How to Interweave Knowledge about Object Structure and Concepts

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Abstract

This article presents a general framework for integrating reasoning about object structure and concept taxonomies. The structural relations in the domain of objects discussed are mereological relations (part, overlap, etc.). Relations defining the concept taxonomies are taken from the standard relations of subsumption hierarchies. A small set of (homogeneous) predicators is introduced to relate the two areas. Predicators can be used to distinguish different modes of predication, corresponding to different types of associated inference patterns. Reasoning about different modes of predication depends both on the theory of objects and the theory of concepts. The theories of objects, concepts, and predicators are formulated in an axiomatic manner. Composition tables can be formally derived from the theories. Consequently, constraint propagation approaches can be easily extended to handle reasoning in the interface of conceptual knowledge and object structure.

Introduction

Initiated by Allen's series of articles on the representation and processing of temporal knowledge (Allen 1983, 1984; Allen and Kautz 1985), networks of constraints and constraint propagation based on composition tables have become an important methodological framework for knowledge representation in the areas of qualitative reasoning. Allen applied this method to reasoning about (temporal) relations between periods of time.

The majority of contributions that focus on the application of constraint-based reasoning and composition tables for specific systems of relations consider the domains of time or space (Nebel and Bürckert 1995; Gotts, Gooday and Cohn 1996; Renz and Nebel 1998, 1999; Egenhofer and Rodríguez 1999). The systems of relations evaluated regarding their computational properties are relations between temporal entities (moments or periods of time) or between spatial entities (points or regions of space).

In addition to the ontological framework of time periods, Allen (1984), following McDermott (1982), introduced reified state types, which he calls 'property', and reified event types, which he calls just 'event', as the basis for representing what was the case or what happened.[1] State types and event types are treated as entities of an ontological status similar to concepts in semantic networks. State types can hold over a period of time and event types can occur at moments or in periods of time. Binary relations between time periods and state types or event types represent such relations. Given that t stands for a period of time and Θ stands for a state type, $\mathsf{HOLDS}(t, \Theta)$ expresses that the state type Θ holds during t. A relation like HOLDS, which relates time periods and state types, will be called 'predicator' in the following.[2]

Galton (1990) elaborated Allen's approach introducing a variety of predicators for state types that differ regarding the interaction between predication and temporal relations. Galton introduced the two predicators called 'HOLDS-ON' and 'HOLDS-IN' to be able to distinguish whether the state type applies to every (moment or period of) time or to some (moment or period of) time within the period. For example, if a car parks in front of my house from noon to mid-night it occupies the same spatial region throughout that period of time. In contrast, if the car (continuously) moves through the street between noon and 5 minutes past noon, it occupies different places within that period and does not stay at any place during any sub-period. Correspondingly, 'the car is in front of my house' is a state type that can be true throughout a period or sometime within a period. These different ways of relating state types to time periods correspond to specific valid inference patterns. If a state type applies throughout a super-period, then the same

[1] Allen's properties are one kind of state type, called 'state of position' by Galton (1990). The concept of 'raining', which is primarily ascribed to time periods and can only indirectly be related to points in time, can also be considered a state type, called 'state of motion' by Galton (1990). Allen calls this kind of state type 'process' and groups processes and event types under the term 'occurrence'.

[2] The view on the relation between time and state types presented here is the view specified in the formalisms of Allen (1984) and Galton (1990). Other approaches argue that state types and event types are basically related to a specific type of entities (sometimes called 'eventuality' (Bach 1986) or 'situation' (Mourelatos 1978)) that have spatial and temporal locations. However, if these entities can be related by the relation part-of and homogeneous predication is suitable, then the general approach outlined here can be applied straightforwardly to predicators connecting situations and situation types.

holds for its sub-periods, and if a state type applies sometime within a sub-period, then it applies sometime within its super-periods. Correspondingly, in Galton's approach, both [F1] and [F2] are valid, where '⊆' stands for the part-of relation among time periods.

[F1] $\forall t\, t'\, \Theta\, [t' \subseteq t \wedge \text{HOLDS-ON}(t, \Theta) \Rightarrow \text{HOLDS-ON}(t', \Theta)]$

[F2] $\forall t\, t'\, \Theta\, [t' \subseteq t \wedge \text{HOLDS-IN}(t', \Theta) \Rightarrow \text{HOLDS-IN}(t, \Theta)]$

The idea of predicators connecting thematic concepts and objects in different ways is vital for every domain where objects and their parts can fall under the same concepts and objects need not be uniform regarding the applicability of a concept to its parts.[1] In the geographic domain we find that regions of space can be uniform or mixed regarding ground coverage or soil quality. Material objects such as cups or cars can be uniform or mixed regarding color or substance.

Eschenbach (1999) presents an explicit transfer of the idea of different types of predication from the temporal to the spatial domain. It is shown that in addition to the part-of relation further structural relations can be used as the basis for distinguishing types of predication. The predicators discussed in the following are called 'homogeneous predicators' in Eschenbach (1999). Further 'heterogeneous' predicators are introduced there as well. However, the inferences based on heterogeneous predicators are powerless as compared to inferences based on the homogeneous predicators.

The representation of concepts and taxonomies traditionally focuses on individuating (sortal) concepts (such as 'man', 'father', 'uncle') that indicate or presuppose a clear separation of the entities to which it applies. For those cases, one type of predication seems sufficient. In the context of description logics, the unique type of predication is mostly represented by a (heterogeneous) predicator called 'instance-of'. However, several concepts employed by humans (such as 'covered by grass', 'red' or 'metallic') do not individuate the objects they apply to. Correspondingly the specification of extensive common-sense ontologies will require the treatment of non-individuating concepts and part-of relations within different domains of objects. In this context, it will be mandatory to distinguish ways of relating thematic concepts to objects and their parts. I will show in the following that taxonomic knowledge of non-individuating concepts, object domains structured by a part-of relation, and different predicators associated with characteristic inference patterns can be smoothly embedded into a common formal framework.

[1] The notion of 'thematic concepts' cannot be defined in this article. But notice that concepts that can be defined on the basis of the structural relations within the domain are not counted among the thematic concepts. For example, it would not make much sense to include the concepts of 'having no proper part' or 'having exactly two atomic parts' in the set of thematic concepts.

A system based on the specification presented below could, for example, perform the following (common-sense) inference task that combines conceptual and structural knowledge in the domain of geographic regions. (On the right you see the symbolic version introduced below. 'd' stands for the concept of dry land, 'w' for the concept of water-covered land. 'j' stands for the concept owned by John. 'A', 'B', and 'C' are names of geographic regions.)

Given that	
Dry land and water-covered land are coverage types that (taken together) are exhaustive.	exh(d, w)
Region A is incompletely covered by water.	MIXED(A, w)
Region B does not have dry land.	NW-IN(B, d)
Region A is part of region C.	P(A, C)
Region C is completely owned by John.	EW-IN(C, j)
The following statements can be concluded:	
Region A is completely owned by John.	EW-IN(A, j)
Region C is incompletely covered by water.	MIXED(C, w)
Region A has (some) dry land.	SW-IN(A, d)
Region B is completely covered by water.	EW-IN(B, w)
Region A is not a part of region B.	¬P(A, B)
Region C is not a part of region B.	¬P(C, B)
Region C has (at least some) dry land.	SW-IN(C, d)
Being owned by John is neither subordinate to nor exclusive with being covered by water.	¬sub(j, w) ¬excl(j, w)
Being owned by John is not subordinate to being dry land.	¬sub(j, d)

The axiomatic characterization is given using predicate logic with two sorts of variables. In addition, I will present sets of mutually exclusive and commonly exhaustive relations and the resulting composition tables as required by constraint based reasoning methods. Constraint-based approaches are well established for reasoning about temporal or spatial relations and can easily be adapted to other systems of relations. The presentation of a constraint approach for the concept relations can be regarded as a simple by-product of the axiomatic specification. For the predicators (and their inverse relations) the constraint-based specification shows how existing mechanisms can be used for reasoning about the interface between concept relations and domain structure.

Relations for Two Sorts of Entities

The basic formalism for the specification is predicate logic with two sorts of variables. Lower case italic Latin characters (*x, y, z*) are variables interpreted in the basic domain, which is structured by the part relation. Greek letters

(Φ, Ψ, Θ) are variables interpreted as (non-individuating) thematic concepts for this domain. If the basic domain is the domain of time periods, the concepts in question are state types. If the basic domain is the range of geographic regions, the concepts in questions are (non-individuating) thematic spatial concepts such as soil quality. The concepts for the domain of chunks of matter include concepts for substance type or color.

According to the two sorts of terms, there are four kinds of binary relations differing in the restrictions on the sorts of their arguments:

(1) structural relations between objects of the domain (for example part-of or temporal relations between time periods)

(2) relations between thematic concepts (taxonomical relations, especially the subsumption hierarchy)

(3) predicators relating objects and thematic concepts and

(4) converse predicators (relating thematic concepts and objects). [Since these relations are nothing but the converse of the relations of the third kind, they will be mostly neglected in the following presentation.]

The constraint graphs of corresponding constraint problems are similarly structured by having two sorts of nodes and four kinds of edges, as displayed in Figure 1.

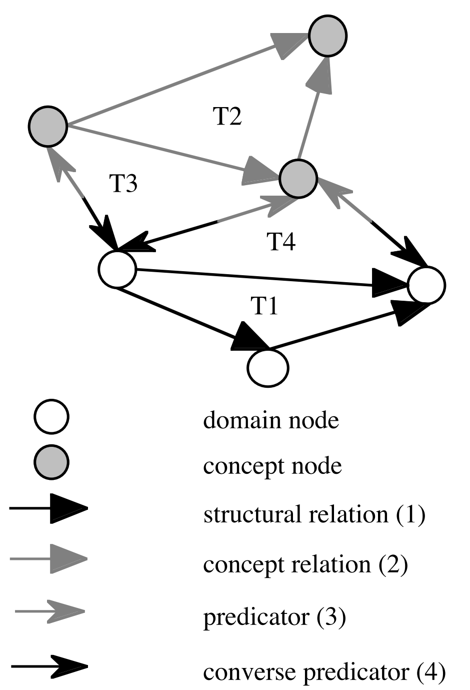

Figure 1. Graph of a constraint problem with two sorts of nodes and three types of edges.

Furthermore, in the display of the constraint network there are four kinds of triangles representing four kinds of relational composition. Triangles formed by three domain nodes (T1) represent reasoning on structural relations as treated by approaches to temporal or spatial relations. Triangles formed by two domain nodes and one concept node (T4) represent reasoning on the interaction of predicators and structural relations. Propagation of constraints in these triangles is justified based on axioms such as [F1] or [F2].

Triangles formed by three concept nodes (T2) represent the interaction of relations between thematic concepts. The concept relations that will be used in the following form a small range of relations that have been studied in more detail in the context of semantic networks and description logics. The constraint-based description of the conceptual relations presented below is not meant to replace other formalizations of conceptual systems. Rather, it is introduced to show which aspects of taxonomic knowledge can be exploited by constraint-based reasoning involving predicators. Correspondingly, we can assume that processing the system of thematic concepts can be done prior to processing triangles involving domain nodes and that the taxonomy is static while the other edges are processed. (Cf. Haarslev, Lutz and Möller (1998) for a different approach to integrating spatial constraint systems into description logics.)

Triangles constituted by two concept nodes and one domain node (T3) stand for the interaction of predicators and the underlying taxonomy. Given that the taxonomy is static, these triangles can be used to transfer information between two predicator edges.

Composition tables for heterogeneous networks can be partitioned according to the different kinds of triangles within the constraint network.

	(1)	(2)	(4)	(3)
(1) structural relation	(1) T1			(3) T4
(2) concept relation		(2) T2	(4) T3	
(3) predicator		(3) T3	(1) T4	
(4) converse predicator	(1) T4			(2) T3

Table 1. Structure of a composition table for a network of domain nodes and concept nodes. For example, the composition of a predicator (3) and a concept relation (2) yield a predicator according to triangle type T3.

The common reasoning procedure for composition tables demands the specification of a set of mutually exclusive and commonly exhaustive binary relations. Such sets will be defined for the different types of relations based on a small set of primitive binary relations specified by axioms. For the sake of brevity, I will call the mutually exclusive and commonly exhaustive binary relations ‘atomic’ relations. They are atomic in the sense that they are sub-relation of every relation that they do not exclude and that is discussed in this article. They need not be atomic relations of a matching relation algebra.

Structural Relations: Mereology

As the basis of demonstrating the general mechanisms, I will take the simple system of mereological relations using the same formalization as in Eschenbach (2001) (cf. Simons 1987; Gotts, Gooday, and Cohn 1996; Varzi 1996; Renz and Nebel 1999). The primitive relations are P (standing for (improper) part) and O (overlap). P is reflexive [A3] and a sub-relation of O [A1], O is symmetric [A2].

Defined relations are P^{-1}, the converse of P, and the five atomic relations equivalence EQ, proper part PP, its converse PP^{-1}, discreteness DR and proper overlap PO.

[Def1] $P^{-1}(x, y) \Leftrightarrow_{def} P(y, x)$
[Def2] $EQ(x, y) \Leftrightarrow_{def} P(x, y) \wedge P^{-1}(x, y)$
[Def3] $PP(x, y) \Leftrightarrow_{def} P(x, y) \wedge \neg P^{-1}(x, y)$
[Def4] $PP^{-1}(x, y) \Leftrightarrow_{def} P^{-1}(x, y) \wedge \neg P(x, y)$
[Def5] $DR(x, y) \Leftrightarrow_{def} \neg O(x, y)$
[Def6] $PO(x, y) \Leftrightarrow_{def} O(x, y) \wedge \neg P(x, y) \wedge \neg P^{-1}(x, y)$

[A1] $\forall x\, y\, [P(x, y) \Rightarrow O(x, y)]$
[A2] $\forall x\, y\, [O(x, y) \Rightarrow O(y, x)]$
[A3] $\forall x\, [P(x, x)]$

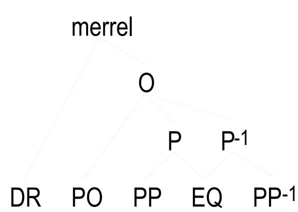

Figure 2. The system of primitive and defined mereological relations.

Two rules of composition based on the primitive relations define the composition table for the mereological relations. They provide the basis for the inferences on constraint triangles of type T1 in Figure 1 (cf. Eschenbach 2001).

[A4] $\forall x\, y\, z\, [P(x, y) \wedge P(y, z) \Rightarrow P(x, z)]$
[A5] $\forall x\, y\, z\, [O(x, y) \wedge P(y, z) \Rightarrow O(x, z)]$

Table 2 presents the composition table for mereological relations derived from the above specification. Multiple labels in a single cell are connected by disjunction. Correspondingly, the cell in the second row and third column is to be read as the theorem [T1].[1] I use '~Rel' to denote the complement of relation 'Rel' (whereas '¬' symbolizes negation of propositions). This notion allows condensing the presentation of the structure of the system of relations and of some composition tables. The symbol 'merrel' marks cells that do not provide more information than that the two objects are mereologically related.

[T1] $\forall x\, y\, z\, [PO(x, y) \wedge PP(y, z) \Rightarrow (PO(x, z) \vee PP(x, z))]$.

v	DR	PO	PP	EQ	PP^{-1}
DR	merrel	$\sim P^{-1}$	$\sim P^{-1}$	DR	DR
PO	~P	merrel	PO PP	PO	~P
PP^{-1}	~P	PO PP^{-1}	O	PP^{-1}	PP^{-1}
EQ	DR	PO	PP	EQ	PP^{-1}
PP	DR	$\sim P^{-1}$	PP	PP	merrel

Table 2. Composition table for mereological relations.

The set of relations containing EQ, PP, PP^{-1}, DR and PO is sometimes called RCC-5, but notice that axiomatic specifications given by different authors can differ. The differences between such proposals are rooted in assumptions on the domain structure and correspondingly concern the existence of sums and parts. For example, the axiomatic system [A1–5] and the Definitions [Def1–6] do not justify the inference that overlapping objects share a part. However, the principles employed here are valid, for example, in Varzi's (1996) system **M** and they are sufficient to prove the commonly assumed composition table of RCC-5. The ontological neutrality of the formalization presented in this article permits its use in various areas independent of their diverging structures (cf. Eschenbach 2001).

Predicators

Primitive Relations

The primitive predicators under consideration in this paper are EW-IN, which is meant to express that the concept is true with respect to every fragment of the object (everywhere in), and NW-IN, which is meant to express that the concept is not true with respect to any fragment of the object (nowhere in). Axiom [A6] expresses that EW-IN and NW-IN exclude each other. Defined predicators that are useful are SW-IN (the concept is true with respect to some fragment; somewhere in), MIXED (there are fragments of both kinds regarding the concept), and HOM (the object is homogeneously regarding the concept). The atomic predicators are EW-IN, NW-IN and MIXED.

[A6] $\forall x\, \Phi\, [\text{EW-IN}(x, \Phi) \Rightarrow \neg\text{NW-IN}(x, \Phi)]$

[Def7] $\text{SW-IN}(x, \Phi) \Leftrightarrow_{def} \neg\text{NW-IN}(x, \Phi)$
[Def8] $\text{MIXED}(x, \Phi) \Leftrightarrow_{def} \neg\text{NW-IN}(x, \Phi) \wedge \neg\text{EW-IN}(x, \Phi)$
[Def9] $\text{HOM}(x, \Phi) \Leftrightarrow_{def} \neg\text{MIXED}(x, \Phi)$

[1] The T in the label of the formula signals that the formula can be proven based on the definitions and axioms (labeled with A) presented in this article. An F signals that the formula is neither an axiom nor provable.

Using EW-IN one can express that an object is completely blue, a region is completely covered by water, or that the car was in front of my house permanently during a period. MIXED is meant to express that an object is partly blue and has some other color as well, that the region is partly but not completely covered by grass, or that the car was sometime but not permanently in front of my house. The natural inferences related to these explanations have to be expressed by axioms on the composition of predicators and mereological relations.

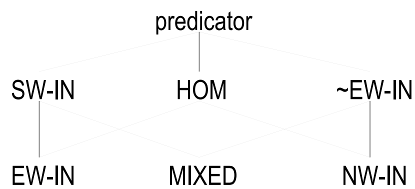

Figure 3. The system of predicators.

Interaction of Predicators and Mereological Relations

In some cases, it can be interesting to express truth at fragments of objects that are not parts in the sense of the formalized relation P. For example using SW-IN we can express that a state holds at a single moment within a period without including moments in the domain of time periods (cf. Galton's (1990) discussion of states of motion and states of position). For this reason, the formalism does not justify certain inferences that might seem plausible at a first glance. For example, given that Φ is true somewhere in x, we are not justified to infer that x has a part for which Φ is true everywhere. However, every part should be a fragment that is considered. As a consequence, everything that has a mixed part regarding Φ is mixed regarding Φ [A7]. Correspondingly, overlapping an object that completely is Φ is sufficient for being somewhat Φ [A8].

[A7] $\forall \Phi\, x\, y\, [P^{-1}(x, y) \wedge \text{MIXED}(y, \Phi) \Rightarrow \text{MIXED}(x, \Phi)]$
[A8] $\forall \Phi\, x\, y\, [O(x, y) \wedge \text{EW-IN}(y, \Phi) \Rightarrow \text{SW-IN}(x, \Phi)]$

Since [A7] and [A8] interrelate objects and concepts, they define the inferences for constraint triangles of type T4 in Figure 1. Correspondingly, these principles determine three composition tables as parts of Table 1. Table 3 specifies the result of combining a mereological relation and a predicator yielding another predicator. The symbol 'predicator' marks cells that do not provide more information than that the two objects are related by a predicator. Table 4 specifies how predicators and there converse restrict the mereological relations between the objects. The third derivable table relates converse predicators and mereological relations yielding converse predicators. It is not presented here since it can be derived via a simple transformation from Table 3.

	EW-IN	MIXED	NW-IN
DR	predicator	predicator	predicator
PO	SW-IN	predicator	~EW-IN
PP^{-1}	SW-IN	MIXED	~EW-IN
=	EW-IN	MIXED	NW-IN
PP	EW-IN	predicator	NW-IN

Table 3. Composition table for mereological relations and predicators.

	EW-IN^{-1}	MIXED^{-1}	NW-IN^{-1}
EW-IN	merrel	$\sim P^{-1}$	DR
MIXED	~P	merrel	~P
NW-IN	DR	$\sim P^{-1}$	merrel

Table 4. Composition table for predicators yielding mereological relations.

Concept Relations

The semantic relations between thematic concepts form the major background knowledge that is needed to draw inferences. If, for example, trees cover a region, then plants cover it as well, but it is not wasteland. If an object is partly made from aluminum, then it is (at least) partly made from metal and not completely wooden. These inferences are due to the assumption that being covered by trees is a special case of being covered by plants, and that wasteland excludes plant coverage. Correspondingly, it is assumed that aluminum is a kind of metal and that metal and wood are distinct kinds of material.

To specify how these kinds of inferences work for the predicators introduced above, I first have to introduce a set of fundamental taxonomic relations and define the set of atomic relations used in the presentation of the composition tables. Based on three primitive relations I will define 15 atomic concept relations. 8 of these relations identify universal or inconsistent concepts.

The specification of the concept relations does not make any assumption regarding the ontological nature of concepts other than captured in the interaction of a specialization hierarchy and concept opposition. It is neutral regarding the question whether identity of concepts is a matter of extension or intension.

Primitive Relations

The central relations between concepts are, on the one hand, the two directions of the ordering relations in the subsumption hierarchy, and, on the other hand, the symmetric relations of exclusion and exhaustion, which are two aspects of concept negation. The following informal explanation of these relations should be interpreted relative to the more common area of heterogeneous predication of the type 'instance-of'. Since in the current approach a predicator mediates the application of a concept to an entity, these informal descriptions will be substantiated in the specification of the interaction of concepts and predicators in the next section.

A concept Φ is sub-ordinate to another concept Ψ (symbolized as '$\mathrm{sub}(\Phi,\Psi)$') if Φ cannot apply without Ψ also applying. A concept Φ is super-ordinate to a concept Ψ (symbolized as '$\mathrm{sup}(\Phi,\Psi)$') if Φ applies whenever Ψ applies. Two concepts Φ, Ψ exclude each other (symbolized as '$\mathrm{excl}(\Phi,\Psi)$') if they cannot apply to the same object. Finally, two concepts Φ, Ψ commonly exhaust the domain of objects (symbolized as '$\mathrm{exh}(\Phi,\Psi)$') if to any object at least one of them applies.

These relations are sufficient for characterizing (Boolean) concept constructors, such as 'conjunction' (intersection, $\cap$) [F3–4], 'negation' (complement, –) [F5], and 'disjunction' (union, $\cup$) [F6–7], and terminological axioms, such as concept inclusion and concept equality.[1]

[F3] $\forall\Phi\,\Psi\,\Theta\,[\mathrm{sub}(\Phi, \Psi \cap \Theta) \Leftrightarrow \mathrm{sub}(\Phi, \Psi) \wedge \mathrm{sub}(\Phi, \Theta)]$
[F4] $\forall\Phi\,\Psi\,\Theta\,[\mathrm{exh}(\Phi, \Psi \cap \Theta) \Leftrightarrow \mathrm{exh}(\Phi, \Psi) \wedge \mathrm{exh}(\Phi, \Theta)]$
[F5] $\forall\Phi\,[\mathrm{excl}(-\Phi, \Phi) \wedge \mathrm{exh}(-\Phi, \Phi)]$
[F6] $\forall\Phi\,\Psi\,\Theta\,[\mathrm{sup}(\Phi, \Psi \cup \Theta) \Leftrightarrow \mathrm{sup}(\Phi, \Psi) \wedge \mathrm{sup}(\Phi, \Theta)]$
[F7] $\forall\Phi\,\Psi\,\Theta\,[\mathrm{excl}(\Phi, \Psi \cup \Theta) \Leftrightarrow \mathrm{excl}(\Phi,\Psi) \wedge \mathrm{excl}(\Phi,\Theta)]$

The relation sup can be defined to be the converse of sub [Def10], while sub, excl and exh are taken here as primitive relations. The four relations sub, sup, excl, and exh will be called 'basic concept relations' in the following.

[Def10] $\mathrm{sup}(\Phi, \Psi) \Leftrightarrow_{def} \mathrm{sub}(\Psi, \Phi)$

According to the intended interpretation of the concept relation, sub is reflexive [A9] and transitive [A12], and excl and exh are symmetric [A10], [A11]. Furthermore, if Φ is sub-ordinate to a concept that excludes Θ, then Φ excludes Θ [A13]. Similarly, if Φ is super-ordinate to a concept that together with Θ is exhaustive, then Φ and Θ are exhaustive [A14]. Finally, if Φ and Ψ are exhaustive, then Φ is super-ordinate to any concept that excludes Ψ [A15].

[A9] $\forall\Phi\,[\mathrm{sub}(\Phi, \Phi)]$
[A10] $\forall\Phi\,\Psi\,[\mathrm{excl}(\Phi, \Psi) \Leftrightarrow \mathrm{excl}(\Psi, \Phi)]$
[A11] $\forall\Phi\,\Psi\,[\mathrm{exh}(\Phi, \Psi) \Leftrightarrow \mathrm{exh}(\Psi, \Phi)]$
[A12] $\forall\Phi\,\Psi\,\Theta\,[\mathrm{sub}(\Phi, \Psi) \wedge \mathrm{sub}(\Psi, \Theta) \Rightarrow \mathrm{sub}(\Phi, \Theta)]$
[A13] $\forall\Phi\,\Psi\,\Theta\,[\mathrm{sub}(\Phi, \Psi) \wedge \mathrm{excl}(\Psi, \Theta) \Rightarrow \mathrm{excl}(\Phi, \Theta)]$
[A14] $\forall\Phi\,\Psi\,\Theta\,[\mathrm{sup}(\Phi, \Psi) \wedge \mathrm{exh}(\Psi, \Theta) \Rightarrow \mathrm{exh}(\Phi, \Theta)]$
[A15] $\forall\Phi\,\Psi\,\Theta\,[\mathrm{exh}(\Phi, \Psi) \wedge \mathrm{excl}(\Psi, \Theta) \Rightarrow \mathrm{sup}(\Phi, \Theta)]$

Relations Concerning Universal and Inconsistent Concepts

There are different options for characterizing universal and inconsistent concepts based on the primitive relations introduced above. For example, a concept (Φ) is universal if it is exhaustive by itself [T3]. More generally, a concept (Φ) is universal if it is super-ordinate to a concept (Ψ) such that Φ and Ψ are exhaustive [Def11]. As the current focus is on the specification of concept relations, I will use the second, relational form of characterizing universal and inconsistent concepts. The relation symbol 'U1' will be used to express that the first concept it combines with is universal [T4]. That the truth of $\mathrm{U1}(\Phi, \Psi)$ is independent of the second argument (Ψ) is enforced by axiom [A16]. U2 is its converse relation [Def12] expressing that the second concept is universal [T5]. A third option of characterizing universal concepts is that a concept is universal if it is super-ordinate to any concept and forms an exhaustive pair with any concept [T6]. In the given framework, this is ensured by [A16].

[Def11] $\mathrm{U1}(\Phi, \Psi) \Leftrightarrow_{def} \mathrm{sup}(\Phi, \Psi) \wedge \mathrm{exh}(\Phi, \Psi)$
[A16] $\forall\Phi\,\Psi\,\Theta\,[\mathrm{U1}(\Phi, \Psi) \Rightarrow \mathrm{U1}(\Phi, \Theta)]$
[Def12] $\mathrm{U2}(\Phi, \Psi) \Leftrightarrow_{def} \mathrm{U1}(\Psi, \Phi)$
[T2] $\forall\Phi\,\Psi\,[\mathrm{U2}(\Phi, \Psi) \Leftrightarrow \mathrm{sub}(\Phi, \Psi) \wedge \mathrm{exh}(\Phi, \Psi)]$

[Def13] $\mathrm{U}(\Phi) \Leftrightarrow_{def} \mathrm{U1}(\Phi, \Phi)$
[T3] $\forall\Phi\,[\mathrm{U}(\Phi) \Leftrightarrow \mathrm{exh}(\Phi, \Phi)]$
[T4] $\forall\Phi\,\Psi\,[\mathrm{U}(\Phi) \Leftrightarrow \mathrm{U1}(\Phi, \Psi)]$
[T5] $\forall\Phi\,\Psi\,[\mathrm{U}(\Psi) \Leftrightarrow \mathrm{U2}(\Phi, \Psi)]$
[T6] $\forall\Phi\,[\mathrm{U}(\Phi) \Leftrightarrow \forall\Psi\,[\mathrm{sup}(\Phi, \Psi) \wedge \mathrm{exh}(\Phi, \Psi)]]$

In contrast to universal concepts, inconsistent concepts are less commonly acknowledged. Nevertheless, I will outline the system of concept relations involving inconsistent concepts in a manner parallel to relations involving universal concepts. This parallels the approach in description logics, where an inconsistent (bottom) concept is introduced for formal reasons (cf. Baader et al. eds. 2003). However, notice that this does not imply or presuppose the existence of inconsistent concepts since the specification of concept relations is ontologically neutral in the sense explained in Eschenbach (2001). The specification of these relations is meant to provide a basis for uncovering inconsistencies in concept definitions. However, it can be augmented by the assumption that no concept is inconsistent if used in a context where this can be guaranteed.

A concept (Φ) that excludes itself cannot be realized on any object and therefore is inconsistent [T8]. More gener-

[1] These relations may not be sufficient to capture all relevant meaning relations among the concepts. For example the complex structure of state types might need further refinement corresponding to the quantificational structure of natural language sentences. Nevertheless, capturing the lattice structure is a mandatory aspect of representing meaning relations between concepts (cf. for example Allen 1984: 130f.).

ally, if a concept (Φ) is sub-ordinate to a concept (Ψ) it excludes, then it must be inconsistent [Def14]. This characterization provides the basis for the relational characterization of inconsistent concepts. The relation symbol ‘I1’ will be used to express that the first concept in its argument list is inconsistent [T9] and I2 for the converse relation [Def15] expressing that the second concept is inconsistent [T10]. To ensure this, I add the axiom [A17]. This also guarantees that the third characterization of inconsistent concepts holds: if a concept is inconsistent, then it is sub-ordinate to and excludes every concept [T11].

[Def14] $\mathrm{I1}(\Phi, \Psi) \Leftrightarrow_{def} \mathrm{sub}(\Phi, \Psi) \wedge \mathrm{excl}(\Phi, \Psi)$
[A17] $\forall\Phi\,\Psi\,\Theta\,[\mathrm{I1}(\Phi, \Psi) \Rightarrow \mathrm{I1}(\Phi, \Theta)]$
[Def15] $\mathrm{I2}(\Phi, \Psi) \Leftrightarrow_{def} \mathrm{I1}(\Psi, \Phi)$
[T7] $\forall\Phi\,\Psi\,[\mathrm{I2}(\Phi, \Psi) \Leftrightarrow \mathrm{sup}(\Phi, \Psi) \wedge \mathrm{excl}(\Phi, \Psi)]$

[Def16] $\mathrm{I}(\Phi) \Leftrightarrow_{def} \mathrm{I1}(\Phi, \Phi)$
[T8] $\forall\Phi\,[\mathrm{I}(\Phi) \Leftrightarrow \mathrm{excl}(\Phi, \Phi)]$
[T9] $\forall\Phi\,\Psi\,[\mathrm{I}(\Phi) \Leftrightarrow \mathrm{I1}(\Phi, \Psi)]$
[T10] $\forall\Phi\,\Psi\,[\mathrm{I}(\Psi) \Leftrightarrow \mathrm{I2}(\Phi, \Psi)]$
[T11] $\forall\Phi\,[\mathrm{I}(\Phi) \Leftrightarrow \forall\Psi\,[\mathrm{sub}(\Phi, \Psi) \wedge \mathrm{excl}(\Phi, \Psi)]]$

If a concept is neither universal nor inconsistent, I will label it with C (contingent).

[Def17] $\mathrm{C1}(\Phi, \Psi) \Leftrightarrow_{def} \neg\mathrm{I1}(\Phi, \Psi) \wedge \neg\mathrm{U1}(\Phi, \Psi)$
[Def18] $\mathrm{C2}(\Phi, \Psi) \Leftrightarrow_{def} \mathrm{C1}(\Psi, \Phi)$

One could add the condition that no concept is inconsistent (or universal) in order to restrict the number of atomic relations between concepts. This is equivalent to assuming that the relations excl and sub exclude each other [T12]. However, this would restrict the option of concept formation. Local problems of inconsistent concept specifications would result in the global problem of an inconsistent knowledge base. Therefore, the following description allows the introduction of inconsistent (and universal concepts) leaving it to the formalism and the inference mechanism to disclose them.

[T12] $\forall\Phi\,[\neg\mathrm{I}(\Phi)] \Leftrightarrow \forall\Phi\,\Psi\,[\mathrm{excl}(\Phi, \Psi) \Rightarrow \neg\mathrm{sub}(\Phi, \Psi)]]$

On the other hand, given the above framework, if a concept is both universal and inconsistent, then every concept is [T14]. In this case, local problems inescapably infect the whole knowledge base. Correspondingly, the assumption that no concept is both universal and inconsistent [A18] reduces the number of concept relations (by one) without introducing avoidable complications. This assumption is equivalent to assuming that no pair of concepts is related by all the basic concept relations [T15].

[T13] $\forall\Phi\,\Psi\,[\mathrm{U1}(\Phi, \Psi) \wedge \mathrm{I1}(\Phi, \Psi) \Leftrightarrow \mathrm{U2}(\Phi, \Psi) \wedge \mathrm{I2}(\Phi,\Psi)]$

[T14] $\exists\Phi\,[\mathrm{U}(\Phi) \wedge \mathrm{I}(\Phi)] \Leftrightarrow \forall\Phi\,[\mathrm{U}(\Phi) \wedge \mathrm{I}(\Phi)]$
[A18] $\forall\Phi\,\Psi\,[\mathrm{U1}(\Phi, \Psi) \Rightarrow \neg\mathrm{I1}(\Phi, \Psi)]$
[T15] $\forall\Phi\,\Psi\,[\neg(\mathrm{sub}(\Phi, \Psi) \wedge \mathrm{sup}(\Phi, \Psi) \wedge \mathrm{excl}(\Phi, \Psi) \wedge \mathrm{exh}(\Phi, \Psi))]$

The four basic concept relations can be combined such that they define 15 atomic concept relations. Eight of these relations identify one of the concepts as inconsistent or universal. According to example [Def19], eight atomic relations (U1U2, U1I2, U1C2, I1U2, I1I2, I1C2, C1U2, C1I2) can be defined that involve at least one inconsistent or universal concept (Figure 4). The remaining 7 atomic relations, which are sub-relations of C1C2, involve contingent concepts and are discussed in the next subsection.

[Def19] $\mathrm{U1I2}(\Phi, \Psi) \Leftrightarrow_{def} \mathrm{U1}(\Phi, \Psi) \wedge \mathrm{I2}(\Phi, \Psi)$
[Def20] $\mathrm{C1C2}(\Phi, \Psi) \Leftrightarrow_{def} \mathrm{C1}(\Phi, \Psi) \wedge \mathrm{C2}(\Phi, \Psi)$

Two inconsistent concepts are related by I1I2, which is an equivalence relation that supports inferences based on substitution as demonstrated in [T16], where REL can be replaced by any concept relation defined in this article. The same is true for U1U2. This can motivate to take them as special cases of identity, allowing for at most one inconsistent and on universal concept. The formalism presented here does neither prohibit nor enforce this additional assumption.

[T16] $\forall\Phi\,\Psi\,\Theta\,[\mathrm{I1I2}(\Phi,\Psi) \Rightarrow (\mathrm{REL}(\Phi,\Theta) \Leftrightarrow \mathrm{REL}(\Psi,\Theta))]$
[T17] $\forall\Phi\,\Psi\,\Theta\,[\mathrm{U1U2}(\Phi,\Psi) \Rightarrow (\mathrm{REL}(\Phi,\Theta) \Leftrightarrow \mathrm{REL}(\Psi,\Theta))]$

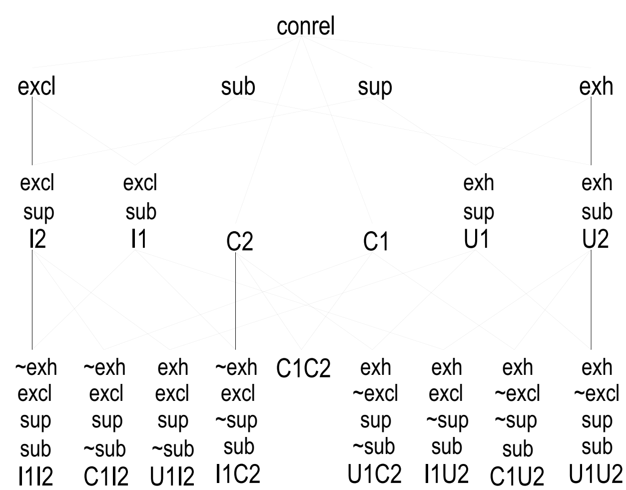

Figure 4. The system of concept relations for inconsistent or universal concepts

Table 5 summarizes the composition of atomic concept relations involving universal and inconsistent concepts. It is derived from the definitions and axiom [A18]. An empty cell signals that the two relations specified by the row and the column cannot be consistently composed. The last line and the last column represent the composition with any of the relations defined in the next section. Correspondingly, Table 6 below can be understood as an elaboration of the rightmost, bottommost cell in Table 5.

v	U1U2	U1I2	U1C2	I1U2	I1I2	I1C2	C1U2	C1I2	C1C2
U1U2	U1U2	U1I2	U1C2						
I1U2	I1U2	I1I2	I1C2						
C1U2	C1U2	C1I2	C1C2						
U1I2				U1U2	U1I2	U1C2			
I1I2				I1U2	I1I2	I1C2			
C1I2				C1U2	C1I2	C1C2			
U1C2							U1U2	U1I2	U1C2
I1C2							I1U2	I1I2	I1C2
C1C2							C1U2	C1I2	C1C2

Table 5. Composition table for concept relations, part 1.

Relations between Contingent Concepts

The remaining seven concept relations that can be defined as combinations of the basic concept relations and their negations are named in the list of definitions below. same expresses that two concept definitions are equivalent. negate relates a concept and its negation. free expresses that two concepts are not related by any of the relations sub, sup, excl and exh. The remaining four relations are the proper versions of the four basic concept relations.

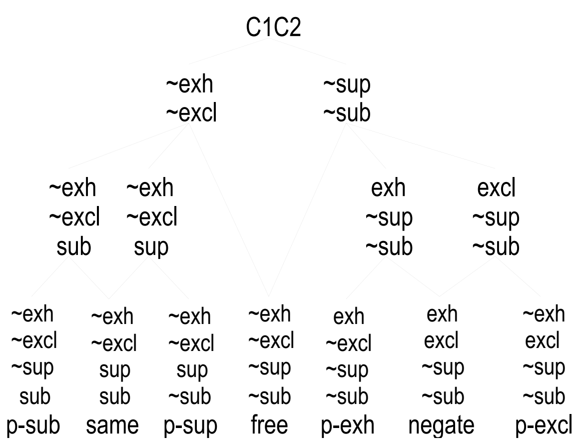

Figure 5. The system of concept relations for consistent and non-universal concepts.

[Def21] same(Φ, Ψ) $\Leftrightarrow_{def}$ sub(Φ, Ψ) ∧ sup(Φ, Ψ) ∧ ¬excl(Φ, Ψ) ∧ ¬exh(Φ, Ψ)

[Def22] negate(Φ, Ψ) $\Leftrightarrow_{def}$ ¬sub(Φ, Ψ) ∧ ¬sup(Φ, Ψ) ∧ excl(Φ, Ψ) ∧ exh(Φ, Ψ)

[Def23] free(Φ, Ψ) $\Leftrightarrow_{def}$ ¬sub(Φ, Ψ) ∧ ¬sup(Φ, Ψ) ∧ ¬excl(Φ, Ψ) ∧ ¬exh(Φ, Ψ)

[Def24] p-sub(Φ, Ψ) $\Leftrightarrow_{def}$ sub(Φ, Ψ) ∧ ¬sup(Φ, Ψ) ∧ ¬excl(Φ, Ψ) ∧ ¬exh(Φ, Ψ)

[Def25] p-sup(Φ, Ψ) $\Leftrightarrow_{def}$ ¬sub(Φ, Ψ) ∧ sup(Φ, Ψ) ∧ ¬excl(Φ, Ψ) ∧ ¬exh(Φ, Ψ)

[Def26] p-excl(Φ, Ψ) $\Leftrightarrow_{def}$ ¬sub(Φ, Ψ) ∧ ¬sup(Φ, Ψ) ∧ excl(Φ, Ψ) ∧ ¬exh(Φ, Ψ)

[Def27] p-exh(Φ, Ψ) $\Leftrightarrow_{def}$ ¬sub(Φ, Ψ) ∧ ¬sup(Φ, Ψ) ∧ ¬excl(Φ, Ψ) ∧ exh(Φ, Ψ)

The composition table (Table 6) can be derived from the axioms [A12]–[A15] and the definitions of the concept relations.

v	p-sub	same	p-sup	free	p-exh	negate	p-excl
p-sup	same p-sup p-sub free p-exh	p-sup	p-sup	p-sup free p-exh	p-exh	p-exh	p-sup free p-excl p-exh negate
same	p-sub	same	p-sup	free	p-exh	negate	p-excl
p-sub	p-sub	p-sub	same p-sup p-sub free p-excl	p-sub free p-excl	p-sub free p-excl p-exh negate	p-excl	p-excl
free	p-sub free p-exh	free	p-sup free p-excl	C1C2	p-sub free p-exh	free	p-sup free p-excl
p-exh	p-exh	p-exh	p-sup free p-excl p-exh negate	p-sup free p-exh	p-sup p-sub free p-exh same	p-sup	p-sup
negate	p-exh	negate	p-excl	free	p-sub	same	p-sup
p-excl	p-sub free p-excl p-exh negate	p-excl	p-excl	p-sub free p-excl	p-sub	p-sub	same p-sup p-sub free p-excl

Table 6. Composition table for concept relations, part 2.

Interaction of Concept Relations and Predicators

The specification of the interaction between predicators and concept relations define the inferences for constraint triangles of type T3 in Figure 1. It is guided by the intended interpretations. If Φ is true for every fragment of x and sub-ordinate to Ψ, then Ψ is true for every fragment of x as well [A19]. For example, if a bowl is completely made from aluminum, then it is completely metallic. If Φ is not true for any fragment of x and super-ordinate to Ψ, then Ψ is not true for any fragment of x either [A20]. Thus, if a cup does not have a metallic fragment, then it does not have a fragment of aluminum as well. If Φ is not true for any fragment of x, and if Φ and Ψ are exhaustive, then Ψ is true for every fragment of x [A21]. Correspondingly, if region B (from the introductory example) does not have dry land, then it is completely covered by water. If Φ is true for every fragment of x and excludes Ψ, then Ψ is not true for any fragment of x [A22]. Thus, a bowl that is completely made from aluminum does not have wooden fragments. Furthermore, inconsistent concepts are not true for any fragment [A23], and universal concepts are true for every fragment [A24].

[A19] $\forall x\, \Phi\, \Psi\, [\text{EW-IN}(x, \Phi) \wedge \text{sub}(\Phi, \Psi) \Rightarrow \text{EW-IN}(x, \Psi)]$
[A20] $\forall x\, \Phi\, \Psi\, [\text{NW-IN}(x, \Phi) \wedge \text{sup}(\Phi, \Psi) \Rightarrow \text{NW-IN}(x, \Psi)]$
[A21] $\forall x\, \Phi\, \Psi\, [\text{NW-IN}(x, \Phi) \wedge \text{exh}(\Phi, \Psi) \Rightarrow \text{EW-IN}(x, \Psi)]$
[A22] $\forall x\, \Phi\, \Psi\, [\text{EW-IN}(x, \Phi) \wedge \text{excl}(\Phi, \Psi) \Rightarrow \text{NW-IN}(x, \Psi)]$
[A23] $\forall x\, \Phi\, \Psi\, [\text{I2}(\Phi, \Psi) \Rightarrow \text{NW-IN}(x, \Psi)]$
[A24] $\forall x\, \Phi\, \Psi\, [\text{U2}(\Phi, \Psi) \Rightarrow \text{EW-IN}(x, \Psi)]$

The composition table for relations involving universal and inconsistent concepts (Table 7) can be derived from [A23] and [A24]. Axioms [A19]–[A22] are substantial for deriving the composition table for the other atomic concept relations (Table 8). The symbol 'pred' marks cells that do not provide more information than that the two objects are related by a predicator.

v	U1U2	U1I2	U1C2	I1U2	I1I2	I1C2	C1U2	C1I2
EW-IN	EW-IN	NW-IN	pred				EW-IN	NW-IN
MIXED							EW-IN	NW-IN
NW-IN				EW-IN	NW-IN	pred	EW-IN	NW-IN

Table 7. Composition table for predicators and concept relations, part 1.

	same	p-sup	p-sub	free	p-excl	p-exh	negate
EW-IN	EW-IN	pred	EW-IN	pred	NW-IN	pred	NW-IN
MIXED	MIXED	~EW-IN	SW-IN	pred	~EW-IN	SW-IN	MIXED
NW-IN	NW-IN	NW-IN	pred	pred	pred	EW-IN	EW-IN

Table 8. Composition table for predicators and concept relations, part 2.

The axioms [A19]–[A24] furthermore lead to the table of restrictions on concept relations based on predications (Table 9).

v	EW-IN	MIXED	NW-IN
EW-IN^{-1}	U1U2 U1C2 C1U2 same p-sup p-sub free p-exh	U1C2 p-sup free p-exh	U1I2 U1C2 C1I2 p-sup free p-excl p-exh negate
MIXED^{-1}	C1U2 p-sub free p-exh	C1C2	C1I2 p-sup free p-excl
NW-IN^{-1}	I1U2 I1C2 C1U2 p-sub free p-excl p-exh negate	I1C2 p-sub free p-excl	I2I2 I1C2 C1I2 same p-sup p-sub free p-excl

Table 9. Disjunctive composition table for predicators yielding concept relations.

Table 10 gives the conjunctive version of this table, since in this case, the conjunctive form presents the inference patterns more clearly. In conjunctive composition tables multiple labels in a single cell are connected by conjunction. Correspondingly, the cell in the first row and second column is to be read as the theorem [T18].

[T18] $\forall x\, \Phi\, \Psi\, [\text{EW-IN}^{-1}(\Phi, x) \wedge \text{MIXED}(x, \Psi) \Rightarrow (\neg\text{excl}(\Phi,\Psi) \wedge \neg\text{sub}(\Phi,\Psi))]$.

∧	EW-IN	MIXED	NW-IN
EW-IN^{-1}	~excl	~excl ~sub	~sub
MIXED^{-1}	~excl ~sup	C1C2	~sub ~exh
NW-IN^{-1}	~sup	~exh ~sup	~exh

Table 10. Conjunctive composition table for predicators yielding concept relations.

You may have noticed that every cell of Table 9 includes the relation free. This corresponds to the fact that from pure observation of the applicability of concepts to objects and their fragments we cannot deduce dependencies between concepts. Observations can lead only to the elimination of potential dependencies.

Where Inferences End

The predicators can be seen as an interface between knowledge and inferences at the level of structural relations on the one hand and at the level of thematic concepts on the other hand. Thus, we should look whether this can lead to inference chains that jump back and forth between the two levels based on a small set of information units. However, the composition tables provide enough information to find that this cannot be the case.

First, the composition of predicators and their converse relations leading to mereological relations (Table 4) shows that this type of inferences can in no case exclude the option that the two entities are disjoint. For example, from knowing that objects share a color or that regions share an owner we cannot conclude that they overlap. Furthermore, if we cannot exclude that two objects are disjoint, we cannot draw inferences regarding concept assignment between them (cf. Table 3). For example, if we know that two objects are disjoint and that one of them is completely green, we cannot infer anything about the color of the other object. Consequently, inferences on concept assignment can be triggered on the mereological level when we can conclude that two objects overlap. However, in the current framework this information cannot be derived from concept assignment.

Second, the composition of converse predicators and predicators leading to concept relations in Table 9 shows that we cannot exclude the relation free on the basis of predicator information. Two concepts are related by free if they are completely independent from each other. For example, the concepts 'red' and 'owned by me' are related by free, since I own both red things and non-red things and there are both red things and non-red things that are not owned by me. In addition, two concepts related by free do not lead to interesting inferences in connection to the predicators, as obvious in Table 8. As a consequence, inferences on concept assignment can be triggered on the conceptual level only if two concepts are not related by free. And, similar to the case before, this information cannot be derived from concept assignment.

These observations are not bound to the general idea of connecting knowledge about object structure and knowledge about concepts by predicators but relative to the choice of predicators presented here. To overcome the first restriction, we could add two primitive predicators to express the facts that outside a region the predicate applies everywhere or does not apply at all. The second predicator would be useful, for example, for expressing that John does not own any region other than region C. In this case, any region owned by John must be part of region C.

Conclusion

The predicators introduced above present a simple set that fits to the structural relations presented on the level of objects. More sophisticated systems of structural relations in the domains of time or space can stimulate the use of more elaborated sets of predicators (cf. Eschenbach 1999). The basic mechanisms of predicator-based inferences and the basic concept relations they interact with are the same. Furthermore, the specification of the interaction of predicators with structural relations and concept relations can easily be embedded in constraint-based reasoning systems.

Constraint-propagation based on composition tables is known as an efficient approach to solve certain reasoning task for systems of binary relations. The discussion of specific relation systems has focused on temporal and spatial relations. However, as demonstrated above, the constraint approach can be used for reasoning about relations between different sorts of objects in the same manner.

The specification of the constraint networks for objects and concepts includes a constraint-based approach to conceptual reasoning. However, the application of the approach to predicators does not require that the same reasoning technique be used for deriving consistency in other parts of the network. The specification rather tells which relations between concepts can be exploited for reasoning about (homogeneous) predicators and how.

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