On-line Qualitative Temporal Reasoning with Explanation

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Abstract

This work is a confluence of three problems in constraint reasoning: qualitative temporal reasoning (QTR), incremental reasoning, and explanation generation. Our primary objective is to detect the cause of inconsistency in an incremental version of the QTR problem.

1. Introduction

Generating explanations for derived assertions is a motivating point behind the long journey in *non-monotonic reasoning* in AI. In this work we propose a version of incremental reasoning problem where a new temporal object is to be inserted in a temporal database, along with the constraints between the new object and the old objects in the database already committed on the time-line. The objective is to find a satisfiable solution for the new object on the time line in case of consistency, or to generate explanation for inconsistency. Gerevini (2003) addressed a similar problem but did not address the explanation-generation issue.

2. Background on Temporal Reasoning

Qualitative point-based temporal reasoning constitutes the simplest form of spatio-temporal reasoning (Vilain and Kautz, 1986). The scheme has three basic qualitative relations *B*: {<, >, =} between any pair of points. Qualitative reasoning with intervals involves thirteen basic Allen's (1983) relations, *B*: {*before(p), after(p^{-1}), meets(m), met-by(m^{-1}), overlaps(o), overlapped-by(o^{-1}), starts(s), started-by(s^{-1}), during(d), contains(d^{-1}), finishes(f), finished-by(f^{-1}), equal(eq)*}, between any pair of intervals. *Qualitative temporal reasoning problem* (QTR(Θ)) is to answer, given a set of intervals and binary constraints between some of them, if a satisfiable assignment for each of the interval exists. Each constraint $R \in \Theta \subseteq P(B)$ is a disjunctive subset of the power set of *B* restricted to Θ that is closed under some operators like composition. The reasoning problem over unrestricted $P(B)$ is known to be NP-hard (Vilain and Kautz, 1986), whereas reasoning with point algebra is a P-class problem. Reasoning over a proper subset Θ may be tractable, some of them being maximal (*Maximal Tractable Subsets*, or *MTS's*)

Ligozat (1996) developed a canonical way of representing time intervals that appears as a useful tool for understanding the MTS's (Fig 1): the starting point of the intervals is *X*-axis, the ending point is *Y*-axis, and the valid space is *Y*>*X*. The topological relationships between the basic relations in this space constitute a lattice (Fig 1b, "*p*" as inferior (0,0) and "p^{-1}" as superior (4,4)).

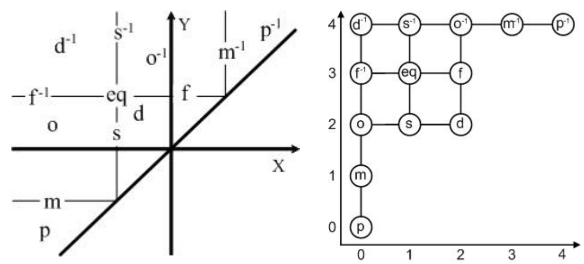

Figure 1a and 1b: Canonical representation of interval-basic relations

We need the following definitions from Ligozat (1996).
Definition 1 (Dimension *dim*(*l*)): For a basic relation *b*, *dim*(*b*) is the dimension of *b* in the Canonical representation in Fig 1a. For any relation *l*, *dim*(*l*) = max{*dim*(*b*) | *b* ∈ *l*}.
Example 1: "*p*" is adjacent to "*m*" and the former is of dimension 2 while the later is of dimension 1.
Definition 2 (Preconvex or Ord-Horn relation): A preconvex relation *l* is an interval on the lattice with some possibly missing relations *r* such that *dim*(*r*) < *dim*(*l*).
Example 2: {*o, s, d,* d^{-1}, o^{-1}} is a preconvex relation, where the missing relations are (*f, eq,* f^{-1},s^{-1}) from the interval [(0,2), (2, 4)] on the lattice.
Set of ORD-Horn relations form a MTS(OH) of $P(B)$.

3. OLQTR Problem Definition

Online qualitative temporal reasoning problem has a total order $T=\{t_1, t_2, \ldots, t_n\}$ as an input. In case of point-based reasoning, each t_i ($1 \le i \le n$) is a time-point and in case of interval-based reasoning each t_i is a boundary point of an interval from the set $I=\{i_1, i_2, \ldots, i_m\}$, $|T|=n \le 2m$. The second input to the problem is a set of constraints *C* between a *new* object *o* (point or interval) and the objects in *T*: $(o\ r_k\ i_k) \in C$ for some $i_k \in I$ in case of intervals

(OLQTR-I), and $i_k \in T$ in case of points (OLQTR-P), where non-null $r_k \in \Theta$. If C is satisfiable the output is a new total order T' including o that satisfies all the constraints in C. OLQTR problem has three components. (1) It decides if the input C is consistent or not; (2) If C is consistent it asks for finding a model (T') of C satisfying all constraints; and (3) In case C is inconsistent it asks for finding a minimal set of constraints $M \subset C$, such that C-M is consistent and $\nexists M' \subset C$ such that $|M'| \leq |M|$, with C-M' being consistent. OLQTR-P and OLQTR-I(Θ), for some subset Θ, including Θ=OH are tractable.

4. OLQTR-P with Points

Algorithm-1D (Mitra et al. 1999) proposed before, solves the first two components of the OLQTR-P. If *Algorithm-1D* finds inconsistency the following algorithm (Fig 2) finds the minimum conflict set M. Let (*degree of conflict*) $DC[r_i]$ be the number of constraints that r_i conflicts with. Initially (*conflict set*) CS collects the set of conflicting constraints (lines 2-6 below). The algorithm then finds the minimal number of those (*MinSet*), removal of which may restore consistency (lines 7-12). In line 4 the routine *conflicts*(r_i, r_j) returns *True* iff r_i conflicts with r_j.

Algorithm FindMinset (Input: T and C)
(1) CS = *null*; *MinSet* = *null*;
(2) $\forall\, r_i \in C$, $DC[r_i] = 0$;
(3) $\forall\, r_i, r_j \in C$ such that $i \neq j$ do
(4) if (r_i is "<", or "≤", or "=")
and (r_j is ">", or "≥", or "=")
and *conflicts*(r_i, r_j) then
(5) $CS = CS \cup \{(r_i, r_j)\}$;
(6) $DC[r_i]$++; $DC[r_j]$++;
(7) while $\Sigma_{CS}\,[DC] \neq 0$ do
(8) let r = a constraint in CS with
the maximum DC;
(9) *MinSet* = *MinSet* $\cup \{r\}$;
(10) for each element $(r_i, r_j) \in CS$ do
(11) $CS = CS - (r_i, r_j)$; $DC[r_i]$--; $DC[r_j]$--;
(12) return *MinSet*.

Figure 2: Culprit detection algorithm for OLQTR-P

5. OLQTR-I(OH) with ORD-Horn MTS

Any OH relation can be expressed as a conjunctive normal formula (CNF) where any clause has at most one positive literal ("≤" or "=") and any number of negative literals involving ("≠") (Nebel et al. 1995). A literal is the boundary-point relations between the related intervals. An interval relation is originally in a disjunctive normal form over the boundary point relations. It is not easy to convert them to CNF in the above form for OH relations. We have developed a *Normalization* algorithm that creates an interval over the lattice for the input OH relation and then adds the formula for the missing basic relations This creates a CNF. We further simplify the CNF using some rules we have developed that creates a CNF with only one positive literal iff the input is OH relation. (detail in https://www.cs.fit.edu/Projects/tech_reports/tr2006.html)
Example 3 (Ligozat): *OH* constraint (i_1 $\{o, d^{-1}, eq\}$ i_2) may be expressed as $\{(i_1^- \leq i_2^-) \wedge (i_2^- \leq i_1^+) \wedge (i_1^+ \neq i_2^-)$ //the 3 unit clauses define the interval $[o, s^{-1}]$ on the lattice// $\wedge\, ((i_2^- \leq i_1^-) \vee (i_1^+ \neq i_2^+))$ //take out f^{-1}// $\wedge\, ((i_1^- \neq i_2^-) \vee (i_1^+ \leq i_2^+))$ //take out s^{-1}// $\wedge\, ((i_1^- \neq i_2^-) \vee (i_2^+ \leq i_1^+))$ //take out s//$\}$. The comments inserted within the CNF above indicate the trick how to create such formula.

Finally, our *Sorter* algorithm parses the CNF for the OLQTR-I(OH) problem C (a conjunct over all constraints involving o) choosing only one point relation from each clause corresponding to the start or end point of o, favoring the least constraining negative literal. Since there is only one positive literal per clause, when that is chosen there is no other alternative for backtracking. *Sorter* creates two lists for start and end points of o. Subsequently *Algorithm-1D* provides a solution to a consistent problem, or *FindMenset* provides *MinSets* for the two boundary points of o.

OH-subset is the most useful of all the MTS's as it contains each of the basic relations. Our immediate following work will address the OLQTR-I problem for all other MTS's available in the literature.

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