

# A Model-based Approach for Merging Prioritized Knowledge Bases in Possibilistic Logic

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## Abstract

This paper presents a new approach for merging prioritized knowledge bases in possibilistic logic. Our approach is semantically defined by a model-based merging operator in propositional logic and the merged result of our approach is a *normal possibility distribution*. We also give an algorithm to obtain the syntactical counterpart of the semantic approach. The logical properties of our approach are considered. Finally, we analyze the computational complexity of our merging approach.

## Introduction

Logic-based merging approaches have received much attention in many computer science fields, such as distributed databases, multi-agent systems, and requirements engineering. In a logical framework, each information source is often considered as a knowledge base, which is a set of logical formulas. One of the key issues for merging multiple knowledge bases is to deal with logical inconsistency. Even though each knowledge base is consistent, by putting them together may give rise to logical contradiction. In propositional logic, it is often required that inconsistency be resolved after merging. There are mainly two families of merging operators in propositional logic: the model-based ones and the formulas-based ones. The former is defined by a binary relation on the set of interpretations and the latter is based on the selection of some preferred consistent subsets of the union of the initial knowledge bases.

It has been widely accepted that priorities play an important role for resolving inconsistency when multiple knowledge bases are merged. Possibilistic logic (Dubois, Lang, & Prade 1994) provides a flexible framework to represent priority information and to deal with inconsistency. At the syntactic level, it is a weighted logic which attaches to each formula with a weight belonging to a totally ordered scale, such as  $[0, 1]$ , where the weight is interpreted as the certainty level of the formula. A *possibilistic knowledge base* is a set of weighted formulas. At the semantic level, it is based on the notion of a *possibility distribution*, which is a mapping from the set of interpretations  $\Omega$  to interval  $[0,1]$ . For each possibilistic knowledge base, there is a unique possibility distribution associated with it.

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Many merging approaches have been proposed in possibilistic logic (Benferhat, Dubois, & Prade 1998; Benferhat et. al. 1999; Benferhat et. al. 2002; Benferhat & Kaci 2003; Qi, Liu, & Glass 2004; Amgoud & Kaci 2005). In (Benferhat, Dubois, & Prade 1998; Benferhat et. al. 1999; Benferhat et. al. 2002; Benferhat & Kaci 2003), given several possibilistic knowledge bases, a semantic combination rule (or merging operator) is applied to aggregate the possibility distributions associated with them into a new possibility distribution. Then the syntactical counterpart of the semantic merging of the possibility distributions is a possibilistic knowledge base whose associated possibility distribution is the new possibility distribution (Benferhat et. al. 2002). Unlike the propositional merging operators, the possibilistic merging operators may result in a knowledge base which is inconsistent. Then the possibilistic consequence relation, which is inconsistency tolerant, is applied to infer nontrivial conclusions from the resulting knowledge base. The problem for these merging operators is that too much information may be lost after merging because the possibilistic consequence suffers from the *drowning problem* (Benferhat, Dubois, & Prade 1998).

Several approaches (such as those in (Qi, Liu, & Glass 2004; Amgoud & Kaci 2005)) have been proposed to improve the semantic combination rules. The approach given in (Qi, Liu, & Glass 2004) weakens the conflicting information to restore consistency. However, it is not advisable to merge possibilistic knowledge bases which are *strongly in conflict*, i.e., the *inconsistency degree* of their union is high. The approach proposed in (Amgoud & Kaci 2005) is based on argumentation. The problem for this approach is that it is computationally hard to build argumentation from an inconsistent possibilistic knowledge base.

This paper presents a new approach to merging prioritized knowledge bases in possibilistic logic. Our approach is semantically defined by a model-based merging operator in propositional logic. The result of merging is still a possibility distribution. The syntactical counterpart of our merging approaches is also given. We show that our approach captures some kind of minimal change and has good logical properties. The complexity of our merging operator is analyzed.

The rest of this paper proceeds as follows. We give some *Preliminaries* and introduce the *Model-based Merging Op-*

erators in propositional logic in the next two sections. After that, we define our *Model-based Merging Approach* in possibilistic logic. The syntactic counterpart of the approach is given. We then consider the *Properties* of our approach. We finally discuss *Related Work* and provide *Conclusion*.

## Preliminaries

### Classical logic

We consider a propositional language  $\mathcal{L}_{PS}$  over a finite set  $PS$  of propositional symbols. The classical consequence relation is denoted as  $\vdash$ . An interpretation is a total function from  $PS$  to  $\{\top, \perp\}$ , where  $\top$  is *true* and  $\perp$  is *false*.  $\Omega$  is the set of all possible interpretations. An interpretation  $\omega$  is a model of a formula  $\phi$  iff  $\omega(\phi) = \top$ . For each formula  $\phi$ , we use  $Mod(\phi)$  to denote its set of models. Let  $M$  be a set of interpretations,  $form(M)$  denotes the logical formula (unique up to logical equivalence) whose models are  $M$ . A (classical) *knowledge base*  $B$  is a finite set of propositional formulas which can be identified with the conjunction of its elements.  $K$  is consistent iff  $Mod(K) \neq \emptyset$ . Two knowledge bases  $K_1$  and  $K_2$  are equivalent, denoted  $K_1 \equiv K_2$ , iff  $Mod(K_1) = Mod(K_2)$ . A *knowledge profile*  $E$  is a multi-set of knowledge bases, i.e. it may contain a knowledge base twice. The sets of knowledge bases and knowledge profiles are denoted by  $\mathcal{B}$  and  $\mathcal{S}$  respectively.

### Possibilistic logic

We give a brief refresher on possibilistic logic (more details can be found in (Dubois, Lang, & Prade 1994)).

The semantics of possibilistic logic is based on the notion of a *possibility distribution*  $\pi$  which is a mapping from the set of interpretations  $\Omega$  to interval  $[0,1]$ . The possibility degree  $\pi(\omega)$  represents the degree of compatibility (resp. satisfaction) of the interpretation  $\omega$  with the available beliefs about the real world. A possibility distribution is said to be *normal* if  $\exists \omega_0 \in \Omega$ , such that  $\pi(\omega_0) = 1$ . From a *possibility distribution*  $\pi$ , two measures can be determined: the possibility degree of formula  $\phi$ ,  $\Pi_\pi(\phi) = \max\{\pi(\omega) : \omega \in \Omega, \omega \models \phi\}$  and the necessity degree of formula  $\phi$ ,  $N_\pi(\phi) = 1 - \Pi_\pi(\neg\phi)$ . The former evaluates to what extent  $\phi$  is consistent with knowledge expressed by  $\pi$  and the latter evaluates to what extent  $\phi$  is entailed by the available knowledge.

At the syntactic level, a formula, called a *possibilistic formula*, is represented by a pair  $(\phi, a)$ , where  $\phi$  is a propositional formula and  $a$  is an element of the semi-open real interval  $(0, 1]$  or of a finite total ordered scale, which means that the necessity degree of  $\phi$  is at least equal to  $a$ , i.e.  $N(\phi) \geq a$ . Then uncertain or prioritized pieces of information can be represented by a *possibilistic knowledge base* which is a finite set of possibilistic formulas of the form  $B = \{(\phi_i, a_i) : i = 1, \dots, n\}$ . The classical base associated with  $B$  is denoted as  $B^*$ , namely  $B^* = \{\phi_i | (\phi_i, a_i) \in B\}$ . A possibilistic knowledge base  $B$  is consistent if and only if its classical base  $B^*$  is consistent. A *possibilistic knowledge profile*  $\mathcal{E}$  is a multi-set of possibilistic knowledge bases.

Given a possibilistic knowledge base  $B$ , a unique *possibility distribution*, denoted by  $\pi_B$ , can be obtained by the

principle of minimum specificity (Dubois, Lang, & Prade 1994). For all  $\omega \in \Omega$ ,

$$\pi_B(\omega) = \begin{cases} 1 & \text{if } \forall (\phi_i, a_i) \in B, \omega \models \phi_i, \\ 1 - \max\{a_i | \omega \not\models \phi_i, (\phi_i, a_i) \in B\} & \text{otherwise.} \end{cases} \quad (1)$$

**Definition 1.** Let  $B$  be a possibilistic knowledge base, and  $a \in [0, 1]$ . The  $a$ -cut (resp. strict  $a$ -cut) of  $B$  is  $B_{\geq a} = \{\phi_i \in B^* | (\phi_i, b_i) \in B \text{ and } b_i \geq a\}$  (resp.  $B_{>a} = \{\phi_i \in B^* | (\phi_i, b_i) \in B \text{ and } b_i > a\}$ ).

**Definition 2.** Let  $B$  be a possibilistic knowledge base. The *inconsistency degree* of  $B$  is:

$$Inc(B) = \max\{a_i : B_{\geq a_i} \text{ is inconsistent}\}.$$

The inconsistency degree of  $B$  is the largest weight  $a_i$  such that the  $a_i$ -cut of  $B$  is inconsistent.

**Definition 3.** Let  $B$  be a possibilistic base. A formula  $\phi$  is said to be a consequence of  $B$  to a degree  $a$ , denoted by  $B \vdash_\pi (\phi, a)$ , iff (i)  $B_{\geq a}$  is consistent; (ii)  $B_{\geq a} \vdash \phi$ ; (iii)  $\forall b > a, B_{\geq b} \not\vdash \phi$ .

According to Definition 3, an inconsistent possibilistic knowledge base can non-trivially infer conclusion, so it is inconsistency tolerant. However, it suffers from the “drowning problem” (Benferhat, Dubois, & Prade 1998). That is, given an inconsistent possibilistic knowledge base  $B$ , formulas whose certainty degrees are not larger than  $Inc(B)$  are completely useless for nontrivial deductions. For instance, let  $B = \{(p, 0.9), (\neg p, 0.8), (r, 0.6), (q, 0.7)\}$ , it is clear that  $B$  is equivalent to  $B = \{(p, 0.9), (\neg p, 0.8)\}$  because  $Inc(B) = 0.8$ . So  $(q, 0.7)$  and  $(r, 0.6)$  are not used in the possibilistic inference.

Two possibilistic knowledge bases  $B$  and  $B'$  are said to be equivalent, denoted by  $B \equiv_s B'$ , iff  $\forall a \in (0, 1], B_{\geq a} \equiv B'_{\geq a}$ . Two possibilistic knowledge profiles  $\mathcal{E}_1$  and  $\mathcal{E}_2$  are said to be equivalent ( $\mathcal{E}_1 \equiv_s \mathcal{E}_2$ ) if and only if there is a bijection between them such that each possibilistic knowledge base of  $\mathcal{E}_1$  is equivalent to its image in  $\mathcal{E}_2$ .

Many operators have been proposed for merging possibilistic knowledge bases. Given a possibilistic knowledge profile  $\{B_1, \dots, B_n\}$  with possibility distributions  $\{\pi_{B_1}, \dots, \pi_{B_n}\}$ , the semantic combination of the possibility distributions by an aggregation function  $\oplus$  (such as the maximum and the minimum) results in a new possibility distribution  $\pi_\oplus(\omega) = (\dots((\pi_1(\omega) \oplus \pi_2(\omega)) \oplus \pi_3(\omega)) \oplus \dots) \oplus \pi_n(\omega)$ . The possibilistic knowledge base associated with  $\pi_\oplus$  is then computed by the following equation (Benferhat et. al. 2002):

$$B_\oplus = \{(D_j, 1 - \oplus(x_1, \dots, x_n)) : j = 1, \dots, n\}, \quad (2)$$

where  $D_j$  are disjunctions of size  $j$  between formulas taken from different  $B_i$ 's ( $i = 1, \dots, n$ ) and  $x_i$  is either equal to  $1 - a_i$  or 1 depending respectively on whether  $\phi_i$  belongs to  $D_j$  or not.

## Model-based merging operators

A merging operator is a function  $\Delta: \mathcal{S} \times \mathcal{B} \rightarrow \mathcal{B}$  in which the first argument is a knowledge profile and the second one

is seen as the integrity constraints. To make the notation simple, we write  $\Delta_\mu(E)$  instead of  $\Delta(E, \mu)$ .

In the following, we introduce the model-based merging operators which is defined by a binary relation  $\leq_E$  (usually  $\leq_E$  is required to be a total pre-order) on the set of interpretations  $\Omega$  as follows:

$$Mod(\Delta_\mu(E)) = \min(Mod(\mu), \leq_E), \quad (3)$$

where  $E = \{K_1, \dots, K_n\}$  is a knowledge profile.

The binary relation  $\leq_E$  can be compactly derived from a pseudo-distance between interpretations and an aggregation function  $*$  (such as the maximum) by the following steps:

1. Compute the local distance  $d(\omega, K_i)$  between a model  $\omega$  of  $\mu$  and each base  $K_i$ :  $d(\omega, K_i) = \min_{\omega' \models K_i} d(\omega, \omega')$ .
2. Compute the distance between a model  $\omega$  of  $\mu$  and knowledge profile using  $*$ :  $d(\omega, E) = *_{K_i \in E} d(\omega, K_i)$ .
3. A total pre-order  $\leq_E$  is defined as follows:  $\omega \leq_E \omega'$  if and only if  $d(\omega, E) \leq d(\omega', E)$ .

The commonly used distance is the Hamming distance between interpretations, i.e.,  $d(\omega, \omega') = |p : \omega(p) \neq \omega'(p)|$ , and the examples of aggregation functions are the *sum*, the *max*, and the *leximax*. The merging operators obtained by the sum, the max and the leximax are called majority operator  $\Delta^\Sigma$ , max operator  $\Delta^{max}$  and generalized max (or Gmax) operator  $\Delta^{Gmax}$  respectively (Konieczny & Pérez 2002).

In (Benferhat et. al. 2002), the syntactic counterparts of some model-based merging operators are provided by encoding them in the possibilistic logic framework. Because of page limit, we only briefly describe the encoding process, and refer to (Benferhat et. al. 2002) for more details. Given a knowledge profile  $E = \{K_1, \dots, K_n\}$  and integrity constraints  $\mu$ , firstly, each  $K_i$  is associated to a possibilistic knowledge base  $B_{K_i}$  such that the possibility distribution  $\pi_{B_{K_i}}$  is a function of the Hamming distance associated with  $K_i$ .  $\mu$  is associated to a possibilistic knowledge base  $B_\mu = \{(\phi_i, 1) : \phi_i \in \mu\}$ . Secondly, for a distance-based merging operator  $\Delta$  applied to  $E$  and  $\mu$  which results in the knowledge base  $\Delta_\mu(E)$ , there is a possibilistic merging operator  $\oplus$  applied to  $\mathcal{E} = \{B_{K_1}, \dots, B_{K_n}\}$  and yielding a possibilistic knowledge base  $\mathcal{E}_\oplus$  such that the set of preferred models in  $\pi_{\mathcal{E}_\oplus \cup B_\mu}$  is equal to  $Mod(\Delta_\mu(E))$ .

## A Model-based Merging Approach in Possibilistic Logic

### Model-based approach

Given a possibilistic knowledge profile  $\mathcal{E} = \{B_1, \dots, B_n\}$  and the integrity constraints  $\mu$ , we propose a model-based approach for obtaining a normal possibility distribution.

Suppose  $B = B_1 \cup \dots \cup B_n = \{(\psi_1, b_1), \dots, (\psi_m, b_m)\}$ . Rearrange the weights of formulas in  $B$  such that  $a_1 > a_2 > \dots > a_l > 0$ , where  $a_i$  ( $i = 1, \dots, l$ ) are all the distinct weights appearing in  $B$ .

Our approach is given by the following algorithm.

#### Algorithm 1

Data: a possibilistic knowledge profile  $\mathcal{E} = \{B_1, \dots, B_n\}$ ; a knowledge base  $\mu$ ; a model-based merging operator  $\Delta$ .

Result: a possibility distribution

**begin**

1.  $IC_1 \leftarrow Mod(\mu), i=1$ ;
2. **for each**  $\omega \not\models \mu, \pi_{\mathcal{E}, \mu, \Delta}(\omega) = 0$ ;
3. **while**  $i \leq l$  **do**
4.  $(B_k)^i = \{\phi_{kj} : (\phi_{kj}, a_{kj}) \in B_k, a_{kj} = a_i\}$ , for  $k = 1, \dots, n$ , and  $B^i = \{(B_1)^i, \dots, (B_n)^i\}$ ;
5.  $IC_{i+1} \leftarrow Mod(\Delta_{form}(IC_i)(B^i))$ ;
6. **for each**  $\omega \in IC_i \setminus IC_{i+1}, \pi_{\mathcal{E}, \mu, \Delta}(\omega) = 1 - a_i$ ;
7.  $i = i + 1$ ;
8. **end-while**
9. **for**  $\omega \in IC_{l+1}, \pi_{\mathcal{E}, \mu, \Delta}(\omega) = 1$ ;
10. **return**  $\pi_{\mathcal{E}, \mu, \Delta}$

**end**

In Algorithm 1, we merge the possibility distributions associated with possibilistic knowledge bases by iteratively using the model-based merging operator  $\Delta$ . Initially, the possibility degrees of those interpretations which do not satisfy the integrity constraints  $\mu$  are assigned to 0 (Step 2). In the first while loop, suppose  $(B_k)^i$  is the set of formulas in  $B_k$  whose weights are  $a_i$ , then the merging operator  $\Delta$  is applied to merge  $(B_k)^1, \dots, (B_k)^n$  under constraints  $form(IC_i)$ . The set of models of the integrity constraints is replaced by its models (Step 5). For those interpretations which satisfy the original integrity constraints and falsify the newly obtained integrity constraints, their possibility degrees are assigned to  $1 - a_i$  (Step 6). We then go to another while loop. After the while loop ends, the possibility degrees of those interpretations which satisfy the integrity constraints  $IC_{l+1}$  are assigned to 1.

*Example 4* Let  $\mathcal{E} = \{B_1, B_2\}$  be a possibilistic knowledge profile and  $\mu$  be a knowledge base, where  $B_1 = \{(\neg p \vee q, 1), (p \vee r, 0.8)\}$ ,  $B_2 = \{(p, 0.8), (r, 0.7)\}$  and  $\mu = \{\neg q\}$ . Let  $\Omega = \{\omega_1, \dots, \omega_8\}$ , where  $\omega_1 = 000, \omega_2 = 001, \omega_3 = 010, \omega_4 = 011, \omega_5 = 100, \omega_6 = 101, \omega_7 = 110, \omega_8 = 111$ . We apply Algorithm 1 to merging  $\mathcal{E}$  under constraints  $\mu$ . Suppose the majority operator  $\Delta^\Sigma$  is chosen. It is easy to check that  $Mod(\mu) = \{\omega_1, \omega_2, \omega_5, \omega_6\}$ , so  $\pi_{\mathcal{E}, \mu, \Delta}(\omega_3) = \pi_{\mathcal{E}, \mu, \Delta}(\omega_4) = \pi_{\mathcal{E}, \mu, \Delta}(\omega_7) = \pi_{\mathcal{E}, \mu, \Delta}(\omega_8) = 0$ .  $B = B_1 \cup B_2 = \{(\neg p \vee q, 1), (p \vee r, 0.8), (p, 0.8), (q, 0.7)\}$ , the weights of formulas in  $B$  are rearranged as  $a_1 > a_2 > a_3$ , where  $a_1 = 1, a_2 = 0.8, a_3 = 0.7$ . So  $l = 3$ . Let  $IC_1 = Mod(\mu)$ . Since  $a_1 = 1$ , we have  $(B_1)^1 = \{(\neg p \vee q, 1)\}$  and  $(B_2)^1 = \emptyset$  and  $B^1 = \{(B_1)^1, (B_2)^1\}$ . It is clear that  $(B_1)^1 \cup (B_2)^1 \cup \mu = \{\neg q, \neg p \vee q\}$  is consistent. By Theorem 4.2 in (Konieczny & Pérez 2002), we have  $\Delta_{form}^\Sigma(IC_1)(B^1) \equiv \{\neg q, \neg p \vee q\}$ . So  $Mod(\Delta_{form}^\Sigma(IC_1)(B^1)) = \{\omega_1, \omega_2\}$ . Let  $IC_2 = Mod(\Delta_{form}^\Sigma(IC_1)(B^1))$ . We then have  $\pi_{\mathcal{E}, \mu, \Delta}(\omega_5) = \pi_{\mathcal{E}, \mu, \Delta}(\omega_6) = 0$ . Let  $i = 2$ . Since  $a_2 = 0.8$ , we have  $(B_1)^2 = \{p \vee r\}$  and  $(B_2)^2 = \{p\}$ . Clearly,  $(B_1)^2 \cup (B_2)^2$  is inconsistent with  $form(IC_2)$ . We apply the majority operator  $\Delta^\Sigma$  to merge  $(B_1)^2$  and  $(B_2)^2$  under constraints  $IC_2$ . The calculation is summarized in Table 1. For each model of  $IC_2$ , we compute the distances between it and the two knowledge bases  $(B_1)^2$  and  $(B_2)^2$ . We also com-

$\omega$	$d(\omega, (B_1)^2)$	$d(\omega, (B_2)^2)$	$d(\omega, B^2)$
$\omega_1$	1	1	2
$\omega_2$	0	1	1

Table 1:  $\Delta^\Sigma$  operator

pute the distance between this model and the knowledge profile  $B^2$  (see column 4 of Table 1). According to Table 1, the model which has minimal distance with  $B^2$  is  $\omega_2$ . So  $Mod(\Delta_{form(IC_2)}^\Sigma(B^2)) = \{\omega_2\}$ . Let  $IC_3 = \{\omega_2\}$ . We then have  $\pi_{\mathcal{E},\mu,\Delta}(\omega_1) = 0.2$ . Let  $i = 3$ . Since  $a_3 = 0.7$ ,  $(B_1)^3 = \emptyset$  and  $(B_2)^3 = \{(r, 0.7)\}$ . Since  $\omega_2 \models r$ , By Theorem 4.2 in (Konieczny & Pérez 2002), we have  $Mod(\Delta_{form(IC_3)}^\Sigma(B^3)) = \{\omega_2\}$ . So  $\pi_{\mathcal{E},\mu,\Delta}(\omega_2) = 1$ . Therefore, we get the following possibility distribution by Algorithm 1:  $\pi_{\mathcal{E},\mu,\Delta}(\omega_1) = 0.2$ ,  $\pi_{\mathcal{E},\mu,\Delta}(\omega_2) = 1$ ,  $\pi_{\mathcal{E},\mu,\Delta}(\omega_i) = 0$  for  $i = 3, \dots, 8$ .  $\triangle$

We give a model-theoretic characterization of our merging approach. In each “while” loop, we use  $B^i$  to denote a knowledge profile consists of  $(B_k)^i$  ( $k = 1, \dots, n$ ), where some  $(B_k)^i$  may be empty sets. Suppose  $\Delta$  is defined by a distance function and an aggregation and  $\leq_{B^i}$  is the total pre-order associated with the operator  $\Delta$  (see Equation 3 for the definition of  $\Delta$ ). Let us define a lexicographic relation  $\leq_{lex,\mathcal{E}}$  as follows:

$\omega \leq_{lex,\mathcal{E}} \omega'$  if and only if  $\omega \leq_{B^i} \omega'$  for all  $i$  or  $\exists i$  such that  $\omega <_{B^i} \omega'$  and  $\omega =_{B_j} \omega'$  for all  $j < i$ .

We have the following proposition.

**Proposition 5** *Given a possibilistic knowledge profile  $\mathcal{E} = \{B_1, \dots, B_n\}$  and the integrity constraints  $\mu$ , suppose  $\pi_{\mathcal{E},\mu,\Delta}$  is the possibility distribution obtained by Algorithm 1. Let  $M = \{\omega : \pi_{\mathcal{E},\mu,\Delta}(\omega) = 1\}$ . Then*

$$M = \min(Mod(\mu), \leq_{lex,\mathcal{E}}).$$

**Proof sketch:** By Algorithm 1,  $M = IC_{l+1}$ , where  $l$  is the index of the smallest weight of formulas in  $B_1 \cup \dots \cup B_n$ . Clearly, we have  $IC_{l+1} \subseteq IC_i$  for all  $i < l + 1$ . Therefore,  $\omega \in M$  iff  $\omega \in IC_i$  for all  $i \leq l + 1$ . According to the definition of model-based operator  $\Delta$ ,  $\omega \in IC_i$  for all  $i \leq l + 1$  iff  $\omega \in Mod(\mu)$  and it is minimal w.r.t  $\leq_{B^i}$  for all  $i < l + 1$ .  $\square$

Proposition 5 shows that the set of models of our merging operator consists of the models of  $\mu$  which is minimal with respect to the lexicographic relation  $\mathcal{E}$ . This is a very nice property which is not shared by most of merging operators in possibilistic logic.

### Syntactical counterpart of model-based approach

Given a possibilistic knowledge profile  $\mathcal{E}$  and integrity constraints  $\mu$ , our model-based approach results in a possibility distribution  $\pi_{\mathcal{E},\mu,\Delta}$ . In practice, we are more interested in the syntactic form of the merging result. Suppose the model-based merging operator  $\Delta$  in Algorithm 1 has the syntactic counterpart, we then have the following algorithm to obtain the possibilistic knowledge base whose associated possibility distribution is  $\pi_{\mathcal{E},\mu,\Delta}$ .

#### Algorithm 2

Input: a possibilistic knowledge profile  $\mathcal{E} = \{B_1, \dots, B_n\}$ ; integrity constraints  $\mu$ ; a model-based merging operator  $\Delta$ .  
Result: a possibilistic knowledge base  $B_{\mathcal{E},\mu,\Delta}$

**begin**

1.  $\mu_1 \leftarrow \mu, i=1$ ;
2.  $B_{\mathcal{E},\mu,\Delta} \leftarrow \{(\phi, 1) : \phi \in \mu_1\}$ ;
3. **while**  $i \leq l$  **do**
4.  $(B_k)^i = \{\phi_{kj} : (\phi_{kj}, a_{kj}) \in B_k, a_{kj} = a_i\}^1$ , for  $i = 1, \dots, n$ , and  $B^i = \{(B_1)^i, \dots, (B_n)^i\}$ ;
5.  $\mu_{i+1} \leftarrow \Delta_{\mu_i}((B_1)^k, \dots, (B_n)^k)$ ;
6.  $B_{\mathcal{E},\mu,\Delta} \leftarrow B_{\mathcal{E},\mu,\Delta} \cup \{(\phi, a_i) : \phi \in \mu_{i+1} \setminus \mu_i\}$ ;
7.  $i = i + 1$ ;
8. **end-while**
9. **return**  $B_{\mathcal{E},\mu,\Delta}$ ;

**end**

The difference between Algorithm 1 and Algorithm 2 is that the resulting knowledge base of the merging operator  $\Delta$  is a knowledge base in Algorithm 2 and a weigh is attached to each formula in the resulting knowledge base.

*Example 6* (Example 4 Continued) Let  $\mu_1 = \{\neg q\}$  and  $B_{\mathcal{E},\mu,\Delta} = \{(\neg q, 1)\}$ . Let  $i = 1$ . Since  $a_1=1$ , we have  $(B_1)^1 = \{\neg p \vee q\}$  and  $(B_2)^1 = \emptyset$ . Let  $B^1 = \{(B_1)^1, (B_2)^1\}$ . It is clear that  $(B_1)^1$  and  $\mu_1$  are consistent. So  $\Delta_{\mu_1}^\Sigma(B^1) = \{\neg q, \neg p \vee q\}$ . Then,  $\mu_2 = \{\neg p \vee q, \neg q\}$  and  $B_{\mathcal{E},\mu,\Delta} = \{(\neg p \vee q, 1), (\neg q, 1)\}$ . Let  $i = 2$ . Since  $a_2 = 0.8$ , we have  $(B_1)^2 = \{p \vee r, 0.8\}$  and  $(B_2)^2 = \{p, 0.8\}$ . Let  $B^2 = \{(B_1)^2, (B_2)^2\}$ . We apply the syntactical counterpart of  $\Delta^\Sigma$  to merge  $(B_1)^2$  and  $(B_2)^2$  under constraints  $\mu_2$ . First, by Lemma 3 and Proposition 13 in (Benferhat et. al. 2002), the possibilistic knowledge bases associated with  $(B_1)^2$  and  $(B_2)^2$  are  $B'_1 = \{(p \vee r, 1 - \epsilon)\}$  and  $B'_2 = \{(p, 1 - \epsilon)\}$  respectively, where  $\epsilon \in (0, 1)$  is a very small number. By Proposition 14 and Proposition 17 in (Benferhat et. al. 2002),  $\Delta_{\mu_2}(B^2) = \{\neg p \vee q, \neg q, p \vee r\}$ . Let  $\mu_3 = \{\neg p \vee q, \neg q, p \vee r\}$  and  $B_{\mathcal{E},\mu,\Delta} = \{(\neg p \vee q, 1), (\neg q, 1), (p \vee r, 0.8)\}$ . Let  $i = 3$ . Since  $a_3 = 0.7$ , we have  $(B_1)^3 = \emptyset$  and  $(B_2)^3 = \{r\}$ . Let  $B^3 = \{(B_2)^3\}$ . It is clear that  $(B_2)^3$  and  $\mu_3$  are consistent, so  $\Delta_{\mu_3}^\Sigma(B^3) = \{\neg p \vee q, \neg q, p \vee r, r\}$ . So  $B_{\mathcal{E},\mu,\Delta} = \{(\neg p \vee q, 1), (\neg q, 1), (p \vee r, 0.8), (r, 0.7)\}$ , which is equivalent to  $\{(\neg p, 1), (\neg q, 1), (p \vee r, 0.8)\}$ .  $\triangle$

It is easy to check that  $\pi_{B_{\mathcal{E},\mu,\Delta}}(\omega_i) = \pi_{\mathcal{E},\mu,\Delta}(\omega_i)$  for all  $i = 1, \dots, 8$ , where  $B_{\mathcal{E},\mu,\Delta}$  is the possibilistic knowledge base obtained by Algorithm 2 in Example 6 and  $\pi_{\mathcal{E},\mu,\Delta}$  is the possibility distribution obtained by Algorithm 1 in Example 4. More generally, we have the following propositions which establish the correspondence between the semantic and syntactic approaches.

**Proposition 7** *For each  $i$ , suppose  $IC_{i+1}$  is the set of models of the resulting knowledge base in the  $i$  round of “while” loop in Algorithm 1 and  $\mu_{i+1}$  is the resulting knowledge base in the  $i$  round of “while” loop in Algorithm 2, then  $IC_{i+1} = Mod(\mu_{i+1})$ .*

The proof of Proposition 7 is easy to show. Based on Proposition 7, we have the following important proposition

<sup>1</sup>Recall that  $a_i$  ( $i = 1, \dots, l$ ) are rearranged weights of formulas in  $B_1 \cup \dots \cup B_n$  (see Section 4.1).

that establishes the relationship between Algorithm 1 and Algorithm 2.

**Proposition 8** Given a possibilistic knowledge profile  $\mathcal{E} = \{B_1, \dots, B_n\}$  and the integrity constraints  $\mu$ , suppose  $\pi_{\mathcal{E}, \mu, \Delta}$  is the possibility distribution obtained by Algorithm 1 and  $B_{\mathcal{E}, \mu, \Delta}$  is the possibilistic knowledge base obtained by Algorithm 2, then  $\pi_{B_{\mathcal{E}, \mu, \Delta}}(\omega) = \pi_{\mathcal{E}, \mu, \Delta}(\omega)$  for all  $\omega \in \Omega$ .

**Proof sketch:** We prove the proposition by induction over the index  $i$  of each “while” loop. Suppose  $i = 0$ , by Definition 1, we have  $\pi_{B_{\mathcal{E}, \mu, \Delta}}(\omega_i) = 0$  for all  $\omega_i \not\models \mu$ . Suppose Proposition 8 holds when  $i = k$  ( $k < l$ ), by Proposition 7, if  $\omega \in IC_i$  and  $\omega \notin IC_{i+1}$ , then  $\omega \not\models \mu_{i+1}$  and  $\omega \models \mu_i$ . So  $\pi_{B_{\mathcal{E}, \mu, \Delta}}(\omega) = 1 - a_i = \pi_{\mathcal{E}, \mu, \Delta}(\omega)$ .  $\square$

Proposition 8 shows that the possibility distribution obtained by Algorithm 1 is the possibility distribution associated with the possibilistic knowledge base obtained by Algorithm 2. That is, Algorithm 2 is the syntactical counterpart of Algorithm 1.

## Properties

### Logical properties

In propositional logic, a set of logical properties (or rationality postulates) is often proposed to characterize merging operators. In possibilistic logic, however, such a set of logical properties does not exist. In this section, we consider the logical properties of our merging approach.

**Proposition 9** Given possibilistic knowledge profiles  $\mathcal{E}$ ,  $\mathcal{E}_1$ ,  $\mathcal{E}_2$ , and the integrity constraints  $\mu$  and  $\mu'$ , we then have

(P1)  $(B_{\mathcal{E}, \mu, \Delta})^* \vdash \mu$ .

(P2) If  $\mu$  is consistent, then  $B_{\mathcal{E}, \mu, \Delta}$  is consistent.

(P3) If  $\mathcal{E}_1 \equiv_s \mathcal{E}_2$  and  $\mu \equiv \mu'$ , then  $B_{\mathcal{E}_1, \mu, \Delta} \equiv_s B_{\mathcal{E}_2, \mu', \Delta}$ .

(P4) If  $(\bigwedge(\mathcal{E}))^* \wedge \mu$  is consistent, then  $B_{\mathcal{E}, \mu, \Delta} \equiv_{\pi} B_1 \cup \dots \cup B_n \cup \{(\phi_i, 1) : \phi_i \in \mu\}$ .

In Proposition 9, (P1) assures that the integrity constraints are satisfied after merging. (P2) states that the result of merging is consistent if the integrity constraints are consistent. (P3) is the principle of syntax independence. (P4) says that if the possibilistic knowledge profile is consistent with  $\mu$ , then the resulting possibilistic knowledge base is the union of original knowledge bases and the possibilistic knowledge base obtained by attaching weight 1 to every formula in  $\mu$ . (P4) is adapted from the postulate (IC7) in (Konieczny & Pérez 2002).

By (P4) in Proposition 9, we have the following corollary.

**Corollary 1** Let  $\mathcal{E} = \{B_1, \dots, B_n\}$  be a possibilistic knowledge profile and  $\mu = \top$ . Let  $B = \{(\top, 1)\} \cup B_1 \cup \dots \cup B_n$ . Suppose  $\text{Inc}(B) = a$ , let  $B^{\geq a} = \{(\phi, b) \in B : b \geq a\}$ , then

$$B_{\mathcal{E}, \mu, \Delta} \vdash_{\pi} (\phi, b), \quad \forall (\phi, b) \in B^{\geq a}.$$

$B^{\geq a}$  is the resulting knowledge base of the minimum-based normalized conjunctive operator (Benferhat et. al. 2002). Corollary 1 tells that our merging approach infers more conclusions than the minimum-based normalized conjunctive operator.

**Proposition 10** Let  $\mathcal{E} = \{B_1, B_2\}$  be a possibilistic knowledge profile, where  $B_1 = \{(\phi, a)\}$  and  $B_2 = \{(\neg\phi, a), (\phi, b)\}$  ( $b < a$ ), and  $\mu = \top$ . Let  $\Delta = \Delta^{\text{max}}$  or  $\Delta^{\Sigma}$ . Then  $B_{\mathcal{E}, \mu, \Delta} \vdash_{\pi} (\phi, b)$ .

The proof of Proposition 10 is clear by considering the logical properties of  $\Delta^{\text{max}}$  operator and  $\Delta^{\Sigma}$  operator (both satisfies (IC4) in (Konieczny & Pérez 2002)). Proposition 10 shows that our merging operator has reinforcement effect. That is,  $(\phi, a)$  in  $B_1$  and  $(\phi, b)$  in  $B_2$  together make  $\phi$  win over  $\neg\phi$  which has degree  $a$ .

### Computational complexity

We consider the following decision problem  $\text{MERGE}(B_{\mathcal{E}, \mu, \Delta})$ :

- **Input:** a four-tuple  $\langle \mathcal{E}, \mu, (\phi, a), \Delta \rangle$ , where  $\mathcal{E} = \{B_1, \dots, B_n\}$  is a possibilistic knowledge profile,  $\mu$  is a knowledge base,  $\phi$  is a formula and  $a \in (0, 1]$ , and  $\Delta$  is a model-based merging operator.
- **Question:** Does  $B_{\mathcal{E}, \mu, \Delta} \vdash_{\pi} (\phi, a)$  holds?

To answer if a possibilistic formula  $(\phi, a)$  is a possibilistic consequence of  $B_{\mathcal{E}, \mu, \Delta}$ , it is not necessary to compute  $B_{\mathcal{E}, \mu, \Delta}$ . According to Definition 3, we need to check if  $\phi$  is inferred from  $(B_{\mathcal{E}, \mu, \Delta})_{\geq a}$ . By Proposition 7, we only need to add an extra step to Algorithm 1 (between Step 4 and Step 5) to check if  $\Delta_{\text{form}(IC_i)}(B^i) \vdash \phi$ , where  $a_i \geq a$ . The sub-problem of  $\text{MERGE}(B_{\mathcal{E}, \mu, \Delta})$  is the decision problem  $\text{MERGE}(\Delta)$  (Konieczny, Lang, & Marquis 2004):

- **Input:** a triple  $\langle E, \mu, \phi \rangle$ , where  $E$  is a knowledge profile,  $\mu$  a knowledge base and  $\phi$  a formula.
- **Question:** Does  $\Delta_{\mu}(E) \vdash \phi$  holds?

It is clear that we have the following proposition.

**Proposition 11** Let  $X$  be the computational complexity of  $\text{MERGE}(\Delta)$ . Let  $a_{k-1} < a \leq a_k$ . Then  $\text{MERGE}(B_{\mathcal{E}, \mu, \Delta})$  is in  $k \cdot X$ , where  $k \cdot X$  means that to solve the decision problem  $\text{MERGE}(B_{\mathcal{E}, \mu, \Delta})$ , we need to call the algorithm for  $\text{MERGE}(\Delta)$  (which has the complexity  $X$ )  $k$  times.

Proposition 11 shows that the complexity of our merging approach is not much harder than that of the corresponding model-based merging operator, i.e. the levels of polynomial hierarchy in which they are located are close. More specifically, we have the following corollary.

**Corollary 2** Let  $\Delta = \Delta^{\text{max}}$  or  $\Delta^{\Sigma}$ , and  $a_{k-1} < a \leq a_k$ . Then  $\text{MERGE}(B_{\mathcal{E}, \mu, \Delta})$  is in  $k \cdot \Theta_2^p$ , where  $\Theta_2^p$  is the class of all languages that can be recognized in polynomial time by a deterministic Turing machine using a number of calls to an NP oracle bounded by a logarithmic function of the size of the input data.

The proof of Corollary 2 is clear according to Proposition 4 in (Konieczny, Lang, & Marquis 2004).

## Related Work

The main difference between our merging approach and existing possibilistic merging approaches discussed in *Introduction* section is that our approach is semantically defined

by iteratively applying a model-based merging operator in propositional logic, whilst existing approaches are semantically defined by aggregating possibility distributions. The advantage of our approach is that it captures some kind of minimal change (see Proposition 5) and at the same time its computational complexity is not very hard (see Proposition 11 and Corollary 2). This work is also related to the approaches for iterated revision and prioritized merging in (Benferhat et. al. 2004; Qi, Liu, & Bell 2005; Delgrande, Dubois, & Lang 2006). In (Benferhat et. al. 2004), the disjunctive maxi-adjustment (DMA) approach is proposed for weakening conflicting information in a stratified propositional knowledge base of the form  $\Sigma = \{K_1, \dots, K_n\}$ , where  $K_i$  ( $i = 1, \dots, n$ ) is a stratum containing propositional formulae of  $\Sigma$  having the same level of priority such that each formula in  $K_i$  is more reliable than formulae of the stratum  $K_j$  for  $j > i$ . This approach is a special case of a revision-based approach given in (Qi, Liu, & Bell 2005). In (Delgrande, Dubois, & Lang 2006), an interesting approach is proposed to merge knowledge bases which are attached with priority levels. Like the revision-based approach, this approach starts resolving inconsistency in the knowledge base with the highest priority level. The resulting knowledge base is considered as the integrity constraints and is used to resolve inconsistency in the knowledge base with lower priority level, and so on. That is, both approaches presume that information with higher priority level has precedence over information with less certainty level when dealing with inconsistency. The difference between the revision-based approach and the prioritized merging approach in (Delgrande, Dubois, & Lang 2006) is that the former deploys a revision operator to resolve inconsistency whilst the latter uses a model-based merging operator. Both the revision-based approach and the prioritized merging approach can be used to merge possibilistic knowledge bases. Given several possibilistic knowledge bases  $B_1, \dots, B_n$ , suppose  $B = B_1 \cup \dots \cup B_n$ , then we can obtain a stratified knowledge base  $\Sigma$  from  $B$ . After that, we can extract a consistent knowledge base from  $\Sigma$  using the revision-based approach or the prioritized merging approach. Our approach is related to these approaches in that we presume that information with higher certainty level has precedence over information with less certainty level. However, our approach, when applied to merge possibilistic knowledge bases, results in a possibilistic knowledge. Whilst both the revision-based approach and the prioritized merging approach results in a classical knowledge base. Furthermore, we do not conjoin the original knowledge bases. Therefore, our approach takes into account the sources of information.

## Conclusion and Future Work

We have proposed a model-based approach to merging prioritized knowledge bases in possibilistic logic. The main contributions of this paper can be summarized as follows.

(1) We presented an algorithm to obtain a normal possibility distribution from a possibilistic profile using a model-based merging operator. The syntactical counterpart of the semantic merging approach was also considered.

(2) The models of the merged knowledge base are minimal models of the knowledge base representing the integrity constraints with respect to a total pre-order.

(3) We have shown that our merging approach is well-behaved with respect to the logical properties.

(4) We analyzed the computational complexity of querying over the resulting possibilistic knowledge base of our merging approach.

For future work, we are going to find a set of logical properties which best characterizes our merging approach.

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