Characterization of Semantics for Argument Systems

Philippe Besnard and **Sylvie Doutre**
IRIT – Université Paul Sabatier
118, route de Narbonne – 31062 Toulouse Cedex 4 – France
{besnard, doutre}@irit.fr

Abstract

We consider Dung's argumentation framework, in which an argument system consists of a set of arguments and a binary relation between arguments representing the notion of a conflict. The semantics given by Dung define (with respect to each argument system) acceptable sets of arguments called extensions. For his so-called stable semantics, Dung also gives an alternative definition in terms of an equation that a set satisfies if and only if that set is a stable extension. However, neither the original definition nor the equation reflect the fact that the stable semantics (similarly to all of Dung's semantics) rely upon the notion of an admissible set. Moreover, none of Dung's other semantics have been characterized by such an equation.

Our first goal is to provide such characterizations for the other semantics: We capture Dung's semantics by means of equations that a set satisfies if and only if it is an extension under the semantics at hand. Not only do we give such equations, but we also take care of providing them as a unified characterization expressing the common grounds of Dung's semantics. Beyond Dung's semantics, we are interested in semantics (within Dung's argumentation framework) relying upon the notion of an admissible set. Our second goal is to show that many of those semantics are captured like Dung's, using the same unified characterization.

Introduction

Argumentation is a reasoning model which amounts to building and evaluating arguments, often conflicting ones. An argument can be seen as a reason supporting some claim. Conflicts between arguments arise for example when the claim or the reason supporting it is contradicted by another argument. Evaluation then aims at selecting the most acceptable arguments.

Modelling argumentation is an important research topic whose main approaches are surveyed in (Chesñevar, Maguitman, & Loui 2000; Prakken & Vreeswijk 2002). One of these approaches is Dung's argumentation framework (Dung 1995) in which argument systems have an abstract structure, thereby allowing it to unify many other approaches proposed for argumentation on the one hand and formalisms modelling non-monotonic reasoning on the other hand.

A Dung argument system consists of a set of arguments and a binary relation between arguments representing the notion of a conflict. An argument is thus an abstract entity whose internal structure, nature, origin (and so on) are not known. Therefore, such a system shifts the focus to conflicts between arguments as well as evaluation of arguments. The semantics given by Dung for acceptability define (with respect to each argument system) one or several acceptable sets of arguments called extensions. All of Dung's semantics rely upon the notion of an admissible set, but this does not show in the original definitions (cf the notion of a stable extension for example).

For his so-called stable semantics, Dung gives a definition in terms of an equation that a set satisfies if and only if the set is an extension under the stable semantics. Such a characterization is rather simple but has not been given for the other semantics. Our first goal in this paper is to give such characterizations for the other semantics, that is, we want to characterize Dung's semantics by means of equations that a set satisfies if and only if it is an extension under the semantics at hand. These equations should stress that admissibility is the core notion of Dung's semantics. Not only do we want to provide such characterizations, but we also want them to be as close as possible to one another. Notice that we do not aim at providing charaterizations computationally efficient.

Beyond our interest in Dung's semantics, we are more generally interested in semantics relying upon the notion of an admissible set. Our second goal is to investigate whether some of those semantics could be characterized like Dung's, with characterizations as similar as possible.

Formal preliminaries

In this section, we give the definition of a Dung argument system and of the various semantics that Dung proposed for the acceptability of sets of arguments.

Definition 1 *(Dung 1995) An* argument system *is a pair* (A, R) *where* A *is a set whose elements are called* arguments *and* R *is a binary relation over* A *(*$R \subseteq A \times A$*). Given two arguments* a *and* b*,* $(a, b) \in R$ *or equivalently* aRb *means that* a attacks b *(*a *is said to be an* attacker *of* b*). Moreover, a set* S *of arguments attacks an argument* a *if some argument in* S *attacks* a*. Finally, a set* S *of arguments attacks a set* S' *of arguments if some argument in* S *attacks some argument in* S'*.*

An argument system can simply be represented as a directed graph whose vertices are the arguments and edges correspond to the elements of R.

In the following definitions, we assume that an argument system (A, R) is given.

Dung gave several *semantics* for acceptability. These diverse semantics induce one or several acceptable sets of arguments, called *extensions*. The most popular semantics is certainly the stable semantics. Its definition is merely based on the notion of an attack:

Definition 2 *(Dung 1995) A subset $S \subseteq A$ is* conflict-free *iff there are no arguments a and b in S such that a attacks b. A conflict-free $S \subseteq A$ is a* stable extension *iff for each argument which is not in S, there exists an argument in S that attacks it.*

The other semantics for acceptability rely upon the notion of defense[1]:

Definition 3 *An argument $a \in A$ is* defended *by a set $S \subseteq A$ (or S* defends *a) iff for each argument b in A that attacks a there exists an argument in S that attacks b.*

According to Dung, an acceptable set of arguments under any semantics must be a conflict-free set which defends all its elements. This constraint is captured by the notion of admissibility:

Definition 4 *(Dung 1995) A conflict-free $S \subseteq A$ is* admissible *iff each argument in S is defended by S.*

Even if the definition of a stable extension does not rely upon the notion of defense, a stable extension is an admissible set. Admissibility has an advantage over stable semantics: given an argument system, there need not be any stable extension but there always exists at least one admissible set (the empty set is always admissible). A drawback of admissibility is that an argument system may have a large number of admissible sets. This is why other notions of acceptability which select only some admissible sets were designed. Besides the stable semantics, three semantics refining admissibility have been introduced by Dung:

Definition 5 *(Dung 1995) A* preferred extension *is a maximal (wrt set inclusion) admissible subset of A. An admissible $S \subseteq A$ is a* complete extension *iff each argument which is defended by S is in S. The least (wrt set inclusion) complete extension is the* grounded extension*.*

Notice that a stable extension is also a preferred extension and a preferred extension is also a complete extension. Stable, preferred and complete semantics admit multiple extensions whereas the grounded semantics ascribes a single extension to a given argument system.

Example 1 *Let (A, R) be the argument system such that*

$$A = \{a, b, c, d, e\} \text{ and}$$

$$R = \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\}.$$

The graph representation of (A, R) is indicated on Figure 1. The admissible sets of (A, R) are $\emptyset$, $\{a\}$, $\{c\}$, $\{d\}$, $\{a, c\}$ and $\{a, d\}$. Dung's semantics induce the following acceptable sets:

- *Stable extension(s):* $\{a, d\}$
- *Preferred extensions:* $\{a, c\}$, $\{a, d\}$
- *Complete extensions:* $\{a, c\}$, $\{a, d\}$, $\{a\}$
- *Grounded extension:* $\{a\}$.

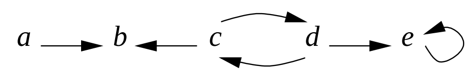

Figure 1: Graph representation of the argument system of Example 1.

Characterizing Dung's semantics

We want to characterize Dung's semantics by means of fixpoint equations expressed in terms of defense and attack, since these two notions are the core of admissibility. A characterization close to the ones we are looking for has been established by Dung himself for the stable semantics (Lemma 14 in (Dung 1995)). Prior to giving this characterization, we introduce[2] some notations: Given an argument system (A, R), for every set $S \subseteq A$:

$$\overline{S} \stackrel{\text{def}}{=} A \setminus S$$

$$Def(S) \stackrel{\text{def}}{=} \{a \in A \mid S \text{ defends } a\}$$

$$R^+(S) \stackrel{\text{def}}{=} \{a \in A \mid S \text{ attacks } a\}$$

$$R^-(S) \stackrel{\text{def}}{=} \{a \in A \mid a \text{ attacks } S\}$$

The notation Def is not primitive according to the following Proposition:

Proposition 1 *(Amgoud & Cayrol 1998) Let (A, R) be an argument system and let $S \subseteq A$. Then $Def(S) = \overline{R^+(\overline{R^+(S)})}$.*

Proposition 2 *(Dung 1995) Let (A, R) be an argument system. $S \subseteq A$ is a stable extension iff $S = \overline{R^+(S)}$.*

The above equation shows that the stable semantics relies upon the notion of an attack while omitting the notion of defense, thereby failing to disclose that the stable semantics is based on admissibility, too.

In order to characterize the stable semantics and Dung's other semantics by way of equations stressing that all these semantics rely upon admissibility, let us first list some properties satisfied by admissible sets.

Recall that an admissible set is a conflict-free set which defends all its elements. Clearly, a set S defends all its elements if and only if $S \subseteq Def(S)$. Let this be supplemented with a property presented in (Amgoud & Cayrol 1998): $S \subseteq A$ is conflict-free if and only if $S \subseteq \overline{R^+(S)}$. Therefore:

[1]What is called *defense* here is called in (Dung 1995) *acceptability of an argument with respect to a set of arguments*.

[2]The notations R^+ and R^- are derived from graph-theoretic terminology in which they stand for *successor* and *predecessor*.

Proposition 3 *Given (A,R), $S \subseteq A$ is an admissible set iff $S \subseteq Def(S) \cap \overline{R^+(S)}$.*

A dual characterization is obtained:

Proposition 4 *Given (A,R), $S \subseteq A$ is an admissible set iff $S \subseteq Def(S \cap \overline{R^-(S)})$.*

One way to characterize complete extensions is to enrich the characterization of admissible sets in order to capture the fact that any argument defended by the extension must belong to the extension. In other words, a set S is a complete extension if and only if S is admissible and $Def(S) \subseteq S$. This leads us to a simple characterization:

Proposition 5 *Given (A,R), $S \subseteq A$ is a complete extension iff $S = Def(S) \cap \overline{R^+(S)}$.*

This characterization can be extended. For any subset X of $Def(S) \cap \overline{R^+(S)}$, $S \subseteq A$ is a complete extension if and only if $S = Def(S \cup X) \cap \overline{R^+(S)}$.

For one thing, if S is a complete extension then for any set X, if $X \subseteq Def(S) \cap \overline{R^+(S)}$ then the equation $S = Def(S \cup X) \cap \overline{R^+(S)}$ holds. Conversely, for any set X, if $X \subseteq Def(S) \cap \overline{R^+(S)}$ and $S = Def(S \cup X) \cap \overline{R^+(S)}$ then S is a complete extension.

A dual characterization can be obtained: For any subset X of $Def(S) \cap \overline{R^+(S)}$, $S \subseteq A$ is a complete extension if and only if $S = Def((S \cup X) \cap \overline{R^-(S)})$.

These two equations characterizing complete extensions also capture stable and preferred extensions, considering a different condition on the set X. The result stating this uniform characterization of Dung's semantics is as follows:

Theorem 1 *Let (A,R) be an argument system. For $S \subseteq A$, the statements below are equivalent:*

- *S is a Dung extension under the t semantics*
- $S = Def(S \cup X) \cap \overline{R^+(S)}$
- $S = Def((S \cup X) \cap \overline{R^-(S)})$

where X is an arbitrary set of arguments such that

- *case t = complete* $\{ \emptyset \subseteq X \subseteq Def(S) \cap \overline{R^+(S)}$
- *case t = preferred* $\begin{cases} \emptyset \subseteq X \subseteq Def(S) \cap \overline{R^+(S)} \\ \text{if } S \text{ is a preferred extension,} \\ \overline{R^+(S)} \subseteq X \subseteq A \text{ otherwise} \end{cases}$
- *case t = stable* $\{ \overline{R^+(S)} \subseteq X \subseteq A$

For the notion of complete or stable extensions, the characterization provided by Theorem 1 is very general due to the wide range allowed for X. Here is an example illustrating that this range cannot be extended:

Counter-example Let (A,R) be the argument system indicated in Figure 2. Consider $S = \{a\}$.
Let us first deal with the complete case. Take $X = \{d\}$ (hence, X is not as required in the theorem because $d \notin Def(S) \cap \overline{R^+(S)}$ due to $Def(S) = \{a\}$). Clearly, S is a complete extension but $\{a\} \neq Def(\{a\} \cup \{d\}) \cap \{a,c,d\}$ (that is, the equation in the theorem fails).
Let us now deal with the stable case. Take $X = \{a,c\}$ (hence, X is not as required in the theorem because $\overline{R^+(S)} \not\subseteq \{a,c\}$ due to $\overline{R^+(S)} = \{a,c,d\}$). Then, $S = Def(\{a\} \cup \{a,c\}) \cap \{a,c,d\}$. That is, $S = Def(S \cup X) \cap \overline{R^+(S)}$ for $X = \{a,c\}$. If it were not for X being a proper subset of $\overline{R^+(S)}$, the equation would be satisfied although S is not a stable extension. ■

Figure 2: Argument system illustrating the restriction of the range of X in Theorem 1.

As far as stable extensions are concerned, it seems that the condition $\overline{R^+(S)} \subseteq X$ rather means $\overline{R^+(S)} \setminus S \subseteq X$ but the latter formulation is more cumbersome while clearly equivalent with the former because stable extensions must be conflict-free.

The clause about preferred extensions in Theorem 1 may seem to exhibit a circularity. Actually, such is not the case. Once S is given, the status of S wrt being a preferred extension is fixed (although we do not know what it is[3]). Accordingly, whatever X is chosen, the status of X wrt being as mentioned in Theorem 1 is fixed (although we do not know what it is). Stated otherwise, it is *not* the result of checking the equation that determines whether X is appropriate or not: There is no circularity.

In other words, when choosing a value for X in order to check the equation, we do not know whether the value is appropriate or not (unless we already found out, by another method, whether S is a preferred extension). Of course, appropriate values do exist but we do not know what they are: In this sense, using the equation to check whether S is a preferred extension is not effective.

Contrast this with the case of using the equation to check whether S is a complete extension for example. Once S is given, we can decide whether an element of A is in $Def(S) \cap \overline{R^+(S)}$ and therefore we can check that a given collection of such elements is an appropriate value for X (i.e., a subset of $Def(S) \cap \overline{R^+(S)}$). Thus, using the equation to check whether S is a complete extension is effective. The same holds for stable extensions.

The grounded semantics has been characterized by Dung as the least fixpoint of Def. This least fixpoint can be computed applying iteratively Def to the empty set. This is not a characterization of the kind we are investigating here because the function to be used is not explicited (even considering only the case that α in $S = Def^{\alpha}(\emptyset)$ is finite, its value is not known in advance).

[3]Very much as when we are given an even natural number, a very large one, the status of that number wrt being a counter-example for Goldbach's conjecture is fixed but we do not know that status.

Other admissibility-based semantics

Dung has proposed four semantics based on admissibility (the grounded, complete, preferred and stable semantics). Theorem 1 exhibits a uniform characterization of Dung's extensions. Could other semantics refining admissibility be captured by the same characterization?

In this section, we focus on such semantics which would give rise to some new notions of extensions intermediate between complete extensions and stable extensions. Given an argument system (A, R), any such semantics characterizes some set $\mathcal{E}$ of admissible sets which are both proper subsets of $\mathcal{C}$ (the set of all complete extensions of (A, R)) and proper supersets of $\mathcal{S}$ (the set of all stable extensions of (A, R)). An example of such a set $\mathcal{E}$ is the one whose elements are exactly the complete extensions which contain a maximum of defenders. In Example 1, this $\mathcal{E}$ collapses with the set of preferred extensions, but it is not the case in the example below:

Example 2 *Let (A, R) be the argument system indicated on Figure 3. Consider the sets $S = \{c, g\}$ and $S' = \{c, g, d\}$. These sets are complete extensions but S does not contain a maximum of defenders because d defends g against f and S can be supplemented with d to form a complete extension (actually, $S' = S \cup \{d\}$). As for a different matter, observe that S' is not a preferred extension.*

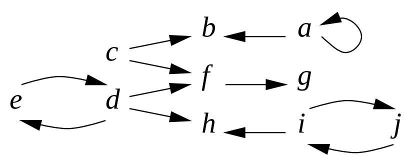

Figure 3: Argument system of Example 2.

The next two results show that such sets $\mathcal{E}$ can be captured by the two equations used in Theorem 1 to characterize Dung's extensions, that is $S = Def(S \cup X) \cap \overline{R^+(S)}$ and $S = Def((S \cup X) \cap \overline{R^-(S)})$, considering that X is the set attached to S by a function $\chi : 2^A \to 2^A$.

Theorem 2 *Let (A, R) be an argument system. Consider $\mathcal{E}$ such that $\mathcal{S} \subseteq \mathcal{E} \subseteq \mathcal{C}$.*

- *There exists $\chi : 2^A \to 2^A$ such that $S \in \mathcal{E}$ iff $S = Def(S \cup \chi(S)) \cap \overline{R^+(S)}$.*
- *There exists $\chi : 2^A \to 2^A$ such that $S \in \mathcal{E}$ iff $S = Def((S \cup \chi(S)) \cap \overline{R^-(S)})$.*

This very general result induces that three of Dung's semantics, namely complete, preferred and stable semantics, and any other set of extensions intermediate between complete and stable extensions are captured by the two equations $S = Def(S \cup \chi(S)) \cap \overline{R^+(S)}$ and $S = Def((S \cup \chi(S)) \cap \overline{R^-(S)})$.

Conversely, the next result shows that for any function χ satisfying some conditions, both equations define a set $\mathcal{E}$ of admissible sets intermediate between complete extensions and stable extensions.

Theorem 3 *Let (A, R) be an argument system.*

- *Consider $\chi : 2^A \to 2^A$ such that for all $S \subseteq A$, if $S = Def(S \cup \chi(S)) \cap \overline{R^+(S)}$ then $S \cup \chi(S)$ is an admissible set.*
  Then, $\{S \subseteq A \mid S = Def(S \cup \chi(S)) \cap \overline{R^+(S)}\}$ is some $\mathcal{E}$ satisfying $\mathcal{S} \subseteq \mathcal{E} \subseteq \mathcal{C}$.
- *Consider $\chi : 2^A \to 2^A$ such that for all $S \subseteq A$, if $S = Def((S \cup \chi(S)) \cap \overline{R^-(S)})$ then $S \cup \chi(S)$ is an admissible set.*
  Then, $\{S \subseteq A \mid S = Def((S \cup \chi(S)) \cap \overline{R^-(S)})\}$ is some $\mathcal{E}$ satisfying $\mathcal{S} \subseteq \mathcal{E} \subseteq \mathcal{C}$.

Of course, not all notions of an extension captured in Theorem 2 as well as Theorem 3 are natural. Some of them are even technically weird. An example is any $\mathcal{E}$ such that there exist $S \in \mathcal{C}$ and $S' \in \mathcal{C}$ which satisfy $S \subseteq S'$ but $S \in \mathcal{E}$ whereas $S' \notin \mathcal{E}$ (accordingly, $\chi(S) \subseteq Def(S) \cap \overline{R^+(S)}$ while $\chi(S') \nsubseteq Def(S') \cap \overline{R^+(S')}$). In order to prevent such cases to arise, it is enough to impose that:

1. if $S \subseteq S'$ for $S \in \mathcal{E}$ and $S' \in \mathcal{C}$ then $S' \in \mathcal{E}$
2. if $S \subseteq S'$ where
   - $S = Def(S \cup \chi(S)) \cap \overline{R^+(S)}$ or $S = Def((S \cup \chi(S)) \cap \overline{R^-(S)})$

   and

   - $S' = Def(S') \cap \overline{R^+(S')}$ or $S' = Def(S' \cap \overline{R^-(S')})$

then $\chi(S') \subseteq Def(S') \cap \overline{R^+(S')}$.

The above results are concerned with existence properties only. A more effective account is as follows. The equations using a function

$$\chi : 2^A \to 2^A$$

such that for a given set S and a given attached[4] set F_S,

$$\chi(S) = F_S \text{ whenever } S \cup F_S \text{ is an admissible set}$$

$$\text{and } \chi(S) = \emptyset \text{ otherwise,}$$

define some $\mathcal{E}$ such that $\mathcal{S} \subseteq \mathcal{E} \subseteq \mathcal{C}$.

Let us consider two such functions χ. First, given (A, R), $\chi_1 : 2^A \to 2^A$ is defined as:

$$\chi_1(S) = \begin{cases} R^-(R^+(S)) & \text{if } S \cup R^-(R^+(S)) \\ & \text{is an admissible set,} \\ \emptyset & \text{otherwise.} \end{cases}$$

Let $\mathcal{E}_1$ denote the set

$$\{S \subseteq A \mid S = Def(S \cup \chi_1(S)) \cap \overline{R^+(S)}\}.$$

Second, consider the following function: Given (A, R), $\chi_2 : 2^A \to 2^A$ is defined as:

$$\chi_2(S) = \begin{cases} R^-(R^-(Def(S))) & \text{if } S \cup R^-(R^-(Def(S))) \\ & \text{is an admissible set,} \\ \emptyset & \text{otherwise.} \end{cases}$$

[4]It is assumed that, with each set S, a set F_S is associated (by whatever means).

Let $\mathcal{E}_2$ denote the set

$$\{S \subseteq A \mid S = Def(S \cup \chi_2(S)) \cap \overline{R^+(S)}\}.$$

Theorem 3 shows that $\mathcal{E}_1$ and $\mathcal{E}_2$ are subsets of the set of complete extensions and supersets of the set of stable extensions. Moreover, $\mathcal{E}_1$ and $\mathcal{E}_2$ are supersets of the set of preferred extensions. In addition, it can be shown that a set $S \in \mathcal{E}_1$ is not always a set which belongs to $\mathcal{E}_2$ and vice versa.

Clearly, any set $\mathcal{E}$ such that $\mathcal{S} \subseteq \mathcal{E} \subseteq \mathcal{C}$ is a set of complete extensions that share a specific property. When $\mathcal{E}$ is described by means of a function χ as illustrated above, it is possible to express that property.

Indeed, the extensions in the sets $\mathcal{E}_1$ and $\mathcal{E}_2$ can be characterized in a more natural way similar to the way Dung originally defined his semantics. The result stating this characterization is the following one:

Proposition 6 *Given (A, R),*

- *$S \in \mathcal{E}_1$ iff S is a complete extension such that if $S \cup R^-(R^+(S))$ is an admissible set then $R^-(R^+(S)) \subseteq S$;*
- *$S \in \mathcal{E}_2$ iff S is a complete extension such that if $S \cup R^-(R^-(Def(S)))$ is an admissible set then $R^-(R^-(Def(S))) \subseteq S$.*

This is a special case of a more general result:

Theorem 4 *Given (A, R), let each $S \subseteq A$ be attached some $F_S \subseteq A$. Define $\chi : 2^A \to 2^A$ to be the function such that*

$$\chi(S) = \begin{cases} F_S & \text{if } S \cup F_S \text{ is an admissible set} \\ \emptyset & \text{otherwise.} \end{cases}$$

For all $S \subseteq A$, the statements below are equivalent:

- *S is a complete extension such that if $S \cup F_S$ is an admissible set then $F_S \subseteq S$*
- *$S = Def(S \cup \chi(S)) \cap \overline{R^+(S)}$*
- *$S = Def((S \cup \chi(S)) \cap \overline{R^-(S)})$*

In Example 1 for instance, $\mathcal{E}_1$ is the set of preferred extensions and $\mathcal{E}_2$ is the set of complete extensions. Consider the following example where $\mathcal{E}_1$ and $\mathcal{E}_2$ collapse neither with the set of preferred extensions nor with the set of complete extensions:

Example 3 (Example 2 continued) *Consider $S = \{c, g\}$. This set is a complete extension and belongs to $\mathcal{E}_1$ but it does not belong to $\mathcal{E}_2$. Actually, $R^-(R^-(Def(S))) = \{c, d\}$ and $S \cup \{c, d\}$ is an admissible set but it is not included in S, so S does not belong to $\mathcal{E}_2$. That is, $\mathcal{E}_2$ does not collapse with the set of all complete extensions of (A, R) since S is not in $\mathcal{E}_2$.*
Consider now the set $S' = \{c, g, d\}$. S' is a complete extension and belongs to $\mathcal{E}_2$ but it does not belong to $\mathcal{E}_1$. Actually, $R^-(R^+(S)) = \{c, d, i\}$ and $S' \cup \{c, d, i\}$ is an admissible set but it is not included in S'. That is, $\mathcal{E}_1$ does not collapse with the set of all complete extensions of (A, R) since S' is not in $\mathcal{E}_1$.
Neither S nor S' are preferred extensions, hence neither $\mathcal{E}_1$ nor $\mathcal{E}_2$ collapse with the set of all preferred extensions (of (A, R)).

Further results

Proposition 3 and Proposition 4 are only two variants for the characterization of admissible sets, here is a longer list:

Theorem 5 *Let (A, R) be an argument system.*

$$\begin{array}{lll} S \subseteq A \text{ is an admissible set} & \text{iff} & S \subseteq Def(S) \cap \overline{R^+(S)} \\ & \text{iff} & S \subseteq Def(S \cap \overline{R^-(S)}) \\ & \text{iff} & S \subseteq Def(S) \cap \overline{R^-(S)} \\ & \text{iff} & S \subseteq Def(S \cap \overline{R^+(S)}). \end{array}$$

The same applies to the case of complete extensions, but the generalization goes even further:

Theorem 6 *Let (A, R) be an argument system. Let $S \subseteq A$. For all $X \subseteq A$ and for all $Y \subseteq A$ such that*

- *$X \subseteq Def(S) \cap \overline{R^-(S)}$ or $X \subseteq Def(S) \cap \overline{R^+(S)}$*

and

- *$\overline{R^-(S)} \cap \overline{R^+(S)} \subseteq Y \subseteq \overline{R^-(S)} \cup \overline{R^+(S)}$*

the following holds:

$$\begin{array}{lll} S \text{ is a complete extension} & \text{iff} & S = Def(S \cup X) \cap Y \\ & \text{iff} & S = Def((S \cup X) \cap Y). \end{array}$$

As regards stable extension, an analogous parameterized characterization is:

Theorem 7 *Let (A, R) be an argument system. Let $S \subseteq A$. For all $X \subseteq A$ and for all $Y \subseteq A$ such that*

- *$\overline{R^-(S)} \cap \overline{R^+(S)} \subseteq X \subseteq \overline{R^-(S)}$ and $\overline{R^+(S)} \subseteq Y \subseteq \overline{R^-(S)} \cup \overline{R^+(S)}$*

  or

- *$X = \overline{R^-(S)} \cap \overline{R^+(S)}$ and $\overline{R^-(S)} \cap \overline{R^+(S)} \subseteq Y \subseteq \overline{R^-(S)} \cup \overline{R^+(S)}$*

  the following holds:

$$\begin{array}{lll} S \text{ is a stable extension} & \text{iff} & S = Def(S \cup X) \cap Y \\ & \text{iff} & S = Def((S \cup X) \cap Y). \end{array}$$

The characterization of stable extensions also includes:

Theorem 8 *Let (A, R) be an argument system. Let $S \subseteq A$.*

- *For all $X \subseteq A$ and for all $Y \subseteq A$ such that at least one of the conditions below is satisfied*
  1. *$\overline{R^-(S)} \cap \overline{R^+(S)} \subseteq X \subseteq \overline{R^-(S)} \cup \overline{R^+(S)}$ and $\overline{R^+(S)} \subseteq Y \subseteq \overline{R^-(S)} \cup \overline{R^+(S)}$*
  2. *$\overline{R^-(S)} \cap \overline{R^+(S)} \subseteq X \subseteq Y \subseteq \overline{R^+(S)}$*

  the following property holds:

  $$S \text{ is a stable extension iff } S = Def(S \cup X) \cap Y$$

- *For all $X \subseteq A$ and for all $Y \subseteq A$ such that at least one of the conditions below is satisfied*
  1. *$\overline{R^+(S)} \cap \overline{R^-(S)} \subseteq X \subseteq \overline{R^-(S)}$ and $\overline{R^+(S)} \cap \overline{R^-(S)} \subseteq Y \subseteq \overline{R^+(S)} \cup \overline{R^-(S)}$*

2. $\overline{R^+(S)} \cap \overline{R^-(S)} \subseteq X$ *and* $\overline{R^+(S)} \cap \overline{R^-(S)} \subseteq Y \subseteq \overline{R^-(S)}$

the following property holds:

$$S \text{ is a stable extension iff } S = Def((S \cup X) \cap Y)$$

Fact 1 *The equivalence*

$$S \text{ is a stable extension iff } S = Def(S \cup X) \cap Y$$

fails in each of the following cases:

(1)	$X = \overline{R^-(S)}$	$Y = \overline{R^+(S)} \cap \overline{R^-(S)}$
(2)	$X = \overline{R^+(S)}$	$Y = \overline{R^+(S)} \cap \overline{R^-(S)}$
(3)	$X = \overline{R^+(S)} \cup \overline{R^-(S)}$	$Y = \overline{R^+(S)} \cap \overline{R^-(S)}$
(4)	$X = \overline{R^-(S)}$	$Y = \overline{R^-(S)}$
(5)	$X = \overline{R^+(S)}$	$Y = \overline{R^-(S)}$
(6)	$X = \overline{R^+(S)} \cup \overline{R^-(S)}$	$Y = \overline{R^-(S)}$

Counter-example. Let (A, R) be as indicated on Figure 4.

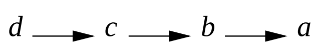

Figure 4: Argument system illustrating a counter-example of Fact 1 points (4) and (6).

Clearly, $S = \{a, d\}$ is not a stable extension. Now, $\overline{R^-(S)} = \{a, c, d\}$ and $\overline{R^+(S)} = \{a, b, d\}$.
(4) $X = \{a, c, d\}$ and $Y = \{a, c, d\}$. Then, $Def(S \cup X) = \{a, b, d\}$. Hence, $Def(S \cup X) \cap Y = S$.
(6) $X = \{a, b, c, d\}$ and $Y = \{a, c, d\}$. Then, $Def(S \cup X) = \{a, b, d\}$. So, $Def(S \cup X) \cap Y = S$.

Let (A, R) be as indicated on Figure 5.

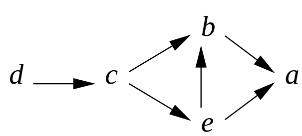

Figure 5: Argument system giving a counter-example for Fact 1 point (1).

Clearly, $S = \{a, d\}$ is not a stable extension. However, $\overline{R^-(S)} = \{a, c, d\}$ and $\overline{R^+(S)} = \{a, b, d, e\}$.
(1) $X = \{a, c, d\}$ and $Y = \{a, d\}$. Then, $Def(S \cup X) = \{a, b, d, e\}$. Therefore, $Def(S \cup X) \cap Y = S$.

Let (A, R) be as indicated on Figure 6.

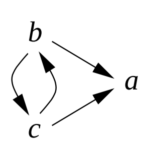

Figure 6: Argument system giving a counter-example for Fact 1 points (2),(3) and (5).

Clearly, $S = \{a\}$ is not a stable extension. However, $\overline{R^-(S)} = \{a\}$ and $\overline{R^+(S)} = \{a, b, c\}$.
(2) (3) (5) $X = \{a, b, c\}$ and $Y = \{a\}$. Thus, $Def(S \cup X) = \{a, b, c\}$. So, $Def(S \cup X) \cap Y = S$. ■

Fact 2 *The equivalence*

$$S \text{ is a stable extension iff } S = Def((S \cup X) \cap Y)$$

fails in each of the following cases:

(1)	$X = \overline{R^+(S)}$	$Y = \overline{R^+(S)}$
(2)	$X = \overline{R^+(S)} \cup \overline{R^-(S)}$	$Y = \overline{R^+(S)}$
(3)	$X = \overline{R^+(S)}$	$Y = \overline{R^+(S)} \cup \overline{R^-(S)}$

Counter-example. Let (A, R) be as indicated on Figure 7. Clearly, $S = \{a, b, g\}$ is not a stable extension. Now, $\overline{R^+(S)} = \{b, c, d\}$ and $\overline{R^-(S)} = \{d, e, f, g\}$.
(1) $X = \{b, c, d\}$ and $Y = \{b, c, d\}$. Then, $(S \cup X) \cap Y = \{b, c, d\}$. Hence, $Def((S \cup X) \cap Y) = S$.
(2) $X = \{b, c, d, e, f, g\}$ and $Y = \{b, c, d\}$. Then, $(S \cup X) \cap Y = \{b, c, d\}$. So, $Def((S \cup X) \cap Y) = S$.
(3) $X = \{b, c, d\}$ and $Y = \{b, c, d, e, f, g\}$. So, $(S \cup X) \cap Y = \{b, c, d\}$ and $Def((S \cup X) \cap Y) = S$. ■

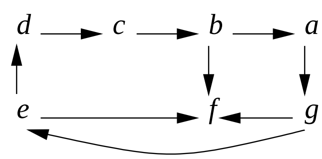

Figure 7: Argument system giving a counter-example for Fact 2.

The equations we provide make no claim to computational value: They are driven by the aim of exhibiting the sameness of the various notions of an extension as introduced by Dung. Anyway, it is unlikely that our equations for stable extensions be of any computational interest. Perhaps, the equations for complete extensions could speed up refuting that a set S is a complete extension (by choosing X to consist of an element that "obviously" prevents the equation to hold) but a careful analysis need to be conducted.

Finally, the results of the previous section extend to the generalized equations just presented.

Conclusion

Here, we have investigated semantics for acceptability in Dung's argument systems, those being semantics that rely upon his notion of admissibility.

The first admissibility-based semantics have been proposed by Dung himself. We have shown that three of them (stable, preferred and complete semantics) can be defined in terms of simple equations that a set satisfies if and only if the set is an extension under this semantics. These equations exhibit that admissibility is the core notion of Dung's semantics. The characterizations obtained are not only very general, but they also are close to one another and actually form a unified characterization. The fourth semantics defined by Dung (the grounded semantics) seems out of the scope of our characterization.

Among the other possibilities for admissibility-based semantics, the ones that induce extensions intermediate between stable extensions and complete extensions are captured by our unified characterization. We have not settled the case of the remaining admissibility-based semantics, but they presumably fall beyond the realm of our characterization.

Some semantics departing from admissibility can be found in the literature. For instance, Jakobovits and Vermeir propose in (Jakobovits & Vermeir 1999) various semantics relying upon a weaker notion of defense. It might be interesting to study whether these semantics could be captured by means of a unified characterization similar to ours.

Appendix

Proposition 1 *(Amgoud & Cayrol 1998) Let* (A, R) *be an argument system and let* $S \subseteq A$*. Then* $Def(S) = \overline{R^+(\overline{R^+(S)})}$.

Proof. (**Sketch**) By definition, all the attackers of an argument defended by S are attacked by S. Consequently, an argument defended by S is not attacked by arguments not attacked by S, that is $Def(S) \subseteq \overline{R^+(\overline{R^+(S)})}$.
Let $a \in \overline{R^+(\overline{R^+(S)})}$. Then no attacker of a belongs to $\overline{R^+(S)}$. Consequently, all the attackers of a are attacked by S, and hence $a \in Def(S)$. ■

Proposition 3 *Given* (A, R), $S \subseteq A$ *is an admissible set iff* $S \subseteq Def(S) \cap \overline{R^+(S)}$.

Proof. (only if direction) Let S be admissible. S is conflict-free. So, $S \subseteq \overline{R^+(S)}$ clearly follows. Since S is admissible, S defends each element of S. I.e., $S \subseteq Def(S)$. Overall, $S \subseteq Def(S) \cap \overline{R^+(S)}$.
(if direction) Let $S \subseteq Def(S) \cap \overline{R^+(S)}$. First, $S \subseteq \overline{R^+(S)}$. This implies that S is conflict-free. Second, $S \subseteq Def(S)$. Hence, S defends each argument in S. Together with S being conflict-free, this means that S is an admissible set. ■

Lemma 1 *Given* (A, R)*, let* $S \subseteq A$ *and* $X \subseteq A$ *be such that* $S \cup X$ *does not attack* S*. Then,* $Def((S \cup X) \cap \overline{R^-(S)}) = Def(S \cup X) \cap \overline{R^+(S)}$.

Proof. Assume that $Def(S \cup X) \not\subseteq \overline{R^+(S)}$: There exists $a \in Def(S \cup X)$ such that $a \notin \overline{R^+(S)}$. That is, $a \in R^+(S)$. Hence, there is $b \in S$ such that bRa holds. Now, $a \in Def(S \cup X)$ means that there must be $c \in S \cup X$ satisfying cRb. However, cRb contradicts the fact that $S \cup X$ does not attack S. Thus, $Def(S \cup X) \subseteq \overline{R^+(S)}$. Accordingly, $Def(S \cup X) = Def(S \cup X) \cap \overline{R^+(S)}$. ■

Proposition 4 *Given* (A, R), $S \subseteq A$ *is an admissible set iff* $S \subseteq Def(S \cap \overline{R^-(S)})$.

Proof. (only if direction) Since S is an admissible set, it is conflict-free. In other words, S does not attack S. Then, apply Lemma 1 (taking $X = \emptyset$) in view of Proposition 3.
(if direction) Let $S \subseteq Def(S \cap \overline{R^-(S)})$. Assume that S fails to be conflict-free. I.e., there exist a and b both in S such that a attacks b. Any $c \in S$ defending b would be in $R^-(S)$ because $a \in S$. So, b cannot be defended by an argument in $S \cap \overline{R^-(S)}$. This means that $b \notin Def(S \cap \overline{R^-(S)})$, contradicting $S \subseteq Def(S \cap \overline{R^-(S)})$. Hence, S is conflict-free. Then, apply Lemma 1 (take $X = \emptyset$) in view of Proposition 3. ■

Lemma 2 *S is a stable extension iff* $S = Def(S \cup X) \cap \overline{R^+(S)}$ *where* $\overline{R^+(S)} \cap \overline{R^-(S)} \subseteq X \subseteq A$.

Proof. (only if direction) Let $S \subseteq A$ be a stable extension. S is an admissible set and Proposition 3 yields $S \subseteq Def(S) \cap \overline{R^+(S)}$. So, $S \subseteq Def(S \cup X) \cap \overline{R^+(S)}$. By Proposition 2, if S is a stable extension then $\overline{R^+(S)} \subseteq S$. Thus, $Def(S \cup X) \cap \overline{R^+(S)} \subseteq S$. To sum up, $S = Def(S \cup X) \cap \overline{R^+(S)}$.
(if direction) Let $S = Def(S \cup X) \cap \overline{R^+(S)}$ and $\overline{R^+(S)} \subseteq X \subseteq A$. So, $S \subseteq \overline{R^+(S)}$. Using Proposition 2 in order to prove that S is a stable extension, there remains to show $\overline{R^+(S)} \subseteq S$. Let $a \in \overline{R^+(S)}$ and assume that $a \notin S$; $a \in \overline{R^+(S)}$ means that either a is not attacked at all, or a is attacked only by arguments which do not belong to S.
(1) If a is not attacked at all, then trivially $a \in Def(S \cup X)$ and $a \in \overline{R^+(S)}$ makes $a \in S$ to ensue: A contradiction arises.
(2) So, a is attacked but only by arguments which do not belong to S. Assume that some attackers of a are not attacked. In other words, assume that there exists some $b \notin S$ such that bRa and b is not attacked. The latter entails both $b \in Def(S \cup X)$ and $b \in \overline{R^+(S)}$, yielding $b \in S$. This is a contradiction. Hence, all the attackers b of a are attacked.
(a) If some attacker b of a (thus, $b \notin S$) has all its attackers in $\overline{S} \cap \overline{X}$, they all are in $R^+(S) \cup R^-(S)$ (cf $\overline{R^+(S)} \cap \overline{R^-(S)} \subseteq X$). Those in $R^+(S)$ are attacked by S hence $S \cup X$, those in $R^-(S)$ are attacked by $S \cup X$ due to $S = Def(S \cup X) \cap \overline{R^+(S)}$. So, $b \in Def(S \cup X)$. That b has all its attackers in $\overline{S} \cap \overline{X}$ yields $b \in \overline{R^+(S)}$. By $S = Def(S \cup X) \cap \overline{R^+(S)}$, $b \in S$ ensues: A contradiction.
(b) So, each attacker of a has at least one attacker in $S \cup X$. Therefore, $a \in Def(S \cup X)$. Then, $a \in \overline{R^+(S)}$ yields $a \in S$ which leads to a contradiction.
To sum up, there exist no argument a which belongs to $\overline{R^+(S)}$ and which does not belong to S. Combined with the fact that $S \subseteq \overline{R^+(S)}$, we have $S = \overline{R^+(S)}$, i.e. S is a stable extension. ■

Lemma 3 *For* X *and for* Y *such that* $\overline{R^+(S)} \cap \overline{R^-(S)} \subseteq Y \subseteq \overline{R^+(S)} \cup \overline{R^-(S)}$*, if S is a stable extension then* $S = Def((S \cup X) \cap Y)$.

Proof. S being conflict-free, $S = S \cap \overline{R^+(S)} \cap \overline{R^-(S)}$. Then, $S \subseteq S \cap Y$. Also, $S \cap Y \subseteq (S \cup X) \cap Y$ trivially holds. So, $S \subseteq (S \cup X) \cap Y$. As Def is monotone increasing,

$Def(S) \subseteq Def((S \cup X) \cap Y)$. Since S is a stable extension, $S = Def(S)$ and $S \subseteq Def((S \cup X) \cap Y)$ follows.
In order to show the converse inclusion, assume that it is false: There exists $a \in Def((S \cup X) \cap Y)$ such that $a \notin S$. Applying Proposition 2, $a \in R^+(S)$. By the definition, bRa for some $b \in S$. In view of $a \in Def((S \cup X) \cap Y)$, there thus exists $c \in (S \cup X) \cap Y$ satisfying cRb. So, $c \in Y$. There are only two possibilities. The first one is $c \in \overline{R^-(S)}$, which contradicts cRb and $b \in S$. The second one is $c \in \overline{R^+(S)}$. Due to cRb and $b \in S = Def(S)$, there must exist $d \in S$ such that dRc. Hence, $c \in R^+(S)$ and a contradiction arises as well. ■

Lemma 4 *For X and Y such that $\overline{R^+(S)} \cap \overline{R^-(S)} \subseteq X$ and $\overline{R^+(S)} \cap \overline{R^-(S)} \subseteq Y \subseteq \overline{R^-(S)}$, if $S = Def((S \cup X) \cap Y)$ then S is a stable extension.*

Proof. Let $S = Def((S \cup X) \cap Y)$. In order to show $S \subseteq \overline{R^+(S)}$, assume that S fails to be conflict-free. There exist $a \in S$ and $b \in S$ such that bRa. However, $S = Def((S \cup X) \cap Y)$ then entails that a must be defended against b by $c \in (S \cup X) \cap Y$. So, cRb and $c \in Y$. By $Y \subseteq \overline{R^-(S)}$, it follows that $c \in \overline{R^-(S)}$. The latter is contradicted by cRb because $b \in S$. Hence, S is conflict-free. Consequently, $S \subseteq \overline{R^+(S)}$. Using Proposition 2, there only remains to apply Lemma 5. ■

Lemma 5 *Let $X \subseteq A$ and $Y \subseteq A$ be such that $\overline{R^+(S)} \cap \overline{R^-(S)} \subseteq X$ and $\overline{R^+(S)} \cap \overline{R^-(S)} \subseteq Y$. If $S = Def((S \cup X) \cap Y)$ and S is conflict-free then $\overline{R^+(S)} \subseteq Def((S \cup X) \cap Y)$.*

Proof. Let $S = Def((S \cup X) \cap Y)$. Let $a \in \overline{R^+(S)}$. Assume $a \notin Def((S \cup X) \cap Y)$. There exists $b \in A$ such that bRa and $c \in \overline{Y} \cup (\overline{S} \cap \overline{X})$ whenever cRb. Of course, $b \notin S$ due to $a \in \overline{R^+(S)}$. So, $b \notin Def((S \cup X) \cap Y)$. Hence, there exists $c \in A$ such that cRb and $d \in (\overline{S} \cap \overline{X}) \cup \overline{Y}$ whenever dRc. As indicated, $c \in \overline{Y} \cup (\overline{S} \cap \overline{X})$. I.e., $c \in \overline{X} \cup \overline{Y}$. By $\overline{R^+(S)} \cap \overline{R^-(S)} \subseteq X$ and $\overline{R^+(S)} \cap \overline{R^-(S)} \subseteq Y$, either $c \in R^-(S)$ or $c \in R^+(S)$. In view of $S = Def((S \cup X) \cap Y)$, the former case requires $d \in (S \cup X) \cap Y$ to exist such that dRc, which is impossible as just shown. The second case implies that there exists $d \in S$ such that dRc. However, $d \in (\overline{S} \cap \overline{X}) \cup \overline{Y}$ (cf above). Then, $d \in \overline{Y}$ and a consequence of $\overline{R^+(S)} \cap \overline{R^-(S)} \subseteq Y$ is that either $d \in R^+(S)$ or $d \in R^-(S)$ holds. Either option is contradicted by $d \in S$ because S is conflict-free. Accordingly, the assumption must be false. Therefore, $\overline{R^+(S)} \subseteq Def((S \cup X) \cap Y)$. ■

Lemma 6 *Let X and Y be such that $\overline{R^+(S)} \cap \overline{R^-(S)} \subseteq X$ and $\overline{R^+(S)} \cap \overline{R^-(S)} \subseteq Y \subseteq \overline{R^-(S)}$. S is a stable extension iff $S = Def((S \cup X) \cap Y)$.*

Proof. Apply Lemma 4 and Lemma 3. ■

Lemma 7 *Given (A, R), $S \subseteq A$ is a preferred extension iff $S = Def(S \cup X) \cap \overline{R^+(S)}$ where $\emptyset \subseteq X \subseteq Def(S) \cap \overline{R^+(S)}$ if S is a preferred extension and $\overline{R^+(S)} \subseteq X \subseteq A$ otherwise.*

Proof. Let $S \subseteq A$ and $X \subseteq A$ be such that $\emptyset \subseteq X \subseteq Def(S) \cap \overline{R^+(S)}$ if S is a preferred extension whereas $\overline{R^+(S)} \subseteq X \subseteq A$ if S is not a preferred extension.
(only if direction) S is a preferred extension. In view of the assumption, $\emptyset \subseteq X \subseteq Def(S) \cap \overline{R^+(S)}$. Now, S is also a complete extension hence Theorem 6 implies that $S = Def(S \cup X) \cap \overline{R^+(S)}$.
(if direction) Consider the case that S is not a preferred extension even though $S = Def(S \cup X) \cap \overline{R^+(S)}$. By the assumption, $\overline{R^+(S)} \subseteq X \subseteq A$. Lemma 2 then implies that $S = Def(S \cup X) \cap \overline{R^+(S)}$ fails and a contradiction arises. ■

Lemma 8 *Given (A, R), $S \subseteq A$ is a preferred extension iff $S = Def((S \cup X) \cap \overline{R^-(S)})$ where $\emptyset \subseteq X \subseteq Def(S) \cap \overline{R^+(S)}$ if S is a preferred extension and $\overline{R^+(S)} \subseteq X \subseteq A$ otherwise.*

Proof. Let $S \subseteq A$ and $X \subseteq A$ be such that $\emptyset \subseteq X \subseteq Def(S) \cap \overline{R^+(S)}$ if S is a preferred extension whereas $\overline{R^+(S)} \subseteq X \subseteq A$ if S is not a preferred extension.
(only if direction) S is a preferred extension. In view of the assumption, $\emptyset \subseteq X \subseteq Def(S) \cap \overline{R^+(S)}$. Now, S is also a complete extension hence Theorem 6 yields $S = Def((S \cup X) \cap \overline{R^-(S)})$.
(if direction) Consider the case that S is not a preferred extension. By the assumption, $\overline{R^+(S)} \subseteq X \subseteq A$. Lemma 4 then implies that $S = Def((S \cup X) \setminus R^-(S))$ fails, which is a contradiction. ■

Theorem 1 *Let (A, R) be an argument system. For $S \subseteq A$, the statements below are equivalent:*

- *S is a Dung extension under the t semantics*
- *$S = Def(S \cup X) \cap \overline{R^+(S)}$*
- *$S = Def((S \cup X) \cap \overline{R^-(S)})$*

where X is an arbitrary set of arguments such that

- *case t = complete* $\{ \emptyset \subseteq X \subseteq Def(S) \cap \overline{R^+(S)}$
- *case t = preferred* $\left\{ \begin{array}{l} \emptyset \subseteq X \subseteq Def(S) \cap \overline{R^+(S)} \\ \text{if } S \text{ is a preferred extension,} \\ \overline{R^+(S)} \subseteq X \subseteq A \text{ otherwise} \end{array} \right.$
- *case t = stable* $\{ \overline{R^+(S)} \subseteq X \subseteq A$

Proof.

- case t = complete: Theorem 6.
- case t = preferred: Lemma 7 and Lemma 8.
- case t = stable: Lemma 2 and Lemma 6.

■

Theorem 2 *Let (A, R) be an argument system. Consider $\mathcal{E}$ such that $\mathcal{S} \subseteq \mathcal{E} \subseteq \mathcal{C}$.*

- *There exists $\chi : 2^A \to 2^A$ such that $S \in \mathcal{E}$ iff $S = Def(S \cup \chi(S)) \cap \overline{R^+(S)}$.*

- *There exists* $\chi : 2^A \rightarrow 2^A$ *such that* $S \in \mathcal{E}$ *iff* $S = Def((S \cup \chi(S)) \cap \overline{R^-(S)})$.

Proof. Consider $\mathcal{E}$ such that $\mathcal{S} \subset \mathcal{E} \subset \mathcal{C}$ (the two cases for improper inclusion have been dealt with in Theorem 1) and we show that there exists $\chi : 2^A \rightarrow 2^A$ which makes the equations to characterize $\mathcal{E}$ as indicated in the statement of the theorem.
For all $S \subseteq A$, define $\chi(S)$ so as to enforce both
(i) $\emptyset \subseteq \chi(S) \subseteq Def(S) \cap \overline{R^+(S)}$ if $S \in \mathcal{E}$ and
(ii) $\overline{R^+(S)} \subseteq \chi(S) \subseteq A$ if $S \notin \mathcal{E}$.
Clearly, there exists at least one such function χ (for (i), take $\chi(S) = \emptyset$ and for (ii), take $\chi(S) = A$). So, let χ be any function from 2^A to 2^A satisfying (i) and (ii). Let $S \subseteq A$.
Consider first the case $S \notin \mathcal{E}$. By the assumption, $\overline{R^+(S)} \subseteq \chi(S) \subseteq A$. Also, S is not a stable extension due to $\mathcal{S} \subset \mathcal{E} \subset \mathcal{C}$. Applying Theorem 1, $S = Def(S \cup \chi(S)) \cap \overline{R^+(S)}$ and $S = Def((S \cup \chi(S)) \setminus R^-(S))$ both fail. By contraposition, if S satisfies any of the equations then $S \in \mathcal{E}$.
Consider now the case $S \in \mathcal{E}$. By the assumption, $\emptyset \subseteq \chi(S) \subseteq Def(S) \cap \overline{R^+(S)}$. As S is a complete extension due to $\mathcal{S} \subset \mathcal{E} \subset \mathcal{C}$, Theorem 1 yields $S = Def(S \cup \chi(S)) \cap \overline{R^+(S)}$ and $S = Def((S \cup \chi(S)) \cap \overline{R^-(S)})$. That is, if $S \in \mathcal{E}$ then S satisfies both equations. ■

Lemma 9 *Whatever* $X \subseteq A$, *if* S *is a stable extension then* $S = Def(S \cup X) \cap \overline{R^+(S)}$.

Proof. Let $S \subseteq A$ be a stable extension. As a consequence, S is an admissible set and Proposition 3 yields $S \subseteq Def(S) \cap \overline{R^+(S)}$. Therefore, it follows that $S \subseteq Def(S \cup X) \cap \overline{R^+(S)}$. By Proposition 2, if S is a stable extension then $\overline{R^+(S)} \subseteq S$. Accordingly, $Def(S \cup X) \cap \overline{R^+(S)} \subseteq S$. To sum up, $S = Def(S \cup X) \cap \overline{R^+(S)}$. ■

Lemma 10 *Let* $X : 2^A \rightarrow 2^A$. *If* S *is a stable extension then* $S = Def(S \cup X(S)) \cap \overline{R^+(S)}$ *and* $S = Def((S \cup X(S)) \setminus R^-(S))$.

Proof. By Lemma 9 and Lemma 3 where $Y = \overline{R^-(S)}$. ■

Lemma 11 *Whatever* $X \subseteq A$, *if* $S = Def(S \cup X) \cap \overline{R^+(S)}$ *or* $S = Def((S \cup X) \cap \overline{R^-(S)})$ *then* S *is conflict-free.*

Proof. Let $S \subseteq A$ such that $S = Def(S \cup X) \cap \overline{R^+(S)}$. Then $S \subseteq \overline{R^+(S)}$, so S is conflict-free.
Let $S \subseteq A$ such that $S = Def((S \cup X) \cap \overline{R^-(S)})$. Assume that S fails to be conflict-free. So, there exist $a \in S$ and $b \in S$ such that aRb. Yet, $S = Def((S \cup X) \setminus R^-(S))$ entails that b is defended against a by $c \in (S \cup X) \setminus R^-(S)$. In symbols, cRa and $c \in S \cup X$ while $c \notin R^-(S)$. The latter is contradicted by cRa because $a \in S$. Thus, S is conflict-free. ■

Lemma 12 *Whatever* $X \subseteq A$, *if* $S \subseteq A$ *is a self-defending set satisfying either* $S = Def(S \cup X) \cap \overline{R^+(S)}$ *or* $S = Def((S \cup X) \setminus R^-(S))$ *then* S *is a complete extension.*

Proof. Let $S \subseteq A$ be such that $S \subseteq Def(S)$ and $S = Def(S \cup X) \cap \overline{R^+(S)}$. I.e., $S \subseteq \overline{R^+(S)}$ and S is conflict-free. Assume that $Def(S) \subseteq \overline{R^+(S)}$ fails: there exists $a \in Def(S)$ while $a \notin \overline{R^+(S)}$. So, $a \in Def(S)$ and $a \in R^+(S)$. By $a \in R^+(S)$, there must exist $b \in S$ such that bRa. By $a \in Def(S)$, there must then exist $c \in S$ such that cRb. As both b and c are in S, this contradicts the fact that S is conflict-free. Therefore, the assumption must be false. So, $Def(S) \subseteq \overline{R^+(S)}$. However, a consequence of Def being monotone increasing is $Def(S) \subseteq Def(S \cup X)$. Combining this with $Def(S) \subseteq \overline{R^+(S)}$ as was just proved, $Def(S) \subseteq Def(S \cup X) \cap \overline{R^+(S)}$. Hence, $Def(S) \subseteq S$. Together with the fact that S is a self-defending set and S is conflict-free, this means that S is a complete extension.
Let $S \subseteq A$ be such that $S \subseteq Def(S)$ and $S = Def((S \cup X) \setminus R^-(S))$. According to Lemma 11, S is conflict-free. As Def is monotone increasing, $Def(S) \subseteq Def(S \cup (X \setminus R^-(S)))$. Due to S being conflict-free, $S \subseteq \overline{R^-(S)}$ and $S = S \setminus R^-(S)$. Therefore, $Def(S) \subseteq Def((S \setminus R^-(S)) \cup (X \setminus R^-(S)))$. I.e., $Def(S) \subseteq Def((S \cup X) \setminus R^-(S))$. Since $S = Def((S \cup X) \setminus R^-(S))$, it follows that $Def(S) \subseteq S$. ■

Theorem 3 *Let* (A, R) *be an argument system.*

- *Consider* $\chi : 2^A \rightarrow 2^A$ *such that for all* $S \subseteq A$, *if* $S = Def(S \cup \chi(S)) \cap \overline{R^+(S)}$ *then* $S \cup \chi(S)$ *is an admissible set.*
  Then, $\{S \subseteq A \mid S = Def(S \cup \chi(S)) \cap \overline{R^+(S)}\}$ *is some* $\mathcal{E}$ *satisfying* $\mathcal{S} \subseteq \mathcal{E} \subseteq \mathcal{C}$.
- *Consider* $\chi : 2^A \rightarrow 2^A$ *such that for all* $S \subseteq A$, *if* $S = Def((S \cup \chi(S)) \cap \overline{R^-(S)})$ *then* $S \cup \chi(S)$ *is an admissible set.*
  Then, $\{S \subseteq A \mid S = Def((S \cup \chi(S)) \cap \overline{R^-(S)})\}$ *is some* $\mathcal{E}$ *satisfying* $\mathcal{S} \subseteq \mathcal{E} \subseteq \mathcal{C}$.

Proof. Consider $\chi : 2^A \rightarrow 2^A$ such that for all $S \subseteq A$, if $S = Def(S \cup \chi(S)) \cap \overline{R^+(S)}$ (resp., $S = Def((S \cup \chi(S)) \cap \overline{R^-(S)})$) then $S \cup \chi(S)$ is an admissible set. We aim at proving that the solutions of the equations form some $\mathcal{E} \subseteq 2^A$ such that $\mathcal{S} \subseteq \mathcal{E} \subseteq \mathcal{C}$.
Let $\mathcal{E}$ denote the set of all $S \subseteq A$ such that $S = Def(S \cup \chi(S)) \cap \overline{R^+(S)}$ (resp., $S = Def((S \cup \chi(S)) \cap \overline{R^-(S)})$). Consider $S \subseteq A$ satisfying the equation at hand. By the property that χ is assumed to enjoy, it follows that $S \cup \chi(S)$ is an admissible set. Applying Lemma 12, S is a complete extension. We have thus shown $\mathcal{E} \subseteq \mathcal{C}$. We now turn to the case that S is a stable extension. By Lemma 10, $S = Def(S \cup \chi(S)) \cap \overline{R^+(S)}$ (resp., $S = Def((S \cup \chi(S)) \cap \overline{R^-(S)})$). Therefore, $\mathcal{S} \subseteq \mathcal{E}$. ■

Theorem 4 *Given* (A, R), *let each* $S \subseteq A$ *be attached some* $F_S \subseteq A$. *Define* $\chi : 2^A \rightarrow 2^A$ *to be the function such that*

$$\chi(S) = \begin{cases} F_S & \text{if } S \cup F_S \text{ is an admissible set} \\ \emptyset & \text{otherwise.} \end{cases}$$

For all $S \subseteq A$, *the statements below are equivalent:*

- S *is a complete extension such that if* $S \cup F_S$ *is an admissible set then* $F_S \subseteq S$

- $S = Def(S \cup \chi(S)) \cap \overline{R^{+}(S)}$
- $S = Def((S \cup \chi(S)) \cap \overline{R^{-}(S)})$

Proof. (only if direction) Let $S = Def(S \cup \chi(S)) \cap \overline{R^{+}(S)}$ where $\chi(S) = F_S$ if $S \cup F_S$ is an admissible set and $\chi(S) = \emptyset$ otherwise. First, if $S \cup F_S$ is not an admissible set then $S = Def(S) \cap \overline{R^{+}(S)}$. According to Theorem 1, S is then a complete extension. Second, if $S \cup F_S$ is an admissible set then $S = Def(S \cup F_S) \cap \overline{R^{+}(S)}$. Let us show that in this case, $F_S \subseteq S$. Since $S \cup F_S$ is an admissible set (and thus a conflict-free set), $S \cup F_S \subseteq \overline{R^{+}(S)}$. Since $S \cup F_S$ is an admissible set, $S \cup F_S$ defends all its elements. In symbols, $S \cup F_S \subseteq Def(S \cup F_S)$. To sum up, $S \cup F_S \subseteq Def(S \cup F_S) \cap \overline{R^{+}(S)}$. I.e., $S \cup F_S \subseteq S$. So, $S \cup F_S = S$. Hence, $S = Def(S) \cap \overline{R^{+}(S)}$. By Theorem 1, S is a complete extension.
Let $S = Def((S \cup \chi(S)) \cap \overline{R^{-}(S)})$ where $\chi(S) = F_S$ if $S \cup F_S$ is an admissible set and $\chi(S) = \emptyset$ otherwise. First, if $S \cup F_S$ is not an admissible set then $S = Def(S \cap \overline{R^{-}(S)})$. According to Theorem 1, S is then a complete extension. Second, if $S \cup F_S$ is an admissible set then $S = Def((S \cup F_S) \cap \overline{R^{-}(S)})$. Let us show that in this case, $F_S \subseteq S$. Since $S \cup F_S$ is an admissible set (and thus a conflict-free set), $S \cup F_S \subseteq \overline{R^{-}(S)}$. So $(S \cup F_S) \cap \overline{R^{-}(S)} = S \cup F_S$. Since $S \cup F_S$ is an admissible set, $S \cup F_S$ defends all its elements. In symbols, $S \cup F_S \subseteq Def(S \cup F_S)$. To sum up, $S \cup F_S \subseteq Def((S \cup F_S) \cap \overline{R^{-}(S)})$. I.e. $S \cup F_S \subseteq S$. So, $S \cup F_S = S$. Hence, $S = Def(S \cap \overline{R^{-}(S)})$. By Theorem 1, S is a complete extension.
(if direction) Let S be a complete extension such that if $S \cup F_S$ is an admissible set then $F_S \subseteq S$. Consider the function $\chi : 2^A \to 2^A$ such that $\chi(S) = F_S$ if $S \cup F_S$ is an admissible set and $\chi(S) = \emptyset$ otherwise. Since S is a complete extension, Theorem 1 yields $S = Def(S) \cap \overline{R^{+}(S)}$ and $S = Def(S \cap \overline{R^{-}(S)})$. If $S \cup F_S$ is an admissible set then $S \cup F_S = S$ and $\chi(S) = F_S$. Therefore, $S = Def(S \cup \chi(S)) \cap \overline{R^{+}(S)}$ and $S = Def((S \cup \chi(S)) \cap \overline{R^{-}(S)})$ hold if $S \cup F_S$ is an admissible set. ∎

Theorem 5 *Let (A, R) be an argument system.*

$S \subseteq A$ is an admissible set	*iff*	$S \subseteq Def(S) \cap \overline{R^{+}(S)}$
	iff	$S \subseteq Def(S \cap \overline{R^{-}(S)})$
	iff	$S \subseteq Def(S) \cap \overline{R^{-}(S)}$
	iff	$S \subseteq Def(S \cap \overline{R^{+}(S)})$.

Proof.

- $S \subseteq A$ is an admissible set iff $S \subseteq Def(S) \cap \overline{R^{+}(S)}$: by Proposition 3.
- $S \subseteq A$ is an admissible set iff $S \subseteq Def(S \cap \overline{R^{-}(S)})$: by Proposition 4.
- $S \subseteq A$ is an admissible set iff $S \subseteq Def(S) \cap \overline{R^{-}(S)}$:
  - (only if direction) Let $S \subseteq A$ be an admissible set. By the definition, S is conflict-free. Hence, $S \subseteq \overline{R^{-}(S)}$. That S is an admissible set yields $S \subseteq Def(S)$. Overall, $S \subseteq Def(S) \cap \overline{R^{-}(S)}$.
  - (if direction) Assume that $S \subseteq Def(S) \cap \overline{R^{-}(S)}$ holds. Therefore, $S \subseteq Def(S)$ (i.e., S defends each element of S) and $S \subseteq \overline{R^{-}(S)}$ (that is, S is conflict-free). So, S is an admissible set.
- $S \subseteq A$ is an admissible set iff $S \subseteq Def(S \cap \overline{R^{+}(S)})$:
  - (only if direction) Let $S \subseteq A$ be an admissible set. Therefore, S is conflict-free: $S \subseteq \overline{R^{+}(S)}$. So, $S = S \cap \overline{R^{+}(S)}$. As S is an admissible set, $S \subseteq Def(S)$. Substituting, $S \subseteq Def(S \cap \overline{R^{+}(S)})$.
  - (if direction) Let $S \subseteq Def(S \cap \overline{R^{+}(S)})$. Trivially, $S \cap \overline{R^{+}(S)} \subseteq S$. Since Def is monotone increasing, $Def(S \cap \overline{R^{+}(S)}) \subseteq Def(S)$. That is, $S \subseteq Def(S)$. Assume now that S fails to be conflict-free: There exist a and b in S such that bRa holds. By $a \in S$ and $S \subseteq Def(S \cap \overline{R^{+}(S)})$, there exists $c \in S$ such that cRb and $c \notin R^{+}(S)$. In view of $b \in S$ and $S \subseteq Def(S \cap \overline{R^{+}(S)})$, there exists $d \in S$ such that dRc. This contradicts $c \notin R^{+}(S)$. So, S is conflict-free. Hence, S is an admissible set.

∎

Lemma 13 *For all $X \subseteq Def(S)$ and for all Y such that $\overline{R^{-}(S)} \cap \overline{R^{+}(S)} \subseteq Y$, if S is a complete extension then $S = Def(S \cup X) \cap Y$ and $S = Def((S \cup X) \cap Y)$.*

Proof. As S is a complete extension, it is conflict-free: $S \subseteq \overline{R^{-}(S)} \cap \overline{R^{+}(S)}$. Then, $S \subseteq Y$. I.e., $S \cap Y = S$. A consequence of S being a complete extension is $S = Def(S)$. Accordingly, $X \subseteq S$. Hence $S \cup X = S$. On the one hand, applying $S = Def(S)$ and $S \cup X = S$ to $S = S \cap Y$ yield $S = Def(S) \cap Y = Def(S \cup X) \cap Y$. On the other hand, applying $S = S \cap Y$ and $S \cup X = S$ to $S = Def(S)$ yield $S = Def(S \cap Y) = Def((S \cup X) \cap Y)$. ∎

Lemma 14 *For $X \subseteq Def(S)$ such that $X \subseteq \overline{R^{-}(S)}$ or $X \subseteq \overline{R^{+}(S)}$ and for $Y \subseteq \overline{R^{-}(S)} \cup \overline{R^{+}(S)}$, if $S = Def((S \cup X) \cap Y)$ then S is conflict-free.*

Proof. Assume that S is not conflict-free: There exist a and b in S such that bRa. So, there exist $c \in S \cup X$ and $c \in Y$ such that cRb. Similarly, there exist $d \in S \cup X$ and $d \in Y$ such that dRc. That $c \in Y$ gives only two possibilities. Observe that $c \in \overline{R^{-}(S)}$ is impossible since cRb and $b \in S$. Then, c must be in $\overline{R^{+}(S)}$. So, $d \notin S$. That is, $d \in X$. Consider first the case $X \subseteq \overline{R^{-}(S)}$. Hence, $d \in \overline{R^{-}(S)}$. It follows that $c \notin S$. So, $c \in X$ and $c \in \overline{R^{-}(S)}$. This is impossible as already noticed. Second, consider the case $X \subseteq \overline{R^{+}(S)}$. Hence, $d \in \overline{R^{+}(S)}$. Whether $c \in X$ or $c \in S$, there exists $e \in S \cup X$ satisfying eRd. Yet, $d \in \overline{R^{+}(S)}$ implies $e \in X$. Also, $d \in X \subseteq Def(S)$ entails fRe for some $f \in S$ and a contradiction arises as $e \in X$ yields $e \in \overline{R^{+}(S)}$. To sum up, S is conflict-free. ∎

Lemma 15 *Let $X \subseteq Def(S)$ and Y be such that $\overline{R^-(S)} \cap \overline{R^+(S)} \subseteq Y$ where S is conflict-free. If $S = Def((S \cup X) \cap Y)$ then S is a complete extension.*

Proof. Let $S = Def((S \cup X) \cap Y)$. Let us first show $X \subseteq Y$. The fact that S is conflict-free and $X \subseteq Def(S)$ clearly make it impossible for an element of X to be attacked by S. In symbols, $X \subseteq \overline{R^+(S)}$. In order to prove $X \subseteq Y$, there only remains to show $X \subseteq \overline{R^-(S)}$. Assume that the contrary holds: bRa for some $a \in S$ and some $b \in X$. By $a \in S = Def((S \cup X) \cap Y)$, there exists $c \in S \cup X$ such that cRb. In view of $b \in X \subseteq Def(S)$, there exists $d \in S$ such that dRc. Hence, $c \notin \overline{R^+(S)}$ and $X \subseteq \overline{R^+(S)}$ entails $c \notin X$. So, $c \in S$. Yet, cRb and $b \in X$ contradict $X \subseteq \overline{R^+(S)}$. As a result, the assumption must be false. I.e., $X \subseteq \overline{R^-(S)}$. Overall, $X \subseteq Y$.
Since S is conflict-free, $S \subseteq \overline{R^-(S)} \cap \overline{R^+(S)}$. As a consequence, $S \subseteq Y$ and $S \cap Y = S$. Clearly, $X \subseteq Y$ yields $X \cap Y = X$. Therefore, $S \cup X = (S \cap Y) \cup (X \cap Y) = (S \cup X) \cap Y$. By substitution, $S = Def(S \cup X)$. As Def is monotone increasing, $Def(S) \subseteq Def(S \cup X)$ hence $Def(S) \subseteq S$. So, $X \subseteq Def(S)$ yields $X \subseteq S$ and $S \cup X = S$. Since $S = Def(S \cup X)$ was just proven, $S = Def(S)$. Now, S is also conflict-free and it follows that S is a complete extension. ■

Lemma 16 *For $X \subseteq Def(S)$ such that $X \subseteq \overline{R^-(S)}$ or $X \subseteq \overline{R^+(S)}$ and for $Y \subseteq \overline{R^-(S)} \cup \overline{R^+(S)}$, if $S = Def(S \cup X) \cap Y$ then S is conflict-free.*

Proof. Let $S = Def(S \cup X) \cap Y$ where $Y \subseteq \overline{R^-(S)} \cup \overline{R^+(S)}$. Assume that S fails to be conflict-free: There exist $a \in S$ and $b \in S$ satisfying bRa. By $S = Def(S \cup X) \cap Y$, it follows that b is in Y. Clearly, $b \notin \overline{R^-(S)}$. In view of $Y \subseteq \overline{R^-(S)} \cup \overline{R^+(S)}$, the only possibility remaining for b to be in Y is $b \in \overline{R^+(S)}$. However, bRa and $a \in S = Def(S \cup X) \cap Y$ imply that there exists $c \in S \cup X$ such that cRb. Then, $b \in \overline{R^+(S)}$ makes c to be in X. Should $X \subseteq \overline{R^-(S)}$, it follows that $c \in \overline{R^-(S)}$ which is a contradiction because cRb and $b \in S$. Should $X \subseteq \overline{R^+(S)}$, it follows that $c \in \overline{R^+(S)}$. By cRb and $b \in S = Def(S \cup X) \cap Y$, there exists $d \in S \cup X$ satisfying dRc. As $c \in X \subseteq Def(S)$, there exists $e \in S$ such that eRd. Now, $c \in \overline{R^+(S)}$ prevents d to be in S. So, $d \in X$ and $d \in \overline{R^+(S)}$. This is a contradiction due to $e \in S$ and eRd. Overall, the assumption must be false. Therefore, S is conflict-free. ■

Lemma 17 *Let $X \subseteq Def(S)$ and Y be such that $\overline{R^-(S)} \cap \overline{R^+(S)} \subseteq Y$ where S is conflict-free. If $S = Def(S \cup X) \cap Y$ then $S = Def(S)$.*

Proof. In order to prove $Def(S) \subseteq Y$, assume that there exists some $a \in Def(S)$ such that $a \notin Y$. According to $\overline{R^-(S)} \cap \overline{R^+(S)} \subseteq Y$, either $a \notin \overline{R^-(S)}$ or $a \notin \overline{R^+(S)}$. I.e., $a \in R^-(S)$ or $a \in R^+(S)$. Consider first $a \in R^-(S)$. By the definition, aRb for some $b \in S$. Due to $S = Def(S \cup X) \cap Y$, there exists $c \in S \cup X$ such that cRa. In view of $a \in Def(S)$, there exists $d \in S$ satisfying dRc. As S is conflict-free, $c \notin S$. Hence, $c \in X \subseteq Def(S)$. There must then be $e \in S$ such that eRd, which contradicts S being conflict-free. Second, consider $a \in R^+(S)$. By the definition, bRa holds for some $b \in S$. By $a \in Def(S)$, there exists $c \in S$ satisfying cRb. A contradiction arises, in view of S being conflict-free. Therefore, $Def(S) \subseteq Y$.
Trivially, $S \subseteq S \cup X$. Therefore, $Def(S) \subseteq Def(S \cup X)$. So, $Def(S) \cap Y \subseteq Def(S \cup X) \cap Y$. By substitution, $Def(S) \cap Y \subseteq S$. In view of $X \subseteq Def(S)$ and $X \subseteq Y$ (which $X \subseteq Def(S)$ and $Def(S) \subseteq Y$ entail), $X \subseteq S$ follows. Hence, $S \cup X = S$. Then, $Def(S \cup X) \cap Y = Def(S) \cap Y$. That is, $S = Def(S) \cap Y$. Using $Def(S) \subseteq Y$ then yields $S = Def(S)$. ■

Theorem 6 *Let (A, R) be an argument system. Let $S \subseteq A$. For all $X \subseteq A$ and for all $Y \subseteq A$ such that*

- $X \subseteq Def(S) \cap \overline{R^-(S)}$ *or* $X \subseteq Def(S) \cap \overline{R^+(S)}$ *and*
- $\overline{R^-(S)} \cap \overline{R^+(S)} \subseteq Y \subseteq \overline{R^-(S)} \cup \overline{R^+(S)}$

the following holds:

S is a complete extension *iff* $S = Def(S \cup X) \cap Y$
iff $S = Def((S \cup X) \cap Y)$.

Proof. S is a complete extension implies both $S = Def(S \cup X) \cap Y$ and $S = Def((S \cup X) \cap Y)$ by Lemma 13. $S = Def(S \cup X) \cap Y$ implies S is a complete extension by Lemma 14 and Lemma 15. $S = Def((S \cup X) \cap Y)$ implies S is a complete extension by Lemma 16 and Lemma 17. ■

Due to lack of space, proofs of Theorem 7 and Theorem 8 are omitted but can be found in (Besnard & Doutre 2003).

References

Amgoud, L., and Cayrol, C. 1998. On the acceptability of arguments in preference-based argumentation. In *14th Conference on Uncertainty in Artificial Intelligence (UAI'98), Madison, Wisconsin*, 1–7. San Francisco, California: Morgan Kaufmann.

Besnard, P., and Doutre, S. 2003. Characterization of semantics for argument systems. Technical Report 2003-24-R, IRIT.

Chesñevar, C.; Maguitman, A.; and Loui, R. 2000. Logical Models of Argument. *ACM Computing Surveys* 32(4):337–383.

Dung, P. 1995. On the acceptability of arguments and its fundamental role in non-monotonic reasoning, logic programming and n-person games. *Artificial Intelligence* 77:321–357.

Jakobovits, H., and Vermeir, D. 1999. Robust Semantics for Argumentation Frameworks. *Journal of Logic and Computation* 9(2):215–261.

Prakken, H., and Vreeswijk, G. 2002. *Handbook of Philosophical Logic*. Dordrecht/Boston/London: Kluwer Academic Publishers, second edition. chapter Logics for Defeasible Argumentation, 219–318.