhang and Foo 2001).

Negotiation is a process of consensus-seeking among two or more agents. Each agent comes to the negotiation table with an initial set of demands or offers. The parties involved proceed with mutual persuasion or argumentation and terminate when they have converged on a mutually acceptable agreement. If we consider the demands(or offers) of parties as their beliefs on the matter in question, the change of the demands of each party reflects the change of its beliefs during the progress of negotiation. The parties who are convinced to accept part of the other parties' demands would perform a belief revision. New belief states of participants represent their revised demands which are normally closer to each other and apt to reach an agreement. We term such kind of belief revision *mutual belief revision.*

Different from the single agent belief revision, each agent in mutual belief revision tries not only to minimize the loss of its beliefs but also to maximize its gain of information from the other agents. Negotiation behaves in a similar manner. A negotiator attempts to keep as many of her demands as possible and typically tends to selectively accept some demands of his opponents in order to avoid failure of negotiation and maximize gains from negotiation table. The logical framework we propose is intended to strike the correct balance between *competition* and *cooperation* among agents. As a starting point of the investigation, we restrict ourselves to the case of two agents. We propose a set of postulates to model rationality of mutual belief revision and negotiation. These postulates are mostly inspired by the frameworks of AGM theory(AGM 1985), Zhang and Foo's multiple belief revision(Zhang and Foo 2001) and Darwiche and Pearl's iterated belief revision(Darwiche and Pearl 1997).

Postulates for mutual belief revision and negotiation

We will work in a propositional language $\mathcal{L}$. The language is that of classical propositional logic with an associated consequence operation Cn in the sense that $Cn(X) = \{A : X \vdash A\}$. A set K of sentences is *logically closed* or called a *belief set* when $K = Cn(K)$. If X, Y are two sets of sentences, $X + Y$ denotes $Cn(X \cup Y)$.

In this section we propose a set of axioms to specify properties of mutual belief revision and negotiation between two agents. The idea is the following. Suppose that K_1 and K_2 are the current belief states of two agents. During the mutual belief revision, each agent accepts part of beliefs from the other agent and minimize her change of belief states to preserve consistency. As a result, the revised belief states of the agents, denoted by $N_1(K_1, K_2)$ and $N_2(K_1, K_2)$ respectively, normally get closer each other.

In negotiation setting, the revision of belief states reflects the changes of agent demands or offers. If X and Y represent the initial demands/offers of two agents, $N_1(X,Y)$ and $N_2(X,Y)$ will be their revised demands/offers after negotiation. $N_1(X,Y) \cap N_2(X,Y)$ will be then the agreement reached in the negotiation.

Formally, a mutual revision or negotiation function is a two-input and two-output function

$$N(X,Y) = (N_1(X,Y), N_2(X,Y))$$

where X and Y represent the initial belief sets or demands of each agent and $N_1(X,Y)$ and $N_2(X,Y)$ the revised belief sets or demands, respectively. Note that we do not assume that X or Y to be logically closed.

Definition 1 A function $N : 2^{\mathcal{L}} \times 2^{\mathcal{L}} \to 2^{\mathcal{L}} \times 2^{\mathcal{L}}$ is a *mutual belief revision* or *negotiation function* if it satisfies the following postulates:

(N1) *Closure:*
$N_1(X,Y) = Cn(N_1(X,Y));\; N_2(X,Y) = Cn(N_2(X,Y))$

(N2) *Inclusion:*
$$N_1(X,Y) \subseteq X+Y;\; N_2(X,Y) \subseteq X+Y$$

(N3) *Vacuity:* If $X \cup Y$ is consistent, then
$$X+Y \subseteq N_1(X,Y);\; X+Y \subseteq N_2(X,Y)$$

(N4) *Inconsistency:*
$N_1(X,Y)$ is inconsistent iff X or Y is inconsistent;
$N_2(X,Y)$ is inconsistent iff X or Y is inconsistent.

(N5) *Extensionality:* If $Cn(X) = Cn(Y)$, then
$$N(X,Z) = N(Y,Z);\; N(Z,X) = N(Z,Y)$$

We call (N1)-(N5) the *basic postulates* for mutual belief revision or negotiation. Intuitively, (N1) says that the resulting belief state of each agent is logically closed. (N2) states that no third party information would be introduced. (N3) says that each agent will accept all the beliefs of the other agent if no conflict arises. (N4) says that mutual belief revision can only happen between rational agents. (N5) assumes that mutual belief revision is syntax-independent, i.e. logically equivalent description of beliefs leads to the same results of mutual belief revision.

Similar interpretation of these postulates can also be given in terms of negotiation. If we localize the belief state of an agent on the matters of a negotiation, its belief set represents its demands in the negotiation, which we call the *demand sets* of the agents. (N1) then states that each negotiator should be aware of that she is responsible to undertake all the items and their consequences of her demands once they are included in an agreement. (N2) assumes that if no conflicts between the demands of two agents, amendments of demands may only be done within the initial demand sets from the negotiators[1]. (N3) reflects the cooperative aspect of negotiation that each agent is willing to accept the other party's demands provided they do not conflict with herself's. (N4) means that no negotiation can proceed from inconsistent demands. (N5) is similar.

[1]Note that if there are conflicts between two agents' demands, the amended demand sets could be anything since $X+Y$ will be inconsistent.

It is easy to see that postulates (N1)-(N5) are counterparts of AGM basic postulates for belief revision(AGM 1985)(see next section for a review of the postulates) except of *the success postulate*. It is unreasonable to assume that one side of negotiation would accept all the demands of the other side.

It is easy to see that for the following two special cases:

- $X \cup Y$ is consistent. In this case, $N(X,Y) = (X+Y, X+Y)$;
- X or Y is inconsistent. In this case, $N(X,Y) = (\mathcal{L}, \mathcal{L})$.

the basic postulates can uniquely determine a negotiation function. However for the most interesting cases when X and Y are consistent but $X \cup Y$ is inconsistent, the postulates do not tell us too much about what the function would be. We need more postulates to capture more characteristics of negotiation behaviors. The following two postulates reflect a common principle in belief revision and negotiation, known as *the principle of information economy*, from two different perspectives: *competitive* and *cooperative*.

(N6) *Consistent Expansion:*
If $X \cup N_1(X,Y)$ is consistent, then $X \subseteq N_1(X,Y)$;
If $Y \cup N_2(X,Y)$ is consistent, then $Y \subseteq N_2(X,Y)$.

This postulate capture the conservative or self-interest feature of agents, which says that if an agent is not going to accept any counter-demands that contradict her own, she does not need to give up any of her demands. The postulate comes from Ferme and Hansson's selective revision (Ferme and Hansson 1999), where the postulate was interpreted as *"previous beliefs are given up only if this is required to avoid inconsistency"*.

The following postulate (N7) reflects the cooperative attitude of negotiation agents. It states that every agent should commit herself to keeping her original demands/offers once they have been are accepted by the other side. We call it *the rule of no recantation*. It is a quite common negotiation protocol and is one of conditions to guarantee the convergency of negotiation process. To understand the postulate, note that $Cn(X) \cap N_2(X,Y)$ represents the demands of agent 1 which have been accepted by agent 2 after negotiation and $Cn(Y) \cap N_1(X,Y)$ the demands of agent 2 which have been accepted by agent 1.

(N7) *No Recantation:*
$Cn(Y) \cap N_1(X,Y) \subseteq N_2(X,Y)$;
$Cn(X) \cap N_2(X,Y) \subseteq N_1(X,Y)$.

We remark that (N7) does not imply the following condition:

Intersection: $K_1 \cap K_2 \subseteq N_1(K_1,K_2) \cap N_2(K_1,K_2)$
which says that common items of initial demands must be included in the last agreement of the negotiation. This condition can be a special negotiation protocol but we can't take it for granted. If both sides decide to give up a common item (in order to keep some more beneficial demands), the item will not be included in the last agreement.

The following postulate describes a property of iterated process of negotiation.

(N8) *Iteration:*
If $F \subseteq Cn(Y) \cap N_1(X,Y)$, then $N(N_1(X,F),Y) = N(X,Y)$.
If $F \subseteq Cn(X) \cap N_2(X,Y)$, then $N(X,N_2(F,Y)) = N(X,Y)$.

It is a typical strategy in real-life negotiation that a negotiator poses its demands in several stages. At each stage, the negotiator reveals part of her demands, hiding something tougher or some alternatives behind tying to push her opponent changing mind step by step. (N8) says that if one can expect that some of her demands will be definitely accepted by the other side, it will be useless to pose this part first. As we will see, this postulate posts a constraint on the stability of agent's negotiation policy.

Multiple belief revision and iterated belief revision

We have seen in the last section that there exists a close relationship between AGM belief revision and mutual belief revision. In this section we show that an AGM revision function is a special case of mutual belief revision. First let's recall some basic facts about multiple belief revision and iterated belief revision in single agent environments.

To best suit the context of mutual belief revision, instead of using the original AGM framework, we shall exploit the multiple version of the AGM theory (Zhang and Foo 2001), which allows us to revise a belief set by another belief set. Formally, for any belief set K and a set F of sentences, $K \otimes F$ stands for the result of belief revision when K is revised by F. The operation is required to satisfy the following postulates:

($\otimes 1$) $K \otimes F = Cn(K \otimes F)$.
($\otimes 2$) $F \subseteq K \otimes F$.
($\otimes 3$) $K \otimes F \subseteq K + F$.
($\otimes 4$) If $F \cup K$ is consistent, $K + F \subseteq K \otimes F$.
($\otimes 5$) $K \otimes F$ is inconsistent if F is inconsistent.
($\otimes 6$) If $Cn(F_1) = Cn(F_2)$, $K \otimes F_1 = K \otimes F_2$.
($\otimes 7$) $K \otimes (F_1 \cup F_2) \subseteq (K \otimes F_1) + F_2$.
($\otimes 8$) If $F_2 \cup (K \otimes F_1)$ is consistent, $(K \otimes F_1) + F_2 \subseteq K \otimes (F_1 \cup F_2)$.

A negotiation process normally consists of several stages of mutual belief revision. To simulate such a process, an iterated mechanism of belief revision is required[2]. The following assumption has been accepted by several different iterated belief revision formalisms:

($\otimes$IBR) $(K \otimes F_1) \otimes (F_1 \cup F_2) = K \otimes (F_1 \cup F_2)$

It is easy to see that ($\otimes$IBR) is the multiple version of the postulate ($C1$) in (Darwiche and Pearl 1997). Based on Lehmann's observation (Theorem 1 in (Lehmann 1995)) we can easily prove that ($\otimes 1$)-($\otimes 6$) and ($\otimes$IBR) implies ($\otimes 7$) and ($\otimes 8$).

The following theorem is an easy generalization of Lehmann's consistency result presented in (Lehmann 1995).

Theorem 1 *($\otimes 1$)-($\otimes$ 6) are consistent with ($\otimes$IBR).*

In the following we will call a revision function an *iterated belief revision* if it satisfies the postulates ($\otimes 1$)-($\otimes 8$) and ($\otimes$IBR). Note that such an iterated revision function is different from Darwiche and Pearl's one since we use the exact AGM postulates. This should be fine because ($\otimes$IBR) goes well with AGM postulates(Zhang 2004).

Now let's consider a special case of mutual belief revision when one agent unconditionally accepts the other agent's beliefs. Formally, a mutual revision function N is a *master-slave* revision if it satisfies

(M-S) $X \subseteq N_2(X, Y)$.

The condition says that the second agent (slave) always accepts beliefs of the first agent (master) with no reservation. Interestingly the following observation shows that under protocol $(N7)$, the first agent can always keep its beliefs.

Lemma 1 *Let N be a master-slave revision. If it satisfies $(N7)$, then for any X and Y, $X \subseteq N_1(X, Y)$.*

The following is the representation theorem for master-slave revision.

Theorem 2 *Let N be a master-slave revision. If we define a revision function $\otimes$ as $K \otimes F \stackrel{def}{=} N_2(F, K)$, then it satisfies the postulates ($\otimes 1$)-($\otimes 6$). If N satisfies $(N8)$, then it satisfies ($\otimes$IBR).*

Conversely, if $\otimes$ is an iterated belief revision function which satisfies ($\otimes 1$)-($\otimes 6$) and ($\otimes$IBR), then the mutual belief revision function defined as follows satisfies (N1)-(N8) and (M-S):

$$N(X, Y) \stackrel{def}{=} (Cn(Y) \otimes X, Cn(Y) \otimes X).$$

This representation theorem shows that (N1)-(N8) plus (M-S) fully specifies a master-slave revision function. Since an iterated revision function always exists, this theorem also shows the consistency of the postulates (N1)-(N8).

In the next section, we will present a general construction of negotiation function, which will allow more "balanced" negotiation.

Construction of negotiation functions

In this section we provide a model for negotiation functions. Firstly, we give a simple model of negotiation function that satisfies the basic postulates for mutual belief revision and negotiation.

Representation theorem for basic postulates

A function $t : 2^{\mathcal{L}} \to 2^{\mathcal{L}}$ is called a *belief transform* if for any set X of sentences,

1. $t(X)$ is a belief set, i.e., $t(X) = Cn(t(X))$.
2. $t(Cn(X)) = t(X)$.
3. if X is consistent, then $t(X)$ is consistent.

Theorem 3 *A negotiation function N satisfies the basic postulates (N1)-(N5) if and only if there are two belief transforms t_1 and t_2 such that*

$$N(X, Y) = \begin{cases} (X + Y, X + Y), & \text{if } X \cup Y \text{ is consistent;} \\ (\mathcal{L}, \mathcal{L}), & \text{if either } X \text{ or } Y \text{ is inconsistent;} \\ (t_1(X), t_2(Y)), & \text{otherwise.} \end{cases}$$

This theorem indicates that the basic postulates does not tell us too much except that agents in a negotiation might change their mind.

[2] This is similar to some other settings, say belief fusion (Maynard-Reid II and Shoham 2001).

Negotiable deals

Now we give a more sophisticated model to capture more properties of negotiation processes. We shall introduce some concepts that are borrowed from game-theory, inspired by the work of (Rosenschein and Zlotkin 1994).

Let X and Y be two sets of sentences, representing the initial demands or offers of two agents, respectively. We call the pair (X, Y) a *negotiation encounter*.

Given two iterated revision functions $\otimes_1$ and $\otimes_2$. A *deal* over an encounter (X, Y) is a pair (Ψ_1, Ψ_2) such that $\Psi_1 \subseteq Cn(X), \Psi_2 \subseteq Cn(Y)$ and satisfies the fix-point condition:

$$Cn(\Psi_1 \cup \Psi_2) = (Cn(X) \otimes_1 \Psi_2) \cap (Cn(Y) \otimes_2 \Psi_1) \quad (1)$$

The set of all possible deals over (X, Y) is called the *negotiation set* of (X, Y), denoted by $\mathcal{NS}(X, Y)$.

Intuitively, Ψ_1 represents all the demands of agent 1 that agent 2 is willing to accept. Similarly Ψ_2 expresses the demands of agent 2 that agent 1 is willing to accept. Then $Cn(X) \otimes_1 \Psi_2$ is going to be the revised demand set of agent 1 and $Cn(Y) \otimes_2 \Psi_1$ the revised demand set of agent 2 after negotiation. A deal then, is an agreement in which both agents agree on Ψ_1 and Ψ_2 (as well as their consequences). Therefore $Cn(\Psi_1 \cup \Psi_2)$ will be the common acceptable conditions, $(Cn(X) \otimes_1 \Psi_2) \cap (Cn(Y) \otimes_2 \Psi_1)$, after the negotiation. Negotiation set consists of all possible deals that both agent might consider.

Obviously negotiation set can never be empty since $(Cn(X) \cap Cn(Y), Cn(X) \cap Cn(Y))$ always belongs to $\mathcal{NS}(X, Y)$. We call this deal *the conflict deal* of the encounter.

Example 1 *Consider an encounter (X, Y) where $X = \{p, q\}$ and $Y = \{\neg p, \neg q\}$. Let $\otimes_1$ and $\otimes_2$ are any iterated revision functions. Then the conflict deal will be $(Cn(\{p \leftrightarrow q\}), Cn(\{p \leftrightarrow q\}))$, which is in the negotiation set. Two other extreme cases, $(Cn(X), Cn(\{p \leftrightarrow q\}))$ and $(Cn(\{p \leftrightarrow q\}), Cn(Y))$, are also negotiable deals. If each agent is going to take some offers from the other, then more "balanced" negotiation could happen. However the result will heavily depend on the evaluation on their demands and counter-demands. Suppose that for agent 1, p is more entrenched than q and for agent 2 $\neg q$ is more entrenched than $\neg p$. Then $(Cn(\{p\}), Cn(\{\neg q\}))$ will be a negotiable alternative whereas $(Cn(\{q\}), Cn(\{\neg p\}))$ is not.* □

It seems to be that we are ready to define the negotiation function as follows:

$$N(X, Y) \stackrel{def}{=} (Cn(X) \otimes_1 \Psi_2,\ Cn(Y) \otimes_2 \Psi_1)$$

where (Ψ_1, Ψ_2) is a deal of (X, Y). However, this is not going to be a valid definition since we might have several deals for each encounter. We need to pick up one deal from all the possible deals as the agreement.

Selection function

We consider that negotiation is a decision-making procedure in which each agent chooses a deal from negotiation set. If both agents choose the same deal, then an agreement is reached; otherwise, the conflict deal will be the result of the negotiation. Formally, a negotiation process is a selection function which chooses a deal from negotiation set. Let γ be a selection function which selects an element from a nonempty set. We will abbreviate $\gamma(\mathcal{NS}(X, Y))$ to $\gamma(X, Y)$. As usual, $\gamma_i(X, Y)$ means the i^{th} component of $\gamma(X, Y)$. Now we define a negotiation function as follows:

Definition 2 *Let $\otimes_1$ and $\otimes_2$ be two iterated revision functions and γ a selection function. Define a negotiation function N as follows: for any encounter (X, Y),*

$$N(X, Y) = \begin{cases} (X + Y, X + Y), & \text{if } X \cup Y \text{ is consistent;} \\ (\mathcal{L}, \mathcal{L}), & \text{if either } X \text{ or } Y \text{ is inconsistent;} \\ (Cn(X) \otimes_1 \Psi_2, Cn(Y) \otimes_2 \Psi_2), & \text{otherwise.} \end{cases}$$

where $(\Psi_1, \Psi_2) = \gamma(X, Y)$.

It is easy to see that given an encounter (X, Y), if the selection function γ selects the conflict deal of the encounter, then $N(X, Y) = (X, Y)$.

The following example shows another extreme case.

Example 2 *Let $\otimes$ be an iterated revision operator and $\ominus$ is the associated contraction operator. For any encounter (X, Y), let $\gamma(X, Y) = (Cn(X), Cn(Y) \ominus X)$. Then γ is a selection function that defines a master-slave mutual belief revision operator where $\otimes_1 = \otimes_2 = \otimes$.* □

Now let's check whether the defined negotiation function satisfies the proposed postulates. The following observation is easy to verify.

Proposition 1 *Any function that is defined by Definition 2 satisfies the basic postulates (N1)-(N5) as well as postulate (N6).*

However, To satisfy the other postulates we need to introduce some restrictions on the selection function.

Rationality and compatibility

A deal δ=(Ψ_1, Ψ_2) over (X, Y) is called *rational* if it satisfies the following conditions:

$Cn(X) \cap (Cn(Y) \otimes_2 \Psi_1) \subseteq Cn(X) \otimes_1 \Psi_2$

$Cn(Y) \cap (Cn(X) \otimes_1 \Psi_2) \subseteq Cn(Y) \otimes_2 \Psi_1$

The set of all rational deals is called *rational negotiation set*.

Obviously these conditions correspond to the postulation (N7). Therefore if the function in Definition 2 defined by a selection function over rational negotiation set, then it satisfies (N7).

(N8) requires a kind of uniformity in the selection mechanism over different negotiation situations. Before we present the model for (N8), let's introduce a game-theoretical concept.

A deal $\delta = (\Psi_1, \Psi_2)$ *dominates* a deal $\delta' = (\Psi'_1, \Psi'_2)$ if $Cn(X) \otimes_1 \Psi'_2 = Cn(X) \otimes_1 \Psi_2$ and $Cn(Y) \otimes_2 \Psi'_1 = Cn(Y) \otimes_2 \Psi_1$ and either

$\Psi'_1 \subseteq \Psi_1$ and $\Psi'_2 \subset \Psi_2$, or $\Psi'_1 \subset \Psi_1$ and $\Psi'_2 \subseteq \Psi_2$.

In the other words, δ dominates δ' if at least one agent agrees to accept more demands from the other agent without sacrificing any agent's profits.

A deal δ is called *pareto optimal* over a set of deals if it is in the set and there do not exist any other deals in the set that dominate δ.

The set of all the deals that are rational and pareto optimal over the rational deals is called *refined negotiation set.*

Lemma 2 *Let (X, Y) be any encounter. For any deal (Ψ_1, Ψ_2) in the refined negotiation set,*
$\Psi_1 = Cn(X) \cap (Cn(Y) \otimes_2 \Psi_1)$ *and* $\Psi_2 = Cn(Y) \cap (Cn(X) \otimes_1 \Psi_2)$.

Definition 3 A selection function γ is *downward compatible* if for any $F_1 \subseteq \gamma_1(X, Y)$ and $F_2 \subseteq \gamma_2(X, Y)$,
$\gamma(Cn(X)\otimes_1 F_2, Cn(Y)\otimes_2 F_1)=(\gamma_1(X, Y)+F_2, \gamma_2(X, Y)+F_1)$.

Example 3 *Let γ be a selection function such that $\gamma(X, Y) = (Cn(X) \cap Cn(Y), Cn(X) \cap Cn(Y))$. Then γ is downward compatible. In other wards, always-standing-still is a uniform behavior of negotiation.*

Now we come to the main result of the paper.

Theorem 4 *If γ is a downward compatible selection function over a refined negotiation set, the negotiation function defined by Definition 2 satisfies (N1)-(N8).*

Since a downward compatible selection function always exists, this theorem shows again the consistency of the negotiation postulates. We remark that downward compatibility is a sufficient condition for postulate (N8) but not necessary. In fact, we can show that any master-slave negotiation function satisfies postulate (N8) but its associated selection function is not necessarily downward compatible. How to find a sufficient and necessary condition for (N8) is open for the future research.

Related work

There have been several streams of research which related to this work. One stream is the work on arbitration, belief merging and knowledge fusion (Liberatore and Schaerf 1998)(Kfir-Dahav and Tennenholtz 1996)(Konieczny and Pino Perez 1998). All these researches deal with conflicts between agents. With our framework, we can also define an "arbitration" operator that satisfies most of Revesz's postulates and Liberatore and Schaerf's postulates for arbitration operation(Liberatore and Schaerf 1998)(Revesz 1997).

Proposition 2 *Let N be a negotiation function. Define an "arbitration operator" $\triangle$ as follows: for any belief set K_1 and K_2,*
$K_1 \triangle K_2 = N_1(K_1, K_2) \cap N_2(K_1, K_2)$
then $\triangle$ satisfies Liberatore and Schaerf's postulates (A1)-(A3)(A5)-(A6) for two-agent case(Liberatore and Schaerf 1998).[3]

However there is a fundamental difference between arbitration and mutual belief revision or negotiation. An arbitration of two knowledge bases should be fair on both information resources, whereas an outcome of negotiation does not necessarily "fair". We allow an agent totally give up her beliefs and takes all information from the other agent.

Another stream of work is that of non-prioritized belief revision(Booth 2001)(Ferme and Hansson 1999)(Hansson 1999)(Hansson *et al.* 2001). A negotiation function can be viewed as two associated selective revision(Ferme and Hansson 1999). A non-prioritized belief revision function can also be defined by negotiation processes(Booth 2001). However the existing operators proposed under these settings capture only the basic properties of negotiation.

There have been several attempts to consider belief revision in the setting of multi-agent systems(Kfir-Dahav and Tennenholtz 1996)(Malheiro *et al.* 1994)(van der Meyden 1994). In (van der Meyden 1994) a concept of mutual belief revision was defined with a totally different setting from ours, where mutual belief revision is referred to the process of belief change by which an agent in a synchronous multi-agent system revises its beliefs about other agents' beliefs.

In terms of logical approach to negotiation, there are numbers of researches on argumentation-based negotiation(Sycara 1990)(Kraus *et al.* 1998)(Parsons *et al.* 1998)(Sadri *et al.* 2001). This work is more concentrated on agent architectures and procedural analysis of negotiation protocols. Although the goal is similar with ours, the emphases and outcomes of the research are quite different from our work (we focus on the axiomatic analysis on negotiation processes).

Conclusion

In this paper we presented a formal framework for describing and modelling rational negotiation behaviors. A set of AGM-style postulates was presented. A representation theorem is given for the basic postulates. The consistency of all the list of postulates was also proved through an explicit construction of negotiation function in which negotiation process was modelled by two related iterated belief revision operations. This model provided a logical method to analyze the competitive and cooperative behaviors of negotiation.

As an initial work towards an axiomatic approach to negotiation, we only presented the basic axiomatic system and its modelling in this paper. In a sequent paper, we will define a notion of preference-based negotiation and concentrate on the construction of negotiation concessions and outcomes(Meyer *et al.* 2004). By then, more computation-friendly examples will be given. There are also many other things to be done. First of all, in this work, we only extended some basic game-theoretical concepts, such as individual rationality and Pareto optimality, into logical form and express a particular equilibrium by using fix-point inference. However, more profound mechanisms of cooperation and competition behind negotiation, especially the large amount of variations of equilibrium concepts in game theory and economics, need to be investigated.

Proofs of Selected Theorems

Due to the limitation of space, we only list the proof of two main theorems.

Proof of Theorem 2: The postulates $(\otimes 1)$ and $(\otimes 3)$-$(\otimes 6)$ are the special case of the postulates (N1)-(N5), respectively. $(\otimes 2)$ is im-

[3] Note that $\triangle$ does not satisfy (A4) since the inconsistency of $K_1 \triangle K_2$ does not require the inconsistency of both K_1 and K_2. However, we do not consider this to be a major difference between arbitration and negotiation.

plied by (M-S). To prove ($\otimes$ IBR), we know that $(K \otimes F_1) \otimes (F_1 \cup F_2) = N_2(F_1 \cup F_2, K \otimes F_1)$. Since $F_1 \subseteq F_1 \cup F_2$, it follows from (M-S) that $F_1 \subseteq N_2(F_1 \cup F_2, K)$. Therefore $F_1 \subseteq (F_1 \cup F_2) \cap N_2(F_1 \cup F_2, K)$. By (N8) we yield $N_2(F_1 \cup F_2, N_2(F_1, K)) = N_2(F_1 \cup F_2, K) = K \otimes (F_1 \cup F_2)$.

Conversely, suppose that $\otimes$ satisfies ($\otimes$1)-($\otimes$6) and ($\otimes$IBR). It is obvious that the defined negotiation function satisfies obviously (N1)-(N5). (N6) is implies by ($\otimes$2) and ($\otimes$4). (N7) is trivial.

To prove (N8), assume $F \subseteq Cn(Y) \cap N_1(X, Y)$. Then $Cn(F) \ominus X \subseteq Cn(F) \subseteq N_1(X, Y) = Cn(Y) \otimes X$. Hence $N_1(N_1(X, F), Y) = Cn(Y) \otimes N_1(X, F) = Cn(Y) \otimes (Cn(F) \otimes X) = Cn(Y) \otimes (X \cup (Cn(F) \ominus X)) = (Cn(Y) \otimes X) + (Cn(F) \ominus X) = Cn(Y) \otimes X = N_1(X, Y)$. Similarly we have $N_2(N_1(X, F), Y) = N_2(X, Y)$.

Again assume $F \subseteq Cn(X) \cap N_2(X, Y)$. Then $N_1(X, N_2(F, Y)) = N_1(X, Cn(Y) \otimes F) = (Cn(Y) \otimes F) \otimes X = Cn(Y) \otimes (F \cup X) = Cn(Y) \otimes X = N_1(X, Y)$. $N_2(X, N_2(F, Y))$ is exactly the same. Therefore (N7) is valid. □

Proof of Theorem 4: We only need to verify (N8). Assume that $F \subseteq Cn(Y) \cap N_1(X, Y)$. Then $F \subseteq Cn(Y) \cap (Cn(X) \otimes_1 \gamma_2(X, Y))$. It follows from Lemma 2 that $F \subseteq \gamma_2(X, Y)$. According to the construction of negotiation function, $N_1(N_1(X, F), Y) = N_1(X, F) \otimes_1 \gamma_2(N_1(X, F), Y) = (Cn(X) \otimes_1 \gamma_2(X, F)) \otimes_1 \gamma_2(Cn(X) \otimes_1 \gamma_2(X, F), Y)$. Let $F_2 = \gamma_2(X, F)$. Thus $F_2 \subseteq Cn(F) \subseteq \gamma_2(X, Y)$. Since γ is downwards compatible, we have $\gamma_2(Cn(X) \otimes_1 F_2, Y) = \gamma_2(X, Y)$. Therefore $N_1(N_1(X, F), Y) = (Cn(X) \otimes_1 F_2) \otimes_1 \gamma_2(Cn(X) \otimes_1 F_2, Y) = (Cn(X) \otimes_1 F_2) \otimes_1 \gamma_2(X, Y)$. By $(\otimes IBR)$ we know $(Cn(X) \otimes_1 F_2) \otimes_1 \gamma_2(X, Y) = Cn(X) \otimes_1 \gamma_2(X, Y) = N_1(X, Y)$. Put them together we yield $N_1(N_1(X, F), Y) = N_1(X, Y)$. For agent 2, similarly we have $N_2(N_1(X, F), Y) = Cn(Y) \otimes_2 \gamma_1(N_1(X, F), Y) = Cn(Y) \otimes_2 \gamma_1(Cn(X) \otimes_1 \gamma_2(X, F), Y) = Cn(Y) \otimes_2 \gamma_1(Cn(X) \otimes_1 F_2, Y)$. By the downward compatibility of γ, we have $\gamma_1(Cn(X) \otimes_1 F_2, Y) = \gamma_1(X, Y) + F_2$. Since $F_2 \subseteq Cn(Y) \otimes_2 \gamma_1(X, Y)$, Therefore, $Cn(Y) \otimes_2 (\gamma_1(X, Y) + F_2) = Cn(Y) \otimes_2 \gamma_1(X, Y)$. These give us $N_2(N_1(X, F), Y) = Cn(Y) \otimes_2 (\gamma_1(X, Y) + F_2) = Cn(Y) \otimes_2 \gamma_1(X, Y) = N_2(X, Y)$. We have proved $N(N_1(X, F), Y) = N(X, Y)$. The other half of the postulate is symmetric. □

References

E. Alchourrón,P. Gärdenfors and Makinson, 1985. On the logic of theory change: partial meet contraction and revision functions, *Journal of Symbolic Logic,* 50(2)(1985), 510-530.

R. Booth,2001. A negotiation-style framework for non-priorised revision, *TARK'01*, 137-150.

A. Darwiche and J. Pearl, 1997. On the logic of iterated belief revision. *Artificial Intelligence,* 89(1-2):1-29.

P. Faratin, C. Sierra and N. R. Jennings, 1998. Negotiation decision functions for autonomous agents, *Robotics and Autonomous Systems*, 24(1998):159–182.

E. Ferm and S. O. Hansson,1999. Selective Revision, *Studia Logica*, 63 (3):331-342.

S. O. Hansson, 1999. A survey of non-prioritized belief revision, *Erkenntnis* 50:413-427.

S. O. Hansson, E. Ferm, J. Cantwell, and M. Falappa, 2001. Credibility-limited revision, *Journal of Symbolic Logic*, 66:1581-1596.

D. Lehmann, 1995. Belief revision, revised, *IJCAI-95,* 1534-1540.

W. Li , A logical framework for evolution of specification. in *Programming Language and Systems*, (ESOP'94), Springer-Verlag, 394-408, 1994.

P. Liberatore and M. Schaerf, 1998. Arbitration (or How to merge knowledge bases), *IEEE Transactions on Knowledge and Data Engineering*, 1(10):76-90.

N. E. Kfir-Dahav and M. Tennenholtz, 1996. Multi-agent belief revision, *TARK'96*, 175-194.

S. Konieczny and R. Pino Perez, 1998. On the logic of merging, *KR-98*, 488-498.

S.Kraus, K.Sycara, A.Evenchik, 1998. Reaching agreements through argumentation: a logical model and implementation, *Artificial Intelligence*, 104(1998), 1-69.

B. Malheiro, N. R. Jennings, and E. Oliveira, 1994. Belief revision in multi-agent systems, *ECAI-94*, 294-298.

P. Maynard-Reid II and Y. Shoham, 2001. Belief fusion: aggregating pedigreed belief states, *Journal of Logic, Language and Information* 10 (2):183-209.

R. van der Meyden, 1994. Mutual belief revision (Preliminary Report), *KR'94,* 595-606.

T. Meyer, N. Foo, R. Kwok and D. Zhang, Logical foundations of negotiation, in this proceedings.

S. Parsons, C. Sierra, N. Jennings, 1998. Agents that reason and negotiate by arguing, Journal of Logic and Computation 8(3): 261-292.

D. G. Pruitt, 1981. *Negotiation Behavour*, Academic Press.

P. Z. Revesz, 1997. On the semantics of arbitration, *International Journal of Algebra and Computation,* 7(2):133-160.

J. S. Rosenschein and G. Zlotkin, 1997. *Rules of Encounter*, MIT Press.

C. Sierra, N. R. Jennings, P. Noriega, and S. Parsons, 1997. A framework for argumentation-based negotiation. ATAL-97: 177-192.

F. Sadri, F.Toni, and P. Torroni, 2001. Dialogues for Negotiation: agent varietirs and dialogue sequences, n *Proc. ATAL-2001*.

K. Sycara, Persuasive arguementation in negotiation, *Theory and Decision*, 28:203-242.

M. Wooldridge and S. Parsons, 2000. Languages for negotiation, in: *ECAI2000*, 393-400.

D. Zhang and N. Foo, 2001. Infinitary belief revision,*Journal of Philosophical Logic*, 30(6):525-570.

D. Zhang, 2004. Properties of iterated belief revision, in: *Proc. LPNMR-04*.