Flexible Decision-Making in Sequential Auctions

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Introduction

It is quite common that items are sold sequentially in a series of auctions. For example, in the Seattle Fur Exchange, approximately eighty percent of pelts are auctioned sequentially (Lambson & Thurston 2003). On eBay, there are thousands of identical or similar items that are sold sequentially.

A sequential auction is a combination of individual auctions. Each individual auction, which we call a component auction, can be any type of auction, such as a first-price, sealed-bid auction. In this thesis, I assume that there are q items for sale in a sequence of auctions, K. There are n agents, $n > q$, competing for these q items. Each agent has single-unit demand and will leave if it wins one item. Agents have incomplete information; each agent knows its own true valuation; however, it knows only a distribution function of other agents' valuations. After each auction, all bidders' bids are revealed, a practice which is common on eBay and other e-commerce sites.

The intention of decision-making in sequential auctions is to find the optimal solutions for agents in the sequence of auctions as a whole. One myopic approach is to treat sequential auctions as a collection of independent auctions. However, as pointed out by Engelbrecht-Wiggans and Weber (Engelbrecht-Wiggans & Weber 1979), this kind of approach may be inappropriate in the general case. A better approach is to model sequential auctions as a game and find the equilibrium strategies.

There is a voluminous theoretical literature on finding equilibria in sequential auctions. However, these models lack flexibility and a re-analysis is necessary even for a slightly different model. On the other hand, the vast number of trading opportunities and the increasingly fluid markets bolster the need for automated trading support in the form of *trading agents*—software programs that participate in electronic markets on behalf of a user. Simple bidding tools, like eSnipe[1] and AuctionBlitz[2], enable bidders to automate submission of bids. However, these tools lack the sophistication that bidders require when faced with a plethora of auctions possibly occurring in a sequence. The need for economic efficiency and computational efficiency leads to the question:

> Can we design a flexible decision-making system for sequential auctions so that an agent can find the equilibrium strategies efficiently and automatically?

I develop a multi-dimensional sequential auction design space based on Wurman, et al.'s classification, which includes three dimensions: bidding rules, clearing policy, and information revelation policy (Wurman, Wellman, & Walsh 2001). The flexible decision-making system is designed to automatically generate strategies for different models with a specification of the parameters. In next section, I present an approach to generate strategies for sequential auctions with discrete strategy spaces. In the following section, I provide two conjectures and discuss an algorithm to compute strategies for sequential auctions with continuous strategy spaces.

Completed Work

The Monte Carlo Approximation (MCA) Approach

For incomplete information games, we may use Harsanyi's transformation (Harsanyi 1967 8) to translate an incomplete information game into an imperfect information game. However, due to the infinite possible valuations for each agent, the computability of the Harsanyi transformation is limited. In this thesis, I investigate the use of Monte Carlo sampling to generate heuristic bidding policies for the incomplete information game with agents having discrete strategy spaces. The approach to the problem can be summarized as follows:

1. Create a sample complete-information game by drawing a set of valuations for other bidders.
2. Solve for a Nash equilibrium of the sample game.
3. Update the agent's bidding policy.

The first step is straightforward Monte Carlo sampling. The second step is the subject of the following two paragraphs. To update the agent's bidding policy, I take the weighted sum of the equilibrium solutions across all sample games (Cai & Wurman 2003).

The default representation of this game in extensive form is to expand each of the leaves with an appropriate subgame. A subgame is any part of the game tree that begins with a singleton information set. However, it is useful to identify

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[1] http://www.esnipe.com

[2] http://www.auctionblitz.com

the game structure of individual auctions. Given that the bidders have complete information, all subgames with the same players remaining have the same solution(s). Thus, a single-unit, sealed-bid (component) auction with n agents has at most n *unique* subgames—one for each possible set of non-winners. Our agent's approach is essentially *dynamic programming*, and equivalent to standard backward induction with caching. The agent solves all possible smallest component games, and recursively constructs higher-order subgames until it solves the root game.

This leveraged structure enables significant computational savings. The component form representation is exponential in the number of agents and the number of bidding choices. However, the total number of nodes in our leveraged structure required to express the game is exponentially less than in the full expansion (Cai & Wurman 2003). For example, five-agent, four-item sequential single-unit auctions with five bid choices and random tie-breaking require only 1931 nodes to encode in the component form, compared to the 4.5 billion required for the naive expansion.

Empirical Results

To evaluate the efficacy of the approach, I simulated several market configurations in which I varied the bidding rules, the functional form of the valuation distributions, the form of the update equation, and the strategies of the other bidders.

In the experiments, I measure the utility for our agent, the social welfare, and the revenue achieved by the seller. The performance of the MCA strategy is quite close to that of the subgame perfect equilibrium both when the other agents play perfectly and when they construct their own MCA strategies. This result indicates that the approximation technique generates policies that perform quite well in this environment (Cai & Wurman 2003).

A perfect Bayesian equilibrium (PBE) is defined in terms of beliefs at decision points in the game, and requires that an equilibrium policy be consistent with those beliefs. The MCA policy at a node implicitly captures the agent's beliefs about which opponent valuations would explain the fact that the agent arrived at a particular decision point in the game tree. I have shown that MCA strategies converge to the average policy of PBE (Cai 2003).

Proposed Work

I expect to complete three threads of related research. The first one involves showing the relevance of different MCAs to PBE. A successful proof of the convergence of MCA strategies to PBE would validate the significance of using the MCA approach to compute equilibria in sequential auctions. In a previous report (Cai 2003), I introduce another policy update function. This function is a weighted update function in which I add utility to the policy update function to allow more weight to those bids which yield more utility. My preliminary analysis lends support to a conjecture:

Conjecture 1. *For a class of sequential-auction games, the weighted MCA strategies converge to PBE.*

The second and third threads are related to sequential auctions with continuous strategy spaces. Weber's model (Weber 1983) is a classic sequential auction model with continuous strategy spaces, in which the auctioneer announces only the winner and its bid. However, solutions change when we simply change the announcement rule. On the basis of some preliminary analyses, I conjecture:

Conjecture 2. *In a single-unit demand, independent private value sequential auction model with incomplete information and a price quote in which the auctioneer announces the bids of all agents after each auction, there does NOT exist an equilibrium in pure strategies.*

Most literature on sequential auctions is aimed at finding equilibria in pure strategies. The second conjecture suggests that there might be some sequential auction models, in which pure strategy equilibria do not exist, and moreover, this is the type of auction prevalent on the Internet.

In the third thread, I plan to design an algorithm to generate strategies for sequential auctions with continuous strategy spaces. One approach is to discretize the continuous strategy spaces so that we may use the previous MCA approaches to approximate a solution. Another approach could be a heuristic algorithm by combining best response and search algorithms under a leveraged structure similar to the MCA approach for models with discrete strategy spaces.

Conclusion and Contribution

This thesis has the potential to contribute to both computer science and economics. The computer science contribution lies chiefly in the development of automatic and efficient algorithms for solving sequential auction games. The economic contribution is the theoretic analysis of the existence of equilibrium in sequential auctions and the relevance of MCAs to PBE for a class of sequential auction models.

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