Using the Genetic Algorithm to Locate Optimal Bi-Phase Waveforms for Pulse Compression Radar

Ross Wainwright
Air Force Research Laboratory
Space Vehicles Directorate
3550 Aberdeen Ave SE
Kirtland AFB, NM 87117
ross.wainwright@kirtland.af.mil

Abstract

The Air Force Research Laboratory is developing a space-based sparse-array aperture system, called TechSat 21. The TechSat 21 experiment is a cluster of three satellites that will fly in formation and operate cooperatively to perform the function of a larger, single satellite. The payload chosen to demonstrate the TechSat 21 concept is radar. Doppler and range ambiguities resulting from the relatively small antennas of each satellite represent significant challenges. The characteristics of transmitted waveforms must be optimized to resolve the anticipated ambiguities. Since the waveforms are bi-phase encoded and can be represented as a binary sequence, the Genetic Algorithm, that uses a binary sequence to represent a gene, was the obvious choice to search for optimal waveforms. The GA approach was first tested against a known standard. The goal was to find waveforms similar to known optimal waveforms. The GA search was then used to locate waveforms that could potentially support the TechSat 21 experiment. One concept to resolve the range and Doppler ambiguities is to transmit and process groups of waveforms. The GA was used to locate pairs of waveforms suitable for this concept.

Keywords

Pulse Compression Radar, Quasi-Orthogonal, Waveform, Genetic Algorithm

Introduction

The Air Force Research Laboratory (AFRL), Space Vehicles Directorate (VS), is developing a space-based, sparse-array aperture system, called TechSat 21 (AFRL 1998). A sparse-array aperture is a group of small antennae that are combined electronically to give the resolution of one large antenna. One well-know example is the Very Large Array (VLA) near Socorro, New Mexico (NRAO 2000). The TechSat 21 experiment moves the sparse-array into near-earth orbit. TechSat will be a cluster of three micro-satellites. They will fly in formation and operate cooperatively to perform the function of a larger, single satellite. The payload that has been selected to demonstrate the TechSat 21 concept is pulse compression radar.

The resolution of an antenna is determined in part by its beam width that is measured as an angle. The sparse array aperture formed by the cluster should have the same resolution as one large antenna that is the same size as the array. Each TechSat and each VLA antenna is small, however, and has a wide beam width. Radio astronomers have developed techniques to combine signals from the VLA antennae and achieve 0.04 arc-seconds resolution (NRAO 2000). TechSat is different because it is actively transmitting and receiving radar pulses. The wide beam width of each satellite causes Doppler and range ambiguities that must be addressed so that the cluster can achieve the theoretical resolution.

Range ambiguities occur when the returns of multiple pulses are received intermixed. The wide beam width translates into a large view area. The time difference between pulses received from far and near range is relatively large. The solution is a low pulse repetition frequency (PRF). Separate the pulse transmissions in time so that all the returns of one pulse are gathered before the next pulse is transmitted. On the other hand, pulses sample the velocity of a moving target. A high PRF is needed to avoid aliasing. To resolve this contradiction, the plan is to transmit groups of pulses. When the pulses are detected and processed in groups, the apparent PRF is low, and range ambiguities are minimized. When the pulses are detected and processed individually, the apparent PRF is high, and Doppler ambiguities are minimized. The challenge is to find acceptable groups of pulses.

Varying frequency or phase of the carrier in a pre-determined pattern controls the shape of the transmitted pulse. ('Pulse' and 'waveform' appear to have the same meaning to radar engineers. This paper will use 'waveform' from this point forward.) The current plan is to use bi-phase encoded waveforms. Bi-phase means that the carrier is phase-shifted using two phases, usually 0 and pi. The sequence of phase shifts can be written down as a binary code with 0 representing 0 phase and 1 representing pi phase. The sequence of phase shifts (also called chips) can be long. A recent airborne radar experiment required pairs of 21 chip waveforms. The search space is 2^{42} or 4,398,046,511,104 possible solutions.

Clearly an automated method is required to search through the space of possible waveforms. Using the Genetic Algorithm was obvious. A sequence of genes and a binary representation of phase shifts are identical.

This paper starts with a brief description of pulse compression radar including what are the criteria that are used to evaluate a waveform. Radar researchers have already identified optimal phase-coded waveforms up to forty chips long (Skolnik 1990). The GA has been used to duplicate portions of this work to establish that it is a valid technique.

The GA fitness function has been adapted to locate pairs of 'quasi-orthogonal' waveforms. These results are presented followed by concluding remarks.

Pulse Compression Radar

Pulse compression radar works by transmitting waveforms at a target region, and then receiving and processing the returned echoes. Pulse compression mathematically transforms relatively long waveforms in the returned signal into narrow impulses. The pulse-compressed data are then processed into the desired final product such as a synthetic aperture radar image. The advantage of pulse compression is that the radar system can transmit long-duration and low-power waveforms and realize the range resolution of short-duration and high-power impulses (Skolnik 1990).

For phase-encoded waveforms, pulse compression is a correlation process (equation 1). The returned signal is correlated with a copy of the transmitted pulse that is called a matched filter. The correlation peak indicates the location of an echoed pulse.

$$C_N = \sum_{i=-l}^{+l} U_i V_{i+N} \qquad (1)$$

Where: C is the cross-correlation at offset N
U is transmitted waveform (matched filter)
V is the returned signal

Radar engineers examine the plot (Figure 1) of a waveform's auto-correlation to determine the utility of the waveform for pulse compression. Good waveforms have one prominent central peak and minimal off-peak (or side-lobe) correlations. The highest side-lobe is called the peak side-lobe or PSL. The lower the PSL (equation 3) is, the easier it is to find the peak correlation, and the better the resolution of the processed data. Another parameter is the integrated side-lobe (ISL). ISL (equation 4) is the sum of the squares of the side-lobes divided by PSL squared (Skolnik 1990).

C_0 = correlation peak

C_l = correlation side-lobe at offset l (2)

$$PSL = 20 * \log 10 \left\{ \max \left[\frac{C_l}{C_0} \right], l \neq 0 \right\} \qquad (3)$$

$$ISL = 10 * \log 10 \sum_{l=-N}^{N} \left\{ \left[\frac{C_l}{C_0} \right]^2, l \neq 0 \right\} \qquad (4)$$

The theoretical minimum PSL for an N chip code is 1, or in dB, 20*log10 (1/N) (Skolnik 1990). The Matlab array in equation 5 is a thirteen bit or chip code. It is one of a special class of binary codes called Barker codes (Skolnik 1990). Barker codes achieve the theoretical minimum PSL and have been identified for codes up to thirteen chips.

X =[1, 1, 1, 1, 1, 0, 0, 1, 1, 0, 1, 0, 1]; (5)

The binary representation above is translated into phases in equation 6.

ϕ =[pi, pi, pi, pi, pi, 0, 0, pi, pi, 0, pi, 0, pi]; (6)

The waveform is computed as in equation 7:

$w_i = \cos(\phi_i),\ i = 1 \text{ to } 13.$ (7)

Linear and dB plots of the above Barker code's auto-correlation are in figures 1 and 2, respectively. For this rest of this paper, linear plots will be presented.

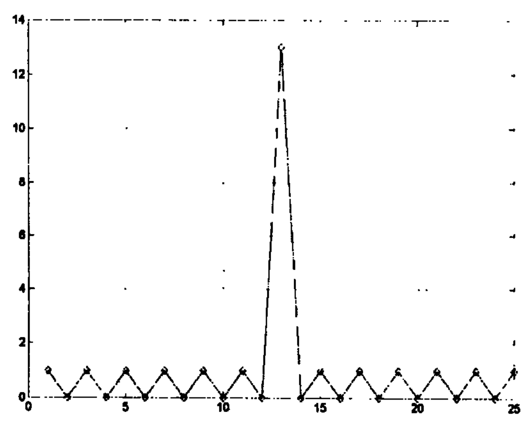

Figure 1. Auto-correlation of 13 Chip Barker Code (linear)

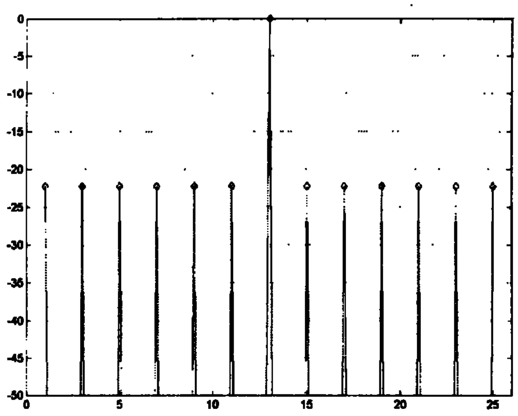

Figure 2. Auto-correlation of 13 Chip Barker Code (dB)

Thirteen chip codes are relatively easy to find. The search space has only 8192 possible combinations.

Evaluate Approach by Finding Single Bi-Phase Waveforms

Achieving the theoretical minimum PSL for codes longer then thirteen chips is nearly impossible. However, minimum PSLs have been identified for codes up to forty chips long. Codes that achieve the minimum PSL are considered optimal.

Equation 8 is a Matlab array of an optimal twenty chip binary code found in the literature0.

X = [1, 1, 1, 1, 1, 0, 1, 1, 0, 0, 1, 0,1,0, 1, 1, 0, 0,0,1]; (8)

The waveform was computed as shown above for the thirteen-chip code. The auto-correlation is plotted in Figure 3. The auto-correlation peak is pronounced and side-lobes are flat with no high peaks. This code is optimal because the auto-correlation achieves the established minimum PSL of 2 for a 20-chip code.

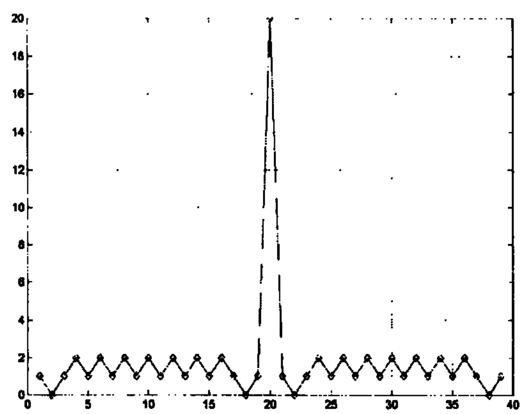

Figure 3 Auto-correlation of 20-Chip Optimal Code

The auto-correlation of a randomly generated 20-chip code is plotted in Figure 4 for comparison. The side-lobes are not flat and the PSL is 8. The presence of high side-lobes will make the peak more difficult to locate and will result in a poor final image.

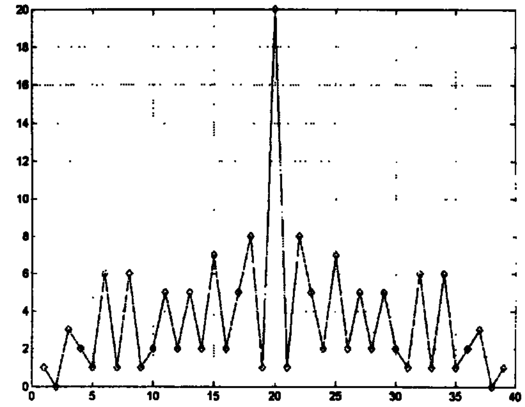

Figure 4. Auto-correlation of 20 Chip Random Code

The Genetic Algorithm approach was evaluated by attempting to find optimal codes. The goals were to locate (1) an optimal twenty-chip code similar to the one shown above, and (2) an optimal forty-chip code. A search for a forty chips was chosen because forty is the longest code found in the Radar Handbook and a significant challenge.

The first component required was a GA program. A robust FORTRAN GA program was located on the Internet (Carroll 1997).

The second component was a fitness function that drove the GA program. A variety of fitness functions were attempted. One attempt used frequencies from the Fast Fourier Transform. (Auto and Cross-Correlations are usually implemented using Fourier Transforms (Numerical Recipes).) Other potential fitness functions used integrated side-lobes, mean side-lobes, and a variety of combinations of side-lobes. The best fitness function was simply (equation 9):

$$\text{Fitness} = 1/(PSL_{autocorr}+ISL_{autocorr}) \qquad (9)$$

PSL and ISL both decrease as codes with lower auto-correlation side-lobes are located. The logarithm was not computed to speed the search.

Twenty Chips

A GA search starts with a random gene sequence, and hopefully, evolves to a gene sequence that maximizes the fitness function. The twenty-chip code in Equation 10 was the result of one GA run. Its auto-correlation is plotted in figure 4.

X = [1, 1, 0, 1, 0, 0, 1, 0, 1, 1, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0]; (10)

This code is optimal since the PSL is 2. According to the Radar Handbook, only six optimal twenty-chip codes exist out of 2^{20} possible codes (Skolnik 1990). Only one of the six is provided in the Handbook. The equation 10 code is not that one, and must be one of the other five.

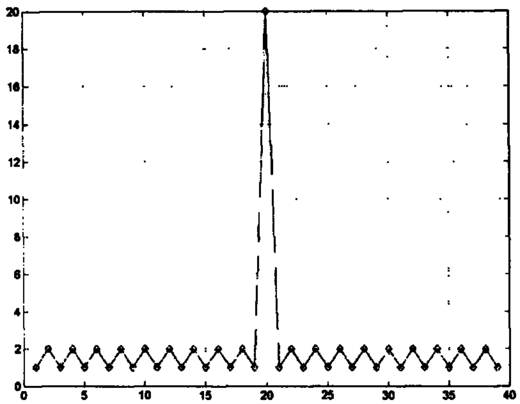

Figure 4. Auto-correlation of GA Optimized 20 Chip Code

Forty Chips

Locating a forty-chip optimal code is much more difficult. The search space is 2^{40} or about 1.0995e+012. The optimal PSL is 3. The GA located a code with PSL equal to 4 that is plotted in figure 5.

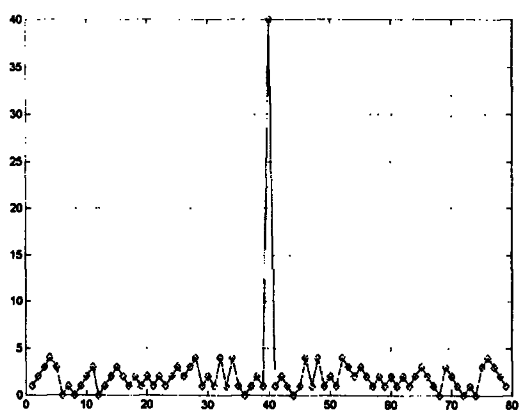

Figure 5. Auto-correlation of GA Optimized 40 Chip Code

The GA program works acceptably. One would prefer to have located an optimal forty-chip code, but according to the Radar Handbook, only 114 such codes exist.

Waveform Pairs

The next challenge was to locate two codes at the same time. Each code's auto-correlation should have a low PSL. The cross-correlation between codes should also have a low PSL. The radar return signal will be pulse compressed twice, once for each code. The result will only be as good as the maximum PSL of the auto and cross correlations. Codes with good auto and cross-correlations are called 'quasi-orthogonal'.

The GA was used to locate a pair of twenty-chip codes. The forty-chip minimum PSL will be used to measure the goodness of a pair of twenty chip codes. An intuitive guess is that the optimal PSL for two twenty-chip codes should be the same as optimal PSL for a forty-chip code, i.e., 3.

The best fitness function, after many attempts, was simply (equation 11):

$$\text{Fitness} = (PSL_{autocorr1} + PSL_{autocorr2} + 2*PSL_{xcorr}). \qquad (11)$$

Finding two twenty-chip codes with low auto-correlation PSL's was not difficult for the GA. Finding two codes with low cross-correlation PSL was more difficult. The GA appeared to use low auto-correlation PSLs to offset a large cross-correlation PSL. Using 2*PSLxcorr in the fitness function forced a low cross –correlation.

The two auto-correlations and the cross-correlation for the quasi-orthogonal code pair located by the GA are plotted in figures 6, 7, and 8. The PSLs are 3 for the auto-correlations and 5 for the cross-correlation. Of course the cross-correlation does not have an auto-correlation central peak.

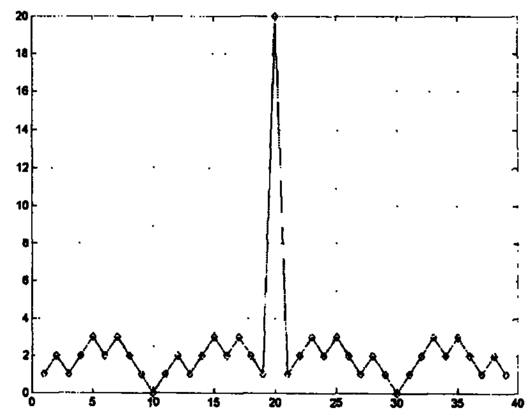

Figure 6. Auto-correlation of First Quasi-Orthogonal 20 Chip Code

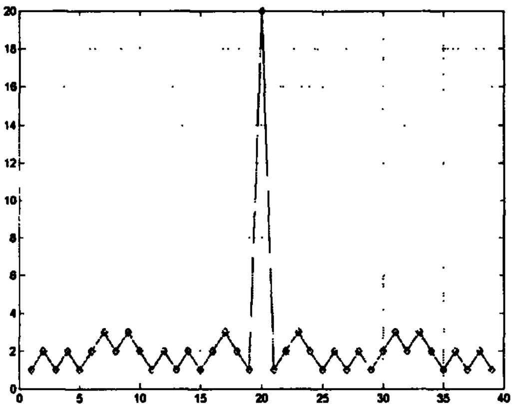

Figure 7. Auto-correlation of Second Quasi-Orthogonal 20 Chip Code

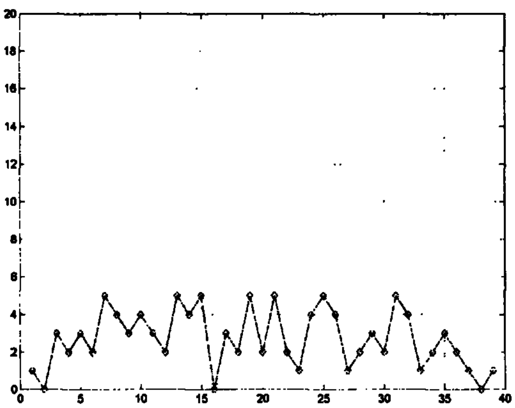

Figure 7. Cross-correlation of First and Second Quasi-Orthogonal 20 Chip Code

Earlier while searching for single codes, the GA located five of the six optimal twenty-chip codes. No two of these had an acceptable cross-correlation PSL. Thus, while searching for pairs of codes, the GA had to resort to codes with greater that optimal PSLs to locate a pair of codes with a low cross-correlation PSL. The cross-correlation of 5 appears to be a 'wall', even after extensive runs of over 10,000 generations.

Conclusions

For this research, the Genetic Algorithm was used to locate nearly optimal bi-phase waveforms for pulse compression radar. The Genetic Algorithm was chosen because the binary representation of a waveform used by radar engineers is identical to the binary representation of a gene used by the Genetic Algorithm. For a short waveform of about twenty chips, the peak side-lobe matched the expected optimal peak side-lobe. For longer waveforms of about forty chips, the peak side-lobe was just greater than optimal, even after extensive searches.

The technique was extended to find pairs of quasi-orthogonal waveforms. The results appear good even though neither of the waveforms was optimal.

Acknowledgements

This research did not include writing a Genetic Algorithm program. A robust FORTRAN GA program was found and downloaded from the Internet. The Genetic Algorithm was written Dr. David Carroll of the University of Illinois (Carroll 1997). This paper's author wrote a variety of FORTRAN fitness functions, that when linked to Dr Carroll's GA program, located nearly optimal waveforms.

The author also acknowledges the radar engineers at AFRL that supplied much needed radar expertise. They are Maj. Scott Berger (AFRL/VSS), Mr. John Garnham (CSC), and Mr. Steve Fiedler (ERIM).

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