Isomorphic Transformations of Uncertainties between the EMYCIN and PROSPECTOR Inexact Reasoning Models *

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The problem solving ability of expert systems is greatly improved by the way of cooperation among different expert systems in a distributed expert system. However, different expert systems in a distributed expert system may use different inexact reasoning models. To achieve cooperation among these expert systems, in our methodology, the first step is to transform the uncertainty of a proposition from one inexact reasoning model to another, then the second step is to synthesize the transformed different results. In other words, the transformation is the foundation for cooperation, and so this is a very important and very interesting problem.

In this paper, first we give criteria to judge which heterogeneous transformations are reasonable. In two different inexact reasoning models,

1) let the sets of uncertainties of B be X and Y, respectively;
2) let its uncertainty be described by $C_1^{(T)}$, $C_2^{(T)}$ when the proposition B is known to be true, and false by $C_1^{(F)}$, $C_2^{(F)}$; and
3) let the operators on $X - \{C_1^{(T)}, C_1^{(F)}\}$, and on $Y - \{C_2^{(T)}, C_2^{(F)}\}$ be $\oplus_1$ and $\oplus_2$, respectively.

We have proved that both $(X - \{C_1^{(T)}, C_1^{(F)}\}, \oplus_1)$ and $(Y - \{C_2^{(T)}, C_2^{(F)}\}, \oplus_2)$ are semi-groups with unit elements.

Since inexact reasoning models are semi-groups with unit elements, the natural criterion for transformations among models is a homomorphic map. Also, two special points, true and false, have corresponding values among models. Formally, we have

Definition A map $\mathcal{F} : X \longrightarrow Y$ is said to be a h-transformation from $(X - \{C_1^{(T)}, C_1^{(F)}\}, \oplus_1)$ to $(Y - \{C_2^{(T)}, C_2^{(F)}\}, \oplus_2)$, if it satisfies

1. $\mathcal{F}(\oplus_1(x_1, x_2)) = \oplus_2(\mathcal{F}(x_1), \mathcal{F}(x_2)), \quad \forall x_1, x_2 \in X - \{C_1^{(T)}, C_1^{(F)}\}$;
2. $\mathcal{F}(C_1^{(T)}) = C_2^{(T)}$;
3. $\mathcal{F}(C_1^{(F)}) = C_2^{(F)}$.

Then we notice that

1) The EMYCIN inexact reasoning model corresponds a group structure $((-1,1), \oplus_1)$, where the operator *oplus*$_1$ is is given by:

$$\oplus_1(x_1, x_2) = \begin{cases} x_1 + x_2 - x_1x_2 & \text{if } x_1 > 0, x_2 > 0 \\ x_1 + x_2 + x_1x_2 & \text{if } x_1 < 0, x_2 < 0 \\ \frac{x_1+x_2}{1-min(|x_1|,|x_2|)} & \text{if } x_1x_2 \le 0 \end{cases}$$

2) The PROSPECTOR inexact reasoning model corresponds a group structure $((0,1), \oplus_3)$ is where operator $\oplus_3$, is defined as : $\forall x_1, x_2 \in (0,1)$

$$\oplus_3(x_1, x_2) = \frac{x_1x_2(1-P(B))}{(1-x_1)(1-x_2)P(B) + x_1x_2(1-P(B))}$$

where $P(B)$ means the prior probability of a proposition B.

Finally, we obtain h-transformations between the EMYCIN inexact reasoning model and the PROSPECTOR reasoning model by the two theorem below.

Theorem 1 *The map*

$$f_{13}(x) = \begin{cases} \frac{P(B)}{1-x\times(1-P(B))} & \text{if } 1 \ge x > 0 \\ \frac{(1+x)P(B)}{1+x\times P(B)} & \text{if } 0 \ge x \ge -1 \end{cases}$$

is an isomorphism from $((-1,1), \oplus_1)$ *to* $((0,1), \oplus_3)$.

Theorem 2 *The map*

$$f_{31}(x) = \begin{cases} \frac{x-P(B)}{x(1-P(B))} & \text{if } x > P(B) \\ \frac{x-P(B)}{(1-x)P(B)} & \text{if } x < P(B) \end{cases}$$

is an isomorphism from $((0,1), \oplus_3)$ *to* $((-1,1), \oplus_1)$.

The isomorphic functions we discover can exactly transform the uncertainties of a proposition between the EMYCIN and PROSPECTOR models for any value of the prior probability of a proposition. This solves one of the key problems in the area of distributed expert systems. These transformation functions have been realized in the prototype HECODES system.

Keywords: algebraic structure, cooperation, distributed expert systems, isomorphic transformation, inexact reasoning, semi-group, uncertainty.