Revision of an Argumentation System

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Abstract

In this paper, we address the problem of revising a Dung-style abstract argumentation system, when we add a new argument which interacts with one previous argument. We study the impact of such an addition on the outcome of the argumentation system, more particularly on the set of its extensions. Different kinds of revision are defined according to the change induced on the number or on the contents of the extensions. Two particular revisions are studied, for which we propose characterization theorems.

Keywords: Argumentation, belief revision, Knowledge Representation, Multi-Agent Systems, Nonmonotonic Reasoning

1 Introduction

When an agent receives a new piece of information, she must adapt its beliefs; this adaptation is not always easy because it may imply to drop some previous knowledge. Choosing the better way to adapt itself to its environment is a very old problem for human being, this is, perhaps, a reason why belief change theory has been so largely studied in the artificial intelligence community. The seminal work of Alchourrón, Gärdenfors and Makinson (AGM) (Alchourrón, Gärdenfors, and Makinson 1985) has settled a formal framework for reasoning about belief change and introduced the concept of "belief revision". Belief revision consists in answering the question of what remains of the old beliefs after the arrival of a new piece of information. In this paper, we transpose this question into argumentation theory, and study the case of the arrival of a new argument into an argumentation system.

Argumentation has become an influential approach to handle AI problems including defeasible reasoning, see e.g. (Dung 1995; Bondarenko et al. 1997; Chesñevar, Maguitman, and Loui 2000; Prakken and Vreeswijk 2002; Amgoud and Cayrol 2002), and modeling agents interactions, see e.g. (Amgoud, Maudet, and Parsons 2000; Kakas and Moraïtis 2003). Argumentation is basically concerned with the exchange of interacting arguments. This set of arguments may come either from a dialogue between several agents but also from the available (and possibly contradictory) pieces of information at the disposal of one unique agent. Usually, the interaction between arguments takes the form of a conflict, called attack. For example, a logical argument can be a pair ⟨set of assumptions, conclusion⟩, where the set of assumptions entails the conclusion according to some logical inference schema. Then a conflict occurs, for instance, if the conclusion of an argument contradicts an assumption of another argument. The main issue for any argumentation system is the selection of acceptable sets of arguments, called "extensions", based on the way arguments interact. The outcome of an argumentation system is often defined by the set of its extensions but, depending on the applications, it may also be defined as the set of arguments that belongs to every extension. Intuitively, an acceptable set of arguments must be in some sense coherent and strong enough (e.g. able to defend itself against all attacking arguments). It is convenient to explore the concept of acceptability through argumentation frameworks, and especially the framework of (Dung 1995), which abstracts from the arguments nature, and represents interaction under the form of a binary relation "attack" on a set of arguments.

When a new argument is added to a set of arguments together with its interactions with the initial set of arguments, the outcome of the argumentation system may change. In this paper, we study the impact of this addition on the set of initial extensions. This leads us to characterize the possible revision operations with respect to the change they induce on the outcome. This study has two main applications, the first one concerns the computation, while the second one belongs to the field of dialogue strategies. On the first hand, the interest for computational processing is that knowledge about the kind of revision that is done may help to deduce what are the changes in the extensions. For instance, knowing that the revision is conservative allows us to deduce that the revision will not change the previous extensions. On the other hand, knowing the impact of adding an argument may help choosing the good one in order to achieve a given goal. For instance, in order to make a dialogue more open, an argument inducing an "expansive revision"[1] must be added (see Section 5).

The paper is organized as follows, Section 2 recalls the basic concepts in argumentation and in revision theory. Section 3 settles a definition of revision in argumentation. In this paper, *we restrict our study to the case of adding one argument having only one interaction with an initial argu-*

[1]The precise definition of this notion is given in Definition 8.

ment. So, the research reported here is a first step towards a study of general revision operators. A typology of revision in argumentation is proposed, based on the impact of the revision under the set of extensions. A particular property for the revision operator is to keep the added argument in each extension. It is called "classical" and the cases when the revision operator is classical are described in Section 3.3. Section 4 is dedicated to the study of two particular revision operators, namely the "decisive" and the "expansive" revision operators. A last section discusses the related approaches in the literature. All the proofs and several important lemmas are given in Appendix.

2 Background

In this section, we recall the necessary background concepts at work in argumentation systems and in revision theory.

2.1 Basic concepts in argumentation frameworks

The present work lies in the frame of the general theory of abstract argumentation frameworks proposed by Dung (Dung 1995). Such an abstract framework assumes that a set of arguments is given, as well as the different conflicts between them, and focuses on the definition of the status of arguments.

Definition 1 *An* argumentation framework *is a pair* $\langle \mathbf{A}, \mathbf{R} \rangle$, *where* $\mathbf{A}$ *is a non-empty set and* $\mathbf{R}$ *is a binary relation on* $\mathbf{A}$, *called* attack relation. *Let* A, $B \in \mathbf{A}$, $(A, B) \in \mathbf{R}$ *or equivalently* $A\mathbf{R}B$ *means that* A *attacks* B, *or* B *is attacked by* A.

In the following, $\langle \mathbf{A}, \mathbf{R} \rangle$ is an argumentation framework, and we assume that the set of arguments $\mathbf{A}$ is finite. It is useful to extend the concept of attack to sets of arguments.

Definition 2 *Let* $A \in \mathbf{A}$ *and* $\mathcal{S} \subseteq \mathbf{A}$. $\mathcal{S}$ attacks A *iff* $\exists B \in \mathcal{S}$ *such that* $B\mathbf{R}A$.

The main issue of any argumentation system is the selection of acceptable sets of arguments. Intuitively, an acceptable set of arguments must be in some sense coherent and strong enough (e.g. able to defend itself against every attacking argument). An argumentation semantics defines the properties required for a set of arguments to be acceptable. The selected sets of arguments under a given semantics are called extensions of that semantics. We recall the basic concepts used for defining usual semantics:

Definition 3 *Let* $A \in \mathbf{A}$ *and* $\mathcal{S} \subseteq \mathbf{A}$.

- $\mathcal{S}$ *is* conflict-free *iff* $\nexists A, B \in \mathcal{S}$ *such that* $A\mathbf{R}B$.
- $\mathcal{S}$ defends A *iff* $\mathcal{S}$ *attacks each argument which attacks* A. *The set of arguments which* $\mathcal{S}$ *defends will be denoted by* $\mathcal{F}(\mathcal{S})$. $\mathcal{F}$ *is called the* characteristic function *of* $\langle \mathbf{A}, \mathbf{R} \rangle$.

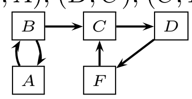
The literature proposes an increasing variety of semantics, refining Dung's traditional ones (Baroni, Giacomin, and Guida 2005; Caminada 2006; Dung, Mancarella, and Toni 2006; Coste-Marquis, Devred, and Marquis 2005). In this paper, only the most well-known semantics are considered: the grounded, preferred and stable semantics.

Definition 4 *Let* $\mathcal{E} \subseteq \mathbf{A}$.

- $\mathcal{E}$ *is* admissible *iff* $\mathcal{E}$ *is conflict-free and defends all its elements (i.e.* $\mathcal{E} \subseteq \mathcal{F}(\mathcal{E})$*).*
- $\mathcal{E}$ *is a* preferred extension *iff* $\mathcal{E}$ *is a maximal (w.r.t. set-inclusion) admissible set.*
- $\mathcal{E}$ *is the* grounded extension *iff* $\mathcal{E}$ *is the least fixed point (w.r.t. set-inclusion) of the characteristic function* $\mathcal{F}$.
- $\mathcal{E}$ *is a* stable extension *iff* $\mathcal{E}$ *is conflict-free and attacks each argument which does not belong to* $\mathcal{E}$.

An argumentation framework can be represented as a directed graph, called attack graph, where nodes are the arguments and edges represent the attack relation. Throughout the paper examples are using this graph representation.

Example 1 $\mathbf{A} = \{A, B, C, D, F\}$ *and* $\mathbf{R} = \{(A, B), (B, A), (B, C), (C, D), (D, F), (F, C)\}$.



The admissible sets are $\varnothing$, $\{A\}$, $\{B\}$ *and* $\{B, D\}$. *The preferred extensions are* $\{A\}$ *and* $\{B, D\}$. *The grounded extension is* $\varnothing$. $\{B, D\}$ *is the unique stable extension.*

Dung (Dung 1995) has proved the following results.

Property 1 *Let* $\langle \mathbf{A}, \mathbf{R} \rangle$ *be an argumentation framework.*

1. *There is at least one preferred extension, always a unique grounded extension, while there may be zero, one or many stable extensions.*
2. *Each admissible set is included in a preferred extension.*
3. *Each stable extension is a preferred extension, the converse is false.*
4. *The grounded extension is included in each preferred extension.*
5. *Each argument which is not attacked belongs to the grounded extension (hence to each preferred and to each stable extension).*
6. *If* $\mathbf{R}$ *is finite, the grounded extension can be computed by iteratively applying the function* $\mathcal{F}$ *from the empty set.*

The presence of cycles in the attack graph has often raised some problems, namely for the stable semantics, for which it may happen that no extension exists. Note that some authors only consider attack graphs without odd-length cycles, arguing that an odd-length cycle carries counterintuitive information. The following results give properties of the preferred, grounded and stable extensions depending on the existence of cycles in the attack graph.

Property 2 *(Dunne and Bench-Capon 2001; 2002) Let* $\mathcal{G}$ *denote the attack graph associated with* $\langle \mathbf{A}, \mathbf{R} \rangle$.

1. *If* $\mathcal{G}$ *contains no cycle,* $\langle \mathbf{A}, \mathbf{R} \rangle$ *has a unique preferred extension, which is also the grounded extension and the unique stable extension.*
2. *If* $\varnothing$ *is the unique preferred extension of* $\langle \mathbf{A}, \mathbf{R} \rangle$, $\mathcal{G}$ *contains an odd-length cycle.*
3. *If* $\langle \mathbf{A}, \mathbf{R} \rangle$ *has no stable extension,* $\mathcal{G}$ *contains an odd-length cycle.*
4. *If* $\mathcal{G}$ *contains no odd-length cycle, preferred and stable extensions coincide.*
5. *If* $\mathcal{G}$ *contains no even-length cycle,* $\langle \mathbf{A}, \mathbf{R} \rangle$ *has a unique preferred extension.*

In the following, we only consider argumentation frameworks such that the attack graph is connected. It does not restrict generality, since any graph can be split into its connected components.

2.2 Basic concepts in revision theory

In the field of belief change theory, the paper of AGM (Alchourrón, Gärdenfors, and Makinson 1985) has introduced the concept of "belief revision". Belief revision consists in answering the question of what remains of the old beliefs after the arrival of a new piece of information. Beliefs are represented by sentences of a formal language. AGM have defined three types of belief change, namely contraction, expansion and revision. Expansion consists only in adding information without checking its consistency with previous beliefs. Contraction is an operation designed for removing information. Revision consists in adding information while preserving consistency. This last operation is the most interesting one since inconsistency leads to un-exploitable information. The main interest of AGM's work is the definition of a set of postulates which should hold for any rational revision operator. As noticed in (Sombé 1994) these postulates are founded on three principles:

- a consistency principle (the result should be consistent),
- a minimum change principle (as few beliefs as possible should be modified),
- priority to the new piece of information principle (the new piece of information should hold after the revision process).

More formally, a revision operator associates to a set of deductively closed formulas K (encoding the initial beliefs[2]) and to a formula p (encoding a new piece of information), another set of beliefs denoted by $K * p$. In order to be "rational" the operator $*$ should satisfy the following AGM postulates:

K* 1 $K * p = Th(K * p)$.
K* 2 $p \in K * p$.
K* 3 $K * p \subseteq Th(K \cup \{p\})$.
K* 4 If $\neg p \notin K$, then $Th(K \cup \{p\}) \subseteq K * p$.
K* 5 $\bot \in K * p$ if and only if $p \leftrightarrow \bot$.
K* 6 If $p \leftrightarrow q$ then $K * p = K * q$.
K* 7 $K * (p \wedge q) \subseteq Th((K * p) \cup \{q\})$.
K* 8 If $\neg q \notin K * p$ then $Th((K * p) \cup \{q\}) \subseteq K * (p \wedge q)$.

K* 1 ensures that the result of the revision is deductively closed. **K* 2** imposes that the new piece of information should belong to the revised beliefs. **K* 3** implies that beliefs after revision should not contain more information than what can be logically derived from K and the new piece of information p. **K* 4** together with **K* 3** means that when the new piece of information is not contradictory with the old beliefs then revision is simply an expansion. **K* 5** says that the revised beliefs set is inconsistent if and only if the new piece of information is itself inconsistent. **K* 6** expresses that belief revision is syntax-independent. These first six postulates are the basic revision postulates and the last two express change minimality. **K* 7** implies that revising by a conjunction $p \wedge q$ should not contain more information that what can be logically derived from the revision of K by p together with the piece of information q. **K* 8** means that, when revising K by $p \wedge q$, every logical deduction from q and $K * p$ should be kept as soon as q is not contradictory with $K * p$.

Note that in the following we are going to limit our study to the case of **K* 2**. And we call "classical" an operator which satisfies **K*2**. However this precise postulate may not always be suitable in the argumentation framework, this is developed in Section 3.3.

A last recall about belief change is the distinction between belief revision and belief update (this was first established in (Winslett 1988)). The difference is in the nature of the new piece of information: either it is completing the knowledge of the world or it informs that there is a change in the world. More precisely, update is a process which takes into account a physical evolution of the system while revision is a process taking into account an epistemic evolution, it is the knowledge about the world that is evolving. In this paper, we suppose that we rather face a revision problem : the agent was not aware of some argument that suddenly appears, it means that the world has not changed but the awareness of the agent has evolved.

3 Revision in argumentation

First, we introduce a formal definition of revision in argumentation. The outcome of a revision process is the set of extensions under a given semantics. Then, by considering how the set of extensions is modified under the revision process, we propose a typology of different revisions.

3.1 Definition

Informally, a revision occurs when a new argument is presented. Note that the case of adding a new argument which is not connected to $\langle \mathbf{A}, \mathbf{R} \rangle$ is trivial. It has only to be added to each preferred extension. Indeed, revision is more interesting when the new argument interacts with previous ones. In this paper, which reports a preliminary study on revision in argumentation, we restrict revision to the addition of *exactly* one argument Z that has *exactly* one interaction, $Z\mathbf{R}X$ or $X\mathbf{R}Z$, where X belongs to $\mathbf{A}$.

In the following, we identify an argumentation framework $\langle \mathbf{A}, \mathbf{R} \rangle$ with its associated attack graph $\mathcal{G}$. We write $X \in \mathcal{G}$ instead of "X is an argument represented by a node of $\mathcal{G}$". The set of extensions of $\langle \mathbf{A}, \mathbf{R} \rangle$ is denoted by $\mathbf{E}$ (with $\mathcal{E}_1$, …, $\mathcal{E}_n$ denoting the extensions).

Revising $\langle \mathbf{A}, \mathbf{R} \rangle$ consists in adding an argument Z which attacks (or is attacked by) an argument X of $\mathbf{A}$. The revision process produces a new framework represented by a graph $\mathcal{G}'$ and a new set of extensions $\mathbf{E}'$ (with $\mathcal{E}'_1$, …, $\mathcal{E}'_p$ denoting the extensions).

$$(\mathcal{G}, \mathbf{E}) \xrightarrow[i=(Z,X) \text{ or } i=(X,Z)]{\text{revision with } Z \text{ and } i} (\mathcal{G}', \mathbf{E}')$$

Definition 5 *Let $\mathcal{G}$ be an attack graph. Let s be a semantics. Let $X \in \mathcal{G}$, $Z \notin \mathcal{G}$ and i a pair of arguments (either (X, Z)*

[2] If BC is a set of formulas encoding these beliefs then $K = Th(BC)$ where Th is the deductive closure operator.

or (Z, X)). Let $\mathcal{G}'$ be the graph obtained from $\mathcal{G}$ by adding the node Z and the edge i. The revision operator Θ *maps* $(Z, i, \mathcal{G}, s)$ *to* $\mathbf{E}'$ *which is the set of extensions of* $\mathcal{G}'$ *under the semantics* s.

Let us mention several results which will be useful in the following. As we revise with only one new argument having only one interaction with an already existing argument, it is easy to prove that:

Property 3

- *If the new interaction is (Z, X), Z is not attacked in $\mathcal{G}'$.*
- *If the new interaction is (X, Z), Z attacks no argument of $\mathcal{G}'$.*
- *The revision process introduces no cycle in $\mathcal{G}'$.*

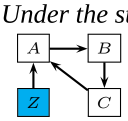
As defined above, revising an argumentation framework may change the set of extensions. Given a semantics, the modifications are more or less important. It depends on the kind of interaction which is added and more precisely on the status of the argument X involved in that interaction. In the next section, we propose a typology of different kinds of revision according to how the set of extensions is modified. The next step is to characterize each kind of revision by providing conditions on the interaction i.

3.2 Typology of revisions

Let $\langle \mathbf{A}, \mathbf{R} \rangle$ be an argumentation framework and $\mathbf{E}$ the set of extensions of $\langle \mathbf{A}, \mathbf{R} \rangle$ under a given semantics s. Different situations may be encountered in the general case. $\mathbf{E}$ may be empty (implying that s is the stable semantics), may be reduced to a singleton $\{\mathcal{E}_1\}$ (where $\mathcal{E}_1$ may be empty), or may contain more than one extension $\{\mathcal{E}_1, \ldots, \mathcal{E}_n\}$. The situation with only one non-empty extension is convenient for the determination of the status of an argument. In contrast, when several extensions exist, different choices are available. We have first considered revisions such that $\mathcal{G}'$ has a unique non-empty extension, while it was not the case for $\mathcal{G}$. Such a revision is called **decisive**.

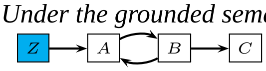
Example 2

1. *Under the stable semantics, with* $i = (Z, A)$



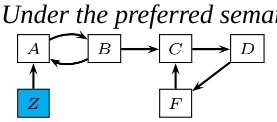
Before revision $\mathbf{E} = \varnothing$,
after revision $\mathbf{E}' = \{\{Z, B\}\}$

2. *Under the grounded semantics, with* $i = (Z, A)$



$\mathbf{E} = \{\{\}\}$, $\mathbf{E}' = \{\{Z, B\}\}$

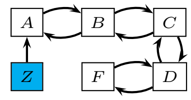
3. *Under the preferred semantics, with* $i = (Z, A)$



$\mathbf{E} = \{\{A\}, \{B, D\}\}$,
$\mathbf{E}' = \{\{Z, B, D\}\}$

A weaker requirement is the decrease of the number of choices. A revision such that $\mathcal{G}'$ has strictly less extensions than $\mathcal{G}$, but still has at least two, is called **selective**. Note that selective revision does not make sense under the grounded semantics, since there is always a unique grounded extension.

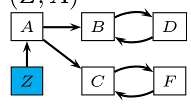
Example 3 *Under the preferred (or stable) semantics, with* $i = (Z, A)$



$\mathbf{E} = \{\{A, C, F\}, \{A, D\}, \{B, D\}, \{B, F\}\}$,
$\mathbf{E}' = \{\{Z, C, F\}, \{Z, B, D\}, \{Z, B, F\}\}$

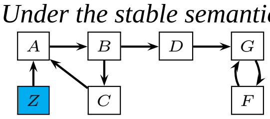
An opposite point of view enables to consider revisions which raise ambiguity, by increasing the number of extensions. This is the case for instance when $\mathcal{G}$ has at least one non-empty extension and $\mathcal{G}'$ has strictly more extensions than $\mathcal{G}$. A slightly different situation occurs when $\mathcal{G}$ has no extension or an empty one, while $\mathcal{G}'$ has more than one extension. In that case, revision brings some information, but is not decisive. Such revisions are called **questioning**. As for selective revision, questioning revision does not make sense under the grounded semantics.

Example 4 *Under the preferred (or stable) semantics, with* $i = (Z, A)$



$\mathbf{E} = \{\{A, D, F\}\}$, $\mathbf{E}' = \{\{Z, B, C\}, \{Z, B, F\}, \{Z, D, C\}, \{Z, D, F\}\}$

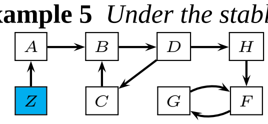
Under the stable semantics, with $i = (Z, A)$



$\mathbf{E} = \varnothing$,
$\mathbf{E}' = \{\{Z, B, F\}, \{Z, B, G\}\}$

Pursuing along the previous line, we consider revisions removing every extension, thus leading to a kind of decisional dead-end. A revision such that $\mathcal{G}'$ has no extension, while $\mathcal{G}$ had at least one, is called **destructive**. Note that destructive decision makes sense only under the stable semantics.

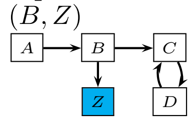
Example 5 *Under the stable semantics, with* $i = (Z, A)$



$\mathbf{E} = \{\{A, D, F\}, \{A, D, G\}\}$,
$\mathbf{E}' = \varnothing$

So far, the considered revisions have an impact on the number of extensions. Now, we are interested in revisions which modify the content of extensions, without modifying the number of extensions. The most interesting situation occurs when each extension of $\mathcal{G}'$ strictly includes one extension of $\mathcal{G}$, the number of extensions being the same. Such revisions are called **expansive**.

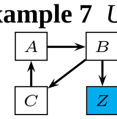
Example 6 *Under the preferred (or stable) semantics, with* $i = (B, Z)$



$\mathbf{E} = \{\{A, C\}, \{A, D\}\}$,
$\mathbf{E}' = \{\{Z, A, C\}, \{Z, A, D\}\}$

When nothing is changed, that is $\mathbf{E} = \mathbf{E}'$, the revision is called **conservative**.

Example 7 *Under the preferred semantics, with* $i = (B, Z)$

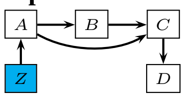


$\mathbf{E} = \{\{\}\}$, $\mathbf{E}' = \{\{\}\}$

Otherwise, it may happen that some extensions (and sometimes all of them) are altered. This is called an **altering** revision. It is the case for instance when each extension of

$\mathcal{G}'$ has a non-empty intersection with (but does not include) an extension of $\mathcal{G}$.

Example 8 *Under the grounded semantics, with $i = (Z, A)$*



$\mathbf{E} = \{\{A, D\}, \mathbf{E}' = \{\{Z, B, D\}\}$

The above discussion can be summarized on the following table.

$\mathbf{E}' =$ / $\mathbf{E} =$	$\varnothing$	$\{\{\}\}$	$\{\mathcal{E}'_1\}$	$\{\mathcal{E}'_1, \ldots, \mathcal{E}'_p\}$ $p \geq 2$
$\varnothing$	conservative	#1	decisive	questioning
$\{\{\}\}$	#2	conservative		
$\{\mathcal{E}_1\}$	destructive	#3	conservative expansive altering	questioning
$\{\mathcal{E}_1, \ldots, \mathcal{E}_n\}$ $n \geq 2$		#4	decisive	$n < p$: questioning $n > p$: selective $n = p$: conservative expansive altering

With $\mathcal{E}_i \neq \varnothing$ and $\mathcal{E}'_i \neq \varnothing$. Each cell of the table contains the name of the corresponding revision. It can be checked that cells with #i correspond to situations which cannot occur:

#1 and #2 The only acceptability semantics in which an argumentation framework may have no extension is the stable semantics. However, with the stable semantics, an argumentation framework cannot have an empty extension when its set of arguments is not empty. And, by assumption, the cases #1 and #2 correspond to argumentation frameworks with non-empty sets of arguments (because at least X belongs to $\mathcal{G}$ and X and Z belong to $\mathcal{G}'$). So these cases cannot occur for all the acceptability semantics used in this paper.

#3 Under the stable semantics, this case cannot occur for the same reason as that given previously (cases #1 and #2).

Under the grounded semantics, as $\mathcal{G}$ has one non-empty extension, there exists at least one unattacked argument W; so, if the added interaction is (X, Z), W is always unattacked and $\mathcal{G}'$ has always one non-empty extension; and, if the added interaction is (Z, X), then Z is unattacked and it belongs to the grounded extension of $\mathcal{G}'$; so, $\mathcal{G}'$ cannot have an empty extension.

Under the preferred semantics, if the added interaction is (Z, X), Z is unattacked and it belongs to the preferred extensions of $\mathcal{G}'$; so these preferred extensions are not empty. And, if the added interaction is (X, Z), then Z does not attack the arguments of $\mathcal{E}_1$; so these arguments also belong to a preferred extension of $\mathcal{G}'$ and the preferred extensions are not empty.

In conclusion, this case cannot occur for all the acceptability semantics used in this paper.

#4 This case could appear only with the preferred semantics (because with the grounded semantics there exists only one extension, and with the stable semantics, an extension cannot be empty since the set of arguments is not empty). If the added interaction is (Z, X), Z can "remove" or "create" extensions, but it belongs to each of them (because it is unattacked), so $\mathcal{G}'$ cannot have an empty extension. And if the added interaction is (X, Z), Z does not attack the arguments of $\mathcal{E}_i$, $\forall i$, so these arguments belong to the preferred extensions of $\mathcal{G}'$ and $\mathcal{G}'$ cannot have an empty extension. Thus, this case cannot occur.

3.3 Classical revision in argumentation

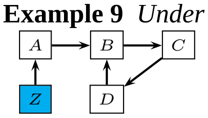
Revising a knowledge base consists in changing its beliefs in a minimal way in order to take into account a new piece of information considered as "prior" (according to AGM **K* 2** postulate). However, the revision operators defined above do not ensure at all that the new argument is accepted in the new graph extensions. In this section, we study when this property (called "classical") holds for a given revision operator.

Definition 6 *The revision $\ominus$ is* classical *iff $\mathcal{G}'$ has at least one extension and the added argument Z belongs to each extension of $\mathcal{G}'$.*

Property 4 *If the added interaction is (Z, X), then the revision is classical under the grounded and the preferred semantics.*

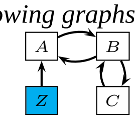
Moreover, if $\mathcal{G}$ has no odd-length cycle then the revision is also classical under the stable semantics.

Example 9 *Under the grounded semantics:*

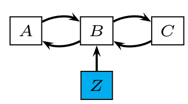


$\mathbf{E} = \{\{A, C\}\}, \mathbf{E}' = \{\{Z\}\}$

Example 10 *Let us compute the extensions of the two following graphs under the preferred semantics:*

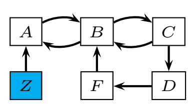


$\mathbf{E} = \{\{A, C\}, \{B\}\}$,
$\mathbf{E}' = \{\{Z, B\}, \{Z, C\}\}$



$\mathbf{E} = \{\{A, C\}, \{B\}\}$,
$\mathbf{E}' = \{\{Z, A, C\}\}$

Example 11 *Under the stable semantics:*



$\mathbf{E} = \{\{A, C, F\}, \{B, D\}\}$,
$\mathbf{E}' = \{\{Z, C, F\}, \{Z, B, D\}\}$

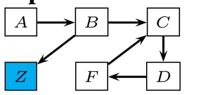
The condition that $\mathcal{G}$ should not have an odd-length cycle ensures the existence of at least one stable extension after adding Z. It is a sufficient but not necessary condition.

Example 9 (continued): *Before revision the stable extension is $\{A, C\}$, and after revision there is no stable extension.*

Property 5 *If the interaction is (X, Z) such that X is attacked by each extension of $\mathcal{G}$ then the revision is classical under the grounded and the preferred semantics.*

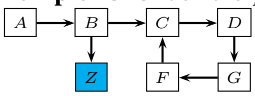
Moreover, if $\mathcal{G}$ has at least one stable extension then the revision is also classical under the stable semantics.

Example 12 *Under the grounded semantics:*



$\mathbf{E} = \{\{A\}\}, \mathbf{E}' = \{\{Z, A\}\}$

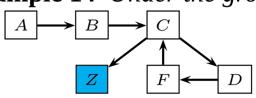
Example 13 *Under the preferred or the stable semantics:*



$\mathbf{E} = \{\{A, C, G\}, \{A, D, F\}\}$,
$\mathbf{E}' = \{\{Z, A, C, G\}, \{Z, A, D, F\}\}$

Note that, under the grounded semantics, X must be attacked and the fact that X is not in the only extension $\mathcal{E}$ of $\mathbf{E}$ does not ensure that the revision is classical :

Example 14 *Under the grounded semantics:*



$\mathcal{E} = \mathcal{E}' = \{A\}$.
So, $\mathbf{E} = \mathbf{E}' = \{\{A\}\}$.

C is not in $\mathcal{E}$, and nevertheless Z does not belong to $\mathcal{E}'$.

As said before, revising a graph by one argument and (one interaction) does not systematically lead to accept this argument. Hence, classicality is not the only property that is worth being studied for revision operators.

4 Case study

In this section, we study two cases: the decisive revision and the expansive revision. In the first case, after the revision there is only one extension (so it is easy to take a decision); in the second case, the number of extensions remains unchanged but each new extension is a superset of an extension which existed before the revision.

4.1 Decisive revision

Decisive revision makes possible a decision: before this revision, in $\mathcal{G}$ there is either no acceptable set of arguments (no possible conclusion), or too many acceptable sets of arguments (so too many possible conclusions), and after this revision there is only one acceptable set of arguments in $\mathcal{G}'$.

Definition 7 *The revision $\ominus$ is* decisive *iff $\ominus$ applied to $\mathcal{G}$, with $\mathbf{E} = \varnothing$, or $\mathbf{E} = \{\{\}\}$, or $\mathbf{E} = \{\mathcal{E}_1, \ldots, \mathcal{E}_n\}$, $n \geq 2$, the result of $\ominus$ is $\mathcal{G}'$ with $\mathbf{E}' = \{\mathcal{E}'\}$, $\mathcal{E}' \neq \varnothing$.*

Property 6 *If a revision is decisive then the added interaction is (Z, X). A decisive revision is classical.*

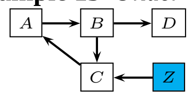
Example 2 (continued):

1. *Under the stable semantics, Example 2.1 illustrates the decisive revision with $\mathbf{E} = \varnothing$ and $\mathbf{E}' = \{\{Z, B\}\}$.*
2. *Under the grounded semantics, Example 2.2 illustrates the decisive revision with $\mathbf{E} = \{\{\}\}$ and $\mathbf{E}' = \{\{Z, B\}\}$.*
3. *Under the preferred semantics, Example 2.3 illustrates the decisive revision with $\mathbf{E} = \{\{A\}, \{B, D\}\}$ and $\mathbf{E}' = \{\{Z, B, D\}\}$.*

Theorem 1 *Under the grounded semantics, if the added interaction is (Z, X) and $\mathbf{E} = \{\{\}\}$, then the revision is decisive.*

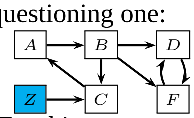
Theorem 2 *Under the preferred semantics, if the added interaction is (Z, X), $\mathbf{E} = \{\{\}\}$ and there is no even-length cycle in $\mathcal{G}$, then the revision is decisive.*

Example 15 *Under the preferred semantics:*



$\mathbf{E} = \{\{\}\}, \mathbf{E}' = \{\{Z, A, D\}\}$

Note that, if even-length cycles exist in the graph, the revision may induce several extensions; this revision would be a questioning one:



$\mathbf{E} = \{\{\}\}$,
$\mathbf{E}' = \{\{Z, A, D\}, \{Z, A, F\}\}$

For this reason, we have considered graphs without even-length cycle in Theorem 2.

Note: under the stable semantics, we have not found any characterization theorem for the decisive revision.

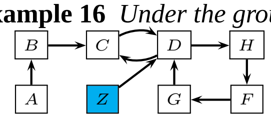
4.2 Expansive revision

A revision is said "expansive" when it does nothing but to add new arguments in the existing extensions.

Definition 8 *The revision $\ominus$ is* expansive *iff $\mathcal{G}$ and $\mathcal{G}'$ have the same number of extensions and each extension of $\mathcal{G}'$ strictly includes an extension of $\mathcal{G}$.*

Property 7 *The expansive revision is classical.*

Example 16 *Under the grounded semantics:*

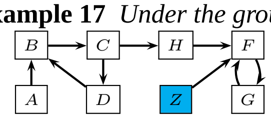


$\mathbf{E} = \{\{A\}\}$,
$\mathbf{E}' = \{\{Z, A, C, H, G\}\}$

Example 6 gives also an illustration of the expansive revision under preferred and stable semantics.

Theorem 3 *Under the grounded semantics with $\mathbf{E} = \{\mathcal{E}\}$, if the added interaction is (Z, X), $X \notin \mathcal{E}$ and $\mathcal{E} \neq \varnothing$, then the revision is expansive.*

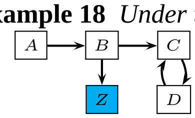
Example 17 *Under the grounded semantics:*



$\mathbf{E} = \{\{A, C\}\}$,
$\mathbf{E}' = \{\{Z, A, C, G\}\}$

Theorem 4 *Under the grounded semantics, if the added interaction is (X, Z), $X \notin \mathcal{E}$ and $\mathcal{E}$ attacks X, then the revision is expansive and $\mathbf{E}' = \{\mathcal{E} \cup \{Z\}\}$.*

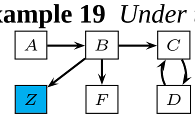
Example 18 *Under the grounded semantics:*



$\mathbf{E} = \{\{A\}\}, \mathbf{E}' = \{\{Z, A\}\}$

Theorem 5 *Under the stable semantics, if the added interaction is (X, Z), if $\mathbf{E} \neq \varnothing$, and $\forall i \geq 1$, $X \notin \mathcal{E}_i$, then the revision is expansive and $\forall i$, $\mathcal{E}'_i = \mathcal{E}_i \cup \{Z\}$.*

Example 19 *Under the stable semantics:*



$\mathbf{E} = \{\{A, C, F\}, \{A, D, F\}\}$,
$\mathbf{E}' = \{\{Z, A, C, F\}, \{Z, A, D, F\}\}$

Note that, in an acyclic graph, Theorem 4 may be applied under the stable semantics. It is a particular case of Theorem 5.

Theorem 6 *Under the preferred semantics, if the added interaction is (X, Z), and $\forall i \geq 1$, $\mathcal{E}_i$ attacks X, then the revision is expansive and $\forall i$, $\mathcal{E}'_i = \mathcal{E}_i \cup \{Z\}$.*

Example 18 (continued): *Under the preferred semantics,* $\mathbf{E} = \{\{A, C\}, \{A, D\}\}$ *and* $\mathbf{E}' = \{\{Z, A, C\}, \{Z, A, D\}\}$

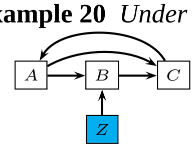
Note that when the initial graph is acyclic, Theorem 4 may be applied under the preferred semantics. It is a particular case of Theorem 6.

If the interaction is (Z, X), weaker results can be obtained. In that case, the revision is not expansive in the sense that $\mathcal{G}'$ may have more extensions than $\mathcal{G}$, however, adding Z to an extension of $\mathcal{G}$ yields an extension of $\mathcal{G}'$.

Property 8 *Under the stable semantics, if the added interaction is (Z, X), and $\forall i \geq 1$, $X \notin \mathcal{E}_i$, then $\forall i$, $\mathcal{E}_i \cup \{Z\}$ is a stable extension of $\mathcal{G}'$.*

However, other stable extensions may appear in $\mathcal{G}'$, see Example 20 for instance. So the revision is not expansive.

Example 20 *Under the stable semantics:*



$\mathbf{E} = \{\{A\}\}$,
$\mathbf{E}' = \{\{Z, A\}, \{Z, C\}\}$

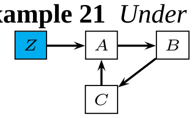
Note that in the particular case when the initial graph is acyclic, Theorem 3 may be applied under the stable semantics. And the obtained result is stronger than the result proposed by Property 8. Another interesting property is:

Property 9 *Under the preferred semantics, if the added interaction is (Z, X) and $\forall i \geq 1$, $X \notin \mathcal{E}_i$, then $\forall i$, $\mathcal{E}'_i = \mathcal{E}_i \cup \{Z\}$ is admissible in $\mathcal{G}'$.*

Moreover, if there is no odd-length cycle in $\mathcal{G}$, $\forall i$, $\mathcal{E}'_i = \mathcal{E}_i \cup \{Z\}$ is a preferred extension in $\mathcal{G}'$.

Example 20 also shows that other preferred extensions may appear in $\mathcal{G}'$. And the following example illustrates the first part of Property 9.

Example 21 *Under the preferred semantics:*



$\mathbf{E} = \{\{\}\}$
$\{\} \cup \{Z\}$ *is admissible in* $\mathcal{G}'$
but $\mathbf{E}' = \{\{Z, B\}\}$.

Note that in the particular case when the initial graph is acyclic, Theorem 3 may also be applied under the preferred semantics and gives a stronger result than the one proposed by Property 9.

5 Discussion and future works

In this paper, we transpose the basic question of revision into argumentation theory. We propose a study of the impact of the arrival of a new argument on the outcome of an argumentation framework. The term "revision" is used by analogy with traditional belief revision. However, there are two main differences.

- The basic underlying formalism is different: in standard belief revision, logical formulas are used for knowledge representation whereas, in this paper, an argumentation framework represents the current knowledge. In the first case, the outcome is a new set of logical formulas, whereas, in the second case, the outcome is a new set of accepted arguments.
- Revision is a task in knowledge representation which is strongly related to concepts such as inference and consistency. The postulates for standard belief revision (AGM) are built on a consistency notion, since it aims at incorporating a new piece of information while preserving consistency. Moreover, "revision" has also been studied in the framework of nonmonotonic theories (Witteveen and van der Hoek 1997). Argumentation theory is linked to nonmonotonicity, but postulates for nonmonotonic theories are also based on consistency and inference notions that are not explicitly present in our framework. So, these postulates are not suited for our problem. Some of the belief revision postulates can be transposed (this is the case for what we call classical revision), but other principles must be proposed.

Our work is a preliminary step towards a formal revision in argumentation frameworks. And it departs from previous work relating argumentation and revision. Indeed, we have chosen to remain at an abstract level in this preliminary study. We do not consider knowledge from which arguments and interactions could be built. More precisely, there are many approaches that deal with adding new pieces of information within an argumentation system. The point of view adopted in this family of works is different because of the status of the new piece of information that is added. For instance, Wassermann (Wassermann 1999), as well as (Falappa, García, and Simari 2004; Paglieri and Castelfranchi 2005), define under which conditions, expressed in terms of arguments, unjustified beliefs should become accepted. The approach of (Pollock and Gillies 2000) studies the properties of knowledge revision under the argumentation point of view, *i.e.*, the problem is to generate a knowledge base in which each piece of information is justified by "good" arguments.

Very recently, (Rotstein et al. 2008) have proposed a warrant-prioritized revision operation, which consists in adding an argument to a theory in such a way that this argument is warranted afterwards. Even if the underlying ideas are similar, this work differs from our approach in at least two points:

- First, in (Rotstein et al. 2008), arguments are given a structure through the subargument relation, and properties such as minimality, consistency and atomicity. And the definition of warranted arguments relies upon an evaluation of argumentation lines. In contrast, our approach remains at the most abstract level, and our sets of accepted arguments are computed with the well-known extension-based semantics.
- Secondly, the warrant-prioritized argument revision is designed in order to satisfy the AGM postulate **K*2**, since the added argument must be warranted in the revised theory. Our work follows another direction. We propose an extensive theoretical study of the impact of an addition on the outcome of an abstract argumentation framework, which enables us to define several kinds of revision.

Note that other crucial cognitive tasks linked to belief change theory have already been transposed in the field of argumentation, see for instance the work on merging presented in (Coste-Marquis et al. 2007).

A promising application of our work could be to design dialogue strategies. Most of the works about dialogue strategies consider that a dialogue is defined by a protocol giving the set of legal moves and that a strategy selects exactly one move (the move which must be done next). For instance, (Bench-Capon 1998) proposes a selection strategy leading to more cooperative dialogues. Other approaches propose dialogue games for answering queries such as: does a given initial argument belong to some extension? In that case, a strategy helps to choose which argument must be defeated in order that the initial argument should be accepted. (Amgoud and Maudet 2002) have proposed heuristics that select the less attackable arguments in a persuasion dialogue. In a similar way, (Riveret et al. 2008) have proposed an optimal strategy in order to win a debate based on the probability of success of the argument and on the cost of this argument for the agent. (Hunter 2004), with a more global approach, has defined a strategy which builds an optimal subtree of arguments maximizing the resonance with the agent goals and minimizing their cost.

Our approach takes another point of view. We do not define any protocol and we do not restrict to a dialogue type. Given a set of arguments which may interact, we are interested in the outcome of the argumentation system, that is the set of extensions under a given semantics. In other words, we study the impact of an argument with respect to the structural change induced on the set of extensions. We do not focus on a particular argument that should be accepted at the end. We just want to act as to modify the form of that outcome (by doing an expansive revision, or a decisive revision for instance). The work reported in this paper enables us to choose the right way of revising (which argument must be affected by the revision, with which kind of interaction) in order to obtain the new outcome. This is why we plan to focus more on strategies for directing a dialogue than on strategies for taking part in it. For instance, if a dialogue arbitrator wants the debate to be more open then she should rather force the next speaker to use arguments appropriate for an expansive revision. If she wants the debate to be more focused then only arguments appropriate for a selective (and even decisive) revision should be accepted.

In order to continue this work, the following directions seem to be of interest:

1. generalize our revision operation to the adding of one argument with several interactions and to the adding of a subgraph of arguments;

2. restate other existing standard belief revision postulates and study the postulates for revision in nonmonotonic systems, in the case where arguments are built from knowledge bases and the outcome of the argumentation framework is a set of formulas;

3. since decisive revision seems to be a "good" kind of revision, it would be interesting to investigate the question "How to make the *minimal change*[3] to a given argumentation framework so that it has a unique non-empty extension?". We thank an anonymous reviewer for this last suggestion.

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[3] In terms of number of edges to add or to remove and/or in terms of number of arguments to add or to remove.

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A Appendix

Lemma 1 *If the added interaction is (Z, X) and $\mathcal{G}'$ has at least one extension $\mathcal{E}'$ then $Z \in \mathcal{E}'$.*

Proof: This result is deduced from Properties 1 and 3. □

Lemma 2 *Under the stable semantics, if the added interaction is (X, Z) and $\mathcal{E}'$ is a stable extension of $\mathcal{G}'$ then $\mathcal{E}' \setminus \{Z\}$ is a stable extension of $\mathcal{G}$.*

Proof:

- $\mathcal{E}'$ is conflict-free in $\mathcal{G}'$ so $\mathcal{E}' \setminus \{Z\}$ is also conflict-free in $\mathcal{G}'$ and in $\mathcal{G}$.
- $\mathcal{E}'$ attacks each argument of $\mathcal{G}'$ which is not in $\mathcal{E}'$. Z attacks no argument, so $\mathcal{E}' \setminus \{Z\}$ attacks all the arguments which are not in $\mathcal{E}'$. So $\mathcal{E}' \setminus \{Z\}$ attacks all the arguments of $\mathcal{G}$ which are not in $\mathcal{E}' \setminus \{Z\}$. So $\mathcal{E}' \setminus \{Z\}$ is a stable extension of $\mathcal{G}$.

□

Lemma 3 *Under the preferred semantics, if the added interaction is (X, Z) and $\mathcal{E}' \neq \varnothing$ is a preferred extension of $\mathcal{G}'$ then $\mathcal{E}' \setminus \{Z\}$ is admissible in $\mathcal{G}$.*

Proof:

- $\mathcal{E}'$ is conflict-free in $\mathcal{G}'$ so $\mathcal{E}' \setminus \{Z\}$ is also conflict-free in $\mathcal{G}'$ and in $\mathcal{G}$.
- Let $Y \in \mathcal{E}' \setminus \{Z\}$. Assume that there is an argument U such that $U\mathbf{R}Y$ then $U \neq Z$ (because Z attacks no argument). $\mathcal{E}'$ is a non-empty preferred extension of $\mathcal{G}'$, so there is an argument $V \in \mathcal{E}'$ such that $V\mathbf{R}U$, $V \neq Z$, (always because Z attacks no argument). So, we have $V \in \mathcal{E}' \setminus \{Z\}$, and $\mathcal{E}' \setminus \{Z\}$ defends all its arguments. So, $\mathcal{E}' \setminus \{Z\}$ is admissible in $\mathcal{G}$.

□

Lemma 4 *If the added interaction is (X, Z) and there is no stable extension in $\mathcal{G}$ then $\mathbf{E} = \mathbf{E}' = \varnothing$.*

Proof:(*reductio ad absurdum*) Assume that there exists a stable extension of $\mathcal{G}'$ denoted by $\mathcal{E}'$. Using Lemma 2, $\mathcal{G}$ would have an extension, which is contradictory with the assumption. □

Lemma 5 *If the added interaction is (X, Z) and $\mathbf{E} = \{\{\}\}$ then $\mathbf{E}' = \{\{\}\}$.*

Proof:(note that this case is impossible under the stable semantics when at least one argument exists)

- under the grounded semantics: $\mathbf{E} = \{\{\}\}$, so there is no unattacked argument in $\mathcal{G}$. Z is attacked so there is also no unattacked argument in $\mathcal{G}'$ and $\mathbf{E}' = \{\{\}\}$.
- under the preferred semantics (*reductio ad absurdum*): Assume that there exists a non-empty extension of $\mathcal{G}'$ denoted by $\mathcal{E}'$. So there exists Y such that $Y \in \mathcal{E}'$. Either $Y = Z$, or $Y \in \mathcal{G}$. In both cases, Y is attacked (because all arguments of $\mathcal{G}$ are attacked and the added interaction is (X, Z)), so $\mathcal{E}'$ must defend Y. If $Y = Z$, $\mathcal{E}'$ cannot be reduced to Y (because Z attacks no argument and cannot defend itself). So $\mathcal{E}' \setminus \{Z\} \neq \varnothing$. If $Y \neq Z$, $Y \in \mathcal{E}' \setminus \{Z\}$, and $\mathcal{E}' \setminus \{Z\} \neq \varnothing$. Due to Lemma 3, $\mathcal{E}' \setminus \{Z\}$ is admissible in $\mathcal{G}$ and so $\mathcal{E}' \setminus \{Z\} \subseteq \mathcal{E}$ with $\mathcal{E}$ being a preferred extension of $\mathcal{G}$. So $\mathcal{G}$ has a non-empty extension, which is in contradiction with the assumption.

□

Lemma 6 *If the added interaction is (X, Z) and there is at least one non-empty extension in $\mathcal{G}$, $\mathbf{E} = \{\mathcal{E}_1, \ldots, \mathcal{E}_n\}$, then $\forall i \geq 1$:*

- *either $\mathcal{E}_i$ is an extension of $\mathcal{G}'$ (it is the case when $\mathcal{E}_i$ does not attack X),*
- *or $\mathcal{E}_i \cup \{Z\}$ is an extension of $\mathcal{G}'$ (it is the case when $\mathcal{E}_i$ attacks X).*

Proof:

- under the grounded semantics ($\mathbf{E} = \{\mathcal{E}\}$ and $\mathbf{E}' = \{\mathcal{E}'\}$): Due to the fact that $\mathbf{R}$ is finite, we have $\mathcal{E} = \cup_{i \geq 1} \mathcal{F}^i(\varnothing)$ and $\mathcal{E}' = \cup_{i \geq 1} \mathcal{F}'^i(\varnothing)$. Let $\mathcal{S}$ be a set of arguments of $\mathcal{G}$. First, as Z attacks no argument of $\mathcal{G}$, it is easy to prove that $\mathcal{F}(\mathcal{S}) \subseteq \mathcal{F}'(\mathcal{S})$. Then, it follows by induction on $i \geq 1$ that $\mathcal{F}^i(\varnothing) \subseteq \mathcal{F}'^i(\varnothing)$. So, $\mathcal{E} \subseteq \mathcal{E}'$. Secondly, as X is the unique attacker of Z, it is easy to check that $\mathcal{S}$ attacks X iff $Z \in \mathcal{F}'(\mathcal{S})$. So we have either $\mathcal{S}$ attacks X and $\mathcal{F}'(\mathcal{S})$ = $\mathcal{F}(\mathcal{S}) \cup \{Z\}$, or $\mathcal{S}$ does not attack X and $\mathcal{F}'(\mathcal{S})$ = $\mathcal{F}(\mathcal{S})$. Now, we consider two cases : either $\mathcal{E}$ attacks X or not. If $\mathcal{E}$ attacks X, $Z \in \mathcal{F}'(\mathcal{E})$, which is included in $\mathcal{F}'(\mathcal{E}')$.

And by definition of the grounded extension, $\mathcal{F}'(\mathcal{E}')$ is $\mathcal{E}'$. So, if $\mathcal{E}$ attacks X, $\mathcal{E} \cup \{Z\} \subseteq \mathcal{E}'$. Conversely, $\mathcal{F}'(\mathcal{E})$= $\mathcal{F}(\mathcal{E}) \cup \{Z\}$ = $\mathcal{E} \cup \{Z\}$. So, $\mathcal{F}'(\mathcal{E} \cup \{Z\})$ = $\mathcal{F}'(\mathcal{E}) \cup \{Z\}$ = $\mathcal{E} \cup \{Z\}$. By definition of the grounded extension of $\mathcal{G}'$ (least fixed point), it follows that $\mathcal{E}' \subseteq \mathcal{E} \cup \{Z\}$. So, we have proved that if $\mathcal{E}$ attacks X, $\mathcal{E}'$ = $\mathcal{E} \cup \{Z\}$. If $\mathcal{E}$ does not attack X, $\mathcal{F}'(\mathcal{E})$ = $\mathcal{F}(\mathcal{E})$ = $\mathcal{E}$. So, by definition of the grounded extension of $\mathcal{G}'$ (least fixed point), $\mathcal{E}' \subseteq \mathcal{E}$. So, we have proved that if $\mathcal{E}$ does not attack X, $\mathcal{E}'$ = $\mathcal{E}$.

- under the stable semantics: $\forall i$, $\mathcal{E}_i$ is conflict-free in $\mathcal{G}'$. Moreover, if $X \in \mathcal{E}_i$ then $Z \notin \mathcal{E}_i$ (because X attacks Z) and $\mathcal{E}_i$ is a stable extension of $\mathcal{G}'$. X is the unique attacker of Z. If $X \notin \mathcal{E}_i$, $\mathcal{E}_i \cup \{Z\}$ is conflict-free. Moreover, in $\mathcal{G}$, $\mathcal{E}_i$ attacks all the arguments which are not in $\mathcal{E}_i$; so, in $\mathcal{G}'$, $\mathcal{E}_i \cup \{Z\}$ attacks all the arguments which are not in $\mathcal{E}_i \cup \{Z\}$; so $\mathcal{E}_i \cup \{Z\}$ is a stable extension of $\mathcal{G}'$.
- under the preferred semantics: Z attacks no argument of $\mathcal{G}$, so $\forall i$, $\mathcal{E}_i$ is admissible in $\mathcal{G}'$. So there exists a preferred extension $\mathcal{E}'_j$ of $\mathcal{G}'$ including $\mathcal{E}_i$. But $Z \notin \mathcal{E}_i$, so $\mathcal{E}_i \subseteq \mathcal{E}'_j \setminus \{Z\}$. Due to Lemma 3, $\mathcal{E}'_j \setminus \{Z\}$ is admissible in $\mathcal{G}$, so there exists $k \geq 1$ such that $\mathcal{E}_i \subseteq \mathcal{E}'_j \setminus \{Z\} \subseteq \mathcal{E}_k$. Using the definition of a preferred extension ($\subseteq$-maximal among the admissible sets), we can conclude that $\mathcal{E}_i = \mathcal{E}'_j \setminus \{Z\} = \mathcal{E}_k$. So, either $\mathcal{E}'_j = \mathcal{E}_i$ (if $Z \notin \mathcal{E}'_j$), or $\mathcal{E}'_j = \mathcal{E}_i \cup \{Z\}$ (if $Z \in \mathcal{E}'_j$). Moreover, if $Z \in \mathcal{E}'_j$, $\mathcal{E}'_j$ defends Z so $\mathcal{E}'_j$ attacks X and so $\mathcal{E}_i$ = $\mathcal{E}'_j \setminus \{Z\}$ attacks X. Conversely, if $\mathcal{E}_i$ attacks X, $\mathcal{E}_i$ defends Z and $\mathcal{E}_i \cup \{Z\}$ is conflict-free in $\mathcal{G}'$. So, $\mathcal{E}_i \cup \{Z\}$ is admissible in $\mathcal{G}'$ and $Z \in \mathcal{E}'_j$.

□

Lemma 7 *If the added interaction is (X, Z), then the number of extensions is preserved by the revision.*

Proof: Due to Lemma 4 (resp. Lemma 5), if $\mathbf{E} = \varnothing$ (resp. $\mathbf{E} = \{\{\}\}$) then the revision does not create new extensions. Let us study the case where $\mathbf{E} = \{\mathcal{E}_1, \ldots, \mathcal{E}_n\}$, $n \geq 1$ and $\forall i$, $\mathcal{E}_i \neq \varnothing$:

- under the stable semantics: Due to Lemma 6, $\forall i \geq 1$ either $X \in \mathcal{E}_i$ and $\mathcal{E}_i$ is an extension of $\mathcal{G}'$, or $X \notin \mathcal{E}_i$ and $\mathcal{E}_i \cup \{Z\}$ is an extension of $\mathcal{G}'$. So, $\mathcal{G}'$ has at least as many extensions as $\mathcal{G}$. Let $\mathcal{E}'_j$ be an extension of $\mathcal{G}'$. Due to Lemma 2, $\mathcal{E}'_j \setminus \{Z\}$ is an extension of $\mathcal{G}$. Let $\mathcal{E}_i$ = $\mathcal{E}'_j \setminus \{Z\}$. Either $Z \in \mathcal{E}'_j$ and then $\mathcal{E}'_j$ is $\mathcal{E}_i \cup \{Z\}$, or $Z \notin \mathcal{E}'_j$ and then $\mathcal{E}'_j$ = $\mathcal{E}_i$ is an extension of $\mathcal{G}$. So, $\mathcal{G}$ and $\mathcal{G}'$ have the same number of extensions.
- under the preferred semantics: Due to Lemma 6, $\forall i \geq 1$ either $\mathcal{E}_i$ does not attack X and $\mathcal{E}_i$ is an extension of $\mathcal{G}'$, or $\mathcal{E}_i$ attacks X and $\mathcal{E}_i \cup \{Z\}$ is an extension of $\mathcal{G}'$. So, $\mathcal{G}'$ has at least as many extensions as $\mathcal{G}$. Let $\mathcal{E}'_j$ be an extension of $\mathcal{G}'$. Due to Lemma 3, $\mathcal{E}'_j \setminus \{Z\}$ is admissible in $\mathcal{G}$. So there exists $\mathcal{E}_i$, an extension of $\mathcal{G}$ such that $\mathcal{E}'_j \setminus \{Z\} \subseteq \mathcal{E}_i$.
  - Either $\mathcal{E}_i$ attacks X, and using Lemma 6, $\mathcal{E}_i \cup \{Z\}$ is an extension of $\mathcal{G}'$. So, $\mathcal{E}'_j \subseteq \mathcal{E}_i \cup \{Z\}$, and as $\mathcal{E}'_j$ is maximal admissible in $\mathcal{G}'$, $\mathcal{E}'_j = \mathcal{E}_i \cup \{Z\}$.
  - Or $\mathcal{E}_i$ does not attack X, and using Lemma 6, $\mathcal{E}_i$ is an extension of $\mathcal{G}'$. As $\mathcal{E}'_j$ is maximal admissible in $\mathcal{G}'$, we have $Z \notin \mathcal{E}'_j$ and $\mathcal{E}'_j = \mathcal{E}'_j \setminus \{Z\} = \mathcal{E}_i$.

  So, $\mathcal{G}$ and $\mathcal{G}'$ have the same number of extensions.

□

Lemma 8 *Let $\mathcal{S}$ be a set of arguments of $\mathcal{G}$ such that $X \notin \mathcal{F}(\mathcal{S})$. If the added interaction is (Z, X), then $\mathcal{F}(\mathcal{S}) \subseteq \mathcal{F}'(\mathcal{S})$*

Proof: Let $Y \in \mathcal{F}(\mathcal{S})$. If Y is not attacked in $\mathcal{G}$ then, due to the fact that $Y \neq X$, Y is not attacked in $\mathcal{G}'$, so $Y \in \mathcal{F}'(\mathcal{S})$. Consider the case when Y is attacked in $\mathcal{G}$ and assume that the argument A is the attacker of Y in $\mathcal{G}'$. Either $A \in \mathcal{G}$ and $\mathcal{S}$ defends Y against A, so $\mathcal{S}$ attacks A and $Y \in \mathcal{F}'(\mathcal{S})$, or $A = Z$ and $X = Y$ which is impossible. □

Proof of Property 4: Due to Property 1 and Lemma 1, each extension of $\mathcal{G}'$ contains Z. If $\mathcal{G}$ has no odd-length cycle, as revision introduces no cycle, then $\mathcal{G}'$ has no odd-length cycle. Hence, $\mathcal{G}'$ has at least one stable extension (Property 2.4). □

Proof of Property 5:

- Grounded semantics: The only extension $\mathcal{E}$ of $\mathcal{G}$ contains at least the argument that attacks X, so $\mathcal{E}$ is a non-empty extension. Lemma 6 may be applied and either $\mathcal{E}' = \mathcal{E}$ or $\mathcal{E}' = \mathcal{E} \cup \{Z\}$. Z is only attacked by X so Z is defended by $\mathcal{E}$ in $\mathcal{G}'$, and then $Z \in \mathcal{F}'(\mathcal{E})$. If $\mathcal{E}' = \mathcal{E}$, $\mathcal{F}'(\mathcal{E}') = \mathcal{F}'(\mathcal{E})$. By definition of $\mathcal{E}'$, we have $\mathcal{E}'$ = $\mathcal{F}'(\mathcal{E}')$, so $\mathcal{E}$ = $\mathcal{E}'$ = $\mathcal{F}'(\mathcal{E})$, which contradicts the fact that $Z \in \mathcal{F}'(\mathcal{E})$. Hence, $\mathcal{E}' = \mathcal{E} \cup \{Z\}$.
- Preferred semantics: Let $\mathcal{E}'_i$ be a preferred extension of $\mathcal{G}'$. Assume that $Z \notin \mathcal{E}'_i$ then $\mathcal{E}'_i$ is conflict-free in $\mathcal{G}$. $\mathcal{E}'_i$ is admissible in $\mathcal{G}$ since Z attacks no argument. Moreover, as $\mathcal{E}'_i$ is an extension of $\mathcal{G}'$, $\mathcal{E}'_i$ is maximal admissible in $\mathcal{G}$ and so is a preferred extension of $\mathcal{G}$. Since X is attacked by each preferred extension of $\mathcal{G}$, we know that $\mathcal{E}'_i$ attacks X, so $X \notin \mathcal{E}'_i$ and so $\mathcal{E}'_i \cup \{Z\}$ is conflict-free in $\mathcal{G}'$. Moreover, $\mathcal{E}'_i$ defends Z. It follows that $\mathcal{E}'_i \cup \{Z\}$ is admissible in $\mathcal{G}'$. That contradicts the fact that $\mathcal{E}'_i$ is a preferred extension of $\mathcal{G}'$ not containing Z. So Z belongs to each preferred extension of $\mathcal{G}'$.
- Stable semantics: Let $\mathcal{E}'_i$ be a stable extension of $\mathcal{G}'$. Assume that $Z \notin \mathcal{E}'_i$ then $\mathcal{E}'_i$ attacks Z. Since X is the only attacker of Z, $X \in \mathcal{E}'_i$. Moreover, as $\mathcal{E}'_i$ is a subset of $\mathcal{G}$, it is also a stable extension of $\mathcal{G}$. That contradicts the fact that each extension of $\mathcal{G}$ attacks X. So Z belongs to each stable extension of $\mathcal{G}'$.

□

Proof of Property 6:

- $\mathbf{E} = \varnothing$ and $\mathbf{E}' = \{\mathcal{E}'\}$ (that case can appear only under the stable semantics): The revision creates an extension, that is impossible if we add an attacked argument [*cf.* Lemma 4]; so the added interaction is (Z, X).
- $\mathbf{E} = \{\{\}\}$ and $\mathbf{E}' = \{\mathcal{E}'\}$ (that case can appear only under the grounded and preferred semantics since at least one argument exists): If $\mathbf{E} = \{\{\}\}$ and the added interaction is (X, Z) then $\mathbf{E}' = \{\{\}\}$ [*cf.* Lemma 5]; so the added interaction is (Z, X).
- $\mathbf{E} = \{\mathcal{E}_1, \ldots, \mathcal{E}_n\}$ and $\mathbf{E}' = \{\mathcal{E}'\}$ (under the stable or preferred semantics): The revision removes some extensions, that is impossible if we add an attacked argument [*cf.* Lemma 7]. So the added interaction is (Z, X).

Now, if the added interaction is (Z, X), each extension of $\mathcal{G}'$ contains Z [*cf.* Lemma 1]. So $Z \in \mathcal{E}'$, and the revision is classical. □

Proof of Theorem 1: In $\mathcal{G}'$, Z is not attacked. So, due to Property 1.5, Z belongs to the grounded extension. So $\mathcal{G}'$ has a grounded extension which is not empty. □

Proof of Theorem 2: The added interaction is (Z, X), so Z is not attacked; it belongs to each preferred extension [*cf.* Property 1.5]. Moreover, there is no even-length cycle in $\mathcal{G}$, so there is no even-length cycle in $\mathcal{G}'$. $\mathcal{G}'$ has only one preferred extension [*cf.* Property 2.5], which is not empty (it contains Z). □

Proof of Property 7:

- If the added interaction is (Z, X): due to Lemma 1, Z belongs to each extension of $\mathcal{G}'$; so the revision is classical.
- If the added interaction is (X, Z): due to Lemma 6, $\forall i$, either $\mathcal{E}'_i = \mathcal{E}_i$, or $\mathcal{E}'_i = \mathcal{E}_i \cup \{Z\}$. Because the revision is expansive, we have $\mathcal{E}'_i = \mathcal{E}_i \cup \{Z\}$; so the revision is classical.

$\square$

Proof of Theorem 3: Due to the fact that $\mathbf{R}$ is finite, we have $\mathcal{E} = \cup_{i\geq 1}\mathcal{F}^i(\varnothing)$ and $\mathcal{E}' = \cup_{i\geq 1}\mathcal{F}'^i(\varnothing)$. We prove by induction on $i \geq 1$ that $\mathcal{F}^i(\varnothing) \subseteq \mathcal{F}'^i(\varnothing)$.

Basic case ($i = 1$): If $Y \in \mathcal{F}(\varnothing)$ then Y is not attacked in $\mathcal{G}$ and due to the fact that $X \notin \mathcal{E}$, we have $Y \neq X$ and Y is not attacked in $\mathcal{G}'$ and $Y \in \mathcal{F}'(\varnothing)$.

Induction hypothesis (for $1 \leq i \leq p$, $\mathcal{F}^i(\varnothing) \subseteq \mathcal{F}'^i(\varnothing)$): let $\mathcal{S} = \mathcal{F}^p(\varnothing)$ and $\mathcal{S}' = \mathcal{F}'^p(\varnothing)$. $\mathcal{S}$ is a set of arguments of $\mathcal{G}$ and $X \notin \mathcal{F}(\mathcal{S})$ (since $X \notin \mathcal{E}$). So Lemma 8 may be applied and we have $\mathcal{F}(\mathcal{S}) \subseteq \mathcal{F}'(\mathcal{S})$. Using the induction hypothesis, we also have $\mathcal{S} \subseteq \mathcal{S}'$. Moreover, by definition $\mathcal{F}'$ is monotonic. So $\mathcal{F}(\mathcal{S}) = \mathcal{F}^{p+1}(\varnothing) \subseteq \mathcal{F}'(\mathcal{S}) \subseteq \mathcal{F}'(\mathcal{S}') = \mathcal{F}'^{p+1}(\varnothing)$.

So, $\mathcal{E} \subseteq \mathcal{E}'$. Moreover, Z is not attacked, so $Z \in \mathcal{E}'$ (*cf* Property 1.5). So $\mathcal{E} \subsetneq \mathcal{E}'$. $\square$

Proof of Theorem 4: $\mathcal{E} \neq \varnothing$ since $\mathcal{E}$ attacks X. So Lemma 6 applies and $\mathcal{E}' = \mathcal{E} \cup \{Z\}$. $\square$

Proof of Theorem 5: X is the only attacker of Z and $X \notin \mathcal{E}_i$; so $\mathcal{E}_i \cup \{Z\}$ is conflict-free. Moreover, in $\mathcal{G}$, $\mathcal{E}_i$ attacks all the arguments which are not in $\mathcal{E}_i$; so, in $\mathcal{G}'$, $\mathcal{E}_i \cup \{Z\}$ attacks all the arguments which are not in $\mathcal{E}_i \cup \{Z\}$; so $\mathcal{E}_i \cup \{Z\}$ is a stable extension of $\mathcal{G}'$. Due to Lemma 7, $\mathcal{G}'$ and $\mathcal{G}$ have the same number of stable extensions. So, the stable extensions of $\mathcal{G}'$ are exactly the $\mathcal{E}_i \cup \{Z\}$. $\square$

Proof of Theorem 6: $\forall i$, $\mathcal{E}_i$ attacks X so $X \notin \mathcal{E}_i$. It follows that $\mathcal{E}_i \cup \{Z\}$ is conflict-free in $\mathcal{G}'$. $\forall i$, $\mathcal{E}_i$ defends Z against X, and $\mathcal{E}_i$ is admissible, so $\mathcal{E}_i \cup \{Z\}$ is admissible in $\mathcal{G}'$. Now, we prove that $\mathcal{E}_i \cup \{Z\}$ is $\subseteq$-maximal admissible in $\mathcal{G}'$. Assume that it is not the case: $\exists \mathcal{E}'_j$ preferred extension of $\mathcal{G}'$ such that $\mathcal{E}'_j \supsetneq \mathcal{E}_i \cup \{Z\}$. Due to Lemma 3, $\mathcal{E}'_j \setminus \{Z\}$ is admissible in $\mathcal{G}$. So, $\mathcal{E}_i \subsetneq \mathcal{E}'_j \setminus \{Z\}$, which is in contradiction with the fact that $\mathcal{E}_i$ is a preferred extension of $\mathcal{G}$. So, $\forall i$, $\mathcal{E}_i \cup \{Z\}$ is a preferred extension of $\mathcal{G}'$. Due to Lemma 7, $\mathcal{G}'$ and $\mathcal{G}$ have the same number of preferred extensions. So, the preferred extensions of $\mathcal{G}'$ are exactly the $\mathcal{E}_i \cup \{Z\}$. $\square$

Proof of Property 8: $\forall i \geq 1$, $X \notin \mathcal{E}_i$, so Z attacks no argument of $\mathcal{E}_i$, and $\mathcal{E}_i \cup \{Z\}$ is conflict-free in $\mathcal{G}'$. Let $A \notin \mathcal{E}_i \cup \{Z\}$. $A \notin \mathcal{E}_i$ and $A \in \mathcal{G}$. $\mathcal{E}_i$ is stable, $\mathcal{E}_i$ attacks A, so $\mathcal{E}_i \cup \{Z\}$ also attacks A. So $\mathcal{E}_i \cup \{Z\}$ is a stable extension of $\mathcal{G}'$. $\square$

Proof of Property 9: $\forall i \geq 1$, $X \notin \mathcal{E}_i$ so $\mathcal{E}_i \cup \{Z\}$ is conflict-free in $\mathcal{G}'$. Let $A \in \mathcal{E}_i \cup \{Z\}$ being attacked in $\mathcal{G}'$. Since no argument attacks Z, $A \neq Z$, so $A \in \mathcal{E}_i$. Since $X \notin \mathcal{E}_i$, $A \neq X$, so A is attacked in $\mathcal{G}$. $\mathcal{E}_i$ is admissible in $\mathcal{G}$, so $\mathcal{E}_i$ defends A and then $\mathcal{E}_i \cup \{Z\}$ also defends A. So, $\mathcal{E}_i \cup \{Z\}$ is admissible in $\mathcal{G}'$.

If there is no odd-length cycle in $\mathcal{G}$, preferred and stable extensions coincide. So, Property 8 may be applied. $\square$