An Abstract Argumentation Framework with Varied-Strength Attacks

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Abstract

In classical abstract argumentation, arguments interact with each other through a single abstract notion of attack. However, several concrete forms of argument conflict are present in the literature, all of them of different nature and strength for a particular context. In this work we define an argumentation framework equipped with a set of abstract attack relations of varied strength. Using this framework, semantic notions dealing with the relative difference of strength are introduced. The focus is put on argument defense, and the study of admissible sets according to the quality of defenders.

Introduction

Abstract argumentation systems (Dung 1995; Vreeswijk 1997; Amgoud & Cayrol 1998; 2002; Martínez, García, & Simari 2006b) are formalisms for argumentation where some components remain unspecified towards the study of pure semantic notions. Most of the existing proposals are based on the single abstract concept of *attack* represented as a binary relation, and according to several rational rules, extensions are defined as sets of possibly accepted arguments. The attack relation is basically a subordinate relation of conflicting arguments. For two arguments $\mathcal{A}$ and $\mathcal{B}$, if $(\mathcal{A}, \mathcal{B})$ is in the attack relation, then the status of acceptance of $\mathcal{B}$ is conditioned by the status of $\mathcal{A}$, but not the other way around. It is said that argument $\mathcal{A}$ attacks $\mathcal{B}$, and it implies a priority between conflicting arguments. One of the most important formalizations on abstract argumentation is the framework defined by Dung in (Dung 1995), where the simplicity of the model allows practical definitions of sets of arguments as possible sets of acceptance. The attack relation in Dung's work is a binary relation between arguments as described previously. However, this relation is not always accurate to model some situations where more detail is needed. In several argumentation scenarios, not every argument conflict has the same weight, as they arise for underlying different reasons.

Consider the following arguments exposing reasons for and against the fairness of a recent increase in agricultural export taxes.

- *IncTaxGood*: The tax on soy bean export is fair, because it makes soy unattractive for local farmers and it promotes other agriculture production.
- *IncTaxBad*: The tax on soy bean export is not fair, as it drastically reduces the farmer's income, and the taxation is assumed to be used by the government for demagogic purposes.
- *GoodUse*: Government affirms the taxation is needed to bring inflation down.
- *GoodIncome*: Government says farmers are having a high income from soy exports anyway.

Although there is a general state of contradiction between these arguments, the nature of this conflict is different from case to case. The argument *IncTaxBad* contradicts argument *IncTaxGood* by the exposure of different reasons for a complementary conclusion. Argument *GoodUse*, in turn, contradicts an assumption of *IncTaxBad* about the destination of the money and thus the assumption seems to be no longer valid. Finally, a new argument called *GoodIncome* establishes a government opinion about farmer's welfare. This argument is now in contradiction with one of the premises of *IncTaxGood*, although this contradiction is clearly based on a subjective point of view. In order to properly analyze the global scenario, it is important to distinguish the difference of strength in every argument conflict. For example, maybe in a formal negotiation argument *GoodIncome* cannot be freely used, and perhaps argument *GoodUse* is more likely to be presented in an informal discussion between government and farmer's Union.

Every argumentation system defines one or more notions of argument conflict, leading to an evolved concept of argument attack. For example, several systems use the notion of *rebut* and *undercut* conflict (Elvang-Goransson & Hunter 1995). The first one is due to contradictory conclusions between arguments, while the second is due to a contradiction between a conclusion of an argument and a premise of the other. In Defeasible Logic Programming (García & Simari 2004) two kinds of attack (defeat) relations are present. These relations are obtained by applying a preference criterion between conflicting arguments, thus obtaining *blocking* and *proper* attacks. An abstract framework capturing this dual interaction is defined in (Martínez, García, & Simari 2006b; 2006a; 2007). In (Tohme, Bodanza, & Simari

2008) the aggregation of different abstract attack relations over a common set of arguments is addressed. These attacks represent diversity of criteria on several rational agents willing to reach on agreement about argument conflicts.

The motivation of this work is to distinguish preferences between attack notions, introducing a structure for semantics elaborations on attacks of different strength, as suggested in the running example. We define an argumentation framework equipped with a set of unquestionable abstract attack relations of varied intrinsic force. The nature and structure of every attack is not specified. It is sufficient to state that these attacks are of varied strengths and some of them can be compared to others under this criteria. Using this novel framework with ordered attacks, the classic notions of acceptability of arguments and admissible sets are applied leading to new formalizations of similar ideas. These extended notions are based on a set of restrictions on attacks.

In the next section the abstract framework with varied strength attacks is introduced. Semantic notions dealing with this relative difference of strength are defined in subsequent sections. Finally, a simple operator to safely add arguments in an extension is presented.

Abstract Framework

Our framework includes a set of arguments and a finite set of binary attack relations denoting conflicts of different nature.

Definition 1 (Framework) *An AF with varied strength attacks (AFV) is a triplet $\langle Args, Atts, R\rangle$ where $Args$ is a set of arguments, $Atts$ is a set of binary attack relations $\{\rightarrow_1, \rightarrow_2, \ldots, \rightarrow_n\}$ defined over $Args$, and R is a binary relation defined over $Atts$.*

The relation $R \subseteq Atts \times Atts$ denotes an order of strength between argument conflicts. Arguments are abstract entities that will be denoted using calligraphic uppercase letters. The set $Atts$ represents different abstract forms of conflicts between arguments, modeled by every $\rightarrow_i \subseteq Args \times Args, 1 \leq i \leq n$. For two arguments $\mathcal{A}$ and $\mathcal{B}$, if $\mathcal{A} \rightarrow_i \mathcal{B}$ then it is said that $\mathcal{A}$ *attacks* $\mathcal{B}$. In this work R it is only assumed to be reflexive.

Definition 2 (Relative strength) *Let $\langle Args, Atts, R\rangle$ be an AFV where $Atts = \{\rightarrow_1, \rightarrow_2, \ldots, \rightarrow_n\}$. For two attack relations $\rightarrow_i$ and $\rightarrow_j$, $1 \leq i, j \leq n$,*

- *if $(\rightarrow_i, \rightarrow_j) \in R$ and $(\rightarrow_j, \rightarrow_i) \notin R$ then it is said that $\rightarrow_i$ is a stronger attack than $\rightarrow_j$, denoted $\rightarrow_i \gg \rightarrow_j$. It may also be said that $\rightarrow_j$ is a weaker attack than $\rightarrow_i$, denoted $\rightarrow_j \ll \rightarrow_i$.*
- *if $(\rightarrow_i, \rightarrow_j) \in R$ and $(\rightarrow_j, \rightarrow_i) \in R$ then it is said that $\rightarrow_i$ and $\rightarrow_j$ are equivalent in force, denoted $\rightarrow_i \approx \rightarrow_j$.*
- *if $(\rightarrow_i, \rightarrow_j) \notin R$ and $(\rightarrow_j, \rightarrow_i) \notin R$ then it is said that $\rightarrow_i$ and $\rightarrow_j$ are of unknown difference force, denoted $\rightarrow_i\ ?\ \rightarrow_j$.*

An attack may be stronger, or equivalent in force, or incomparable to other attacks. Being R reflexive, then $\rightarrow_i \approx \rightarrow_i$ for any attack $\rightarrow_i$[1]. We will only explicitly mention those ?-pairs of attacks which are relevant for the particular case. Clearly, there may be more attacks not related under R, but they will be omitted for simplicity.

Example 1 *Following the introductory example of tax increase, the framework is $\Phi_{tax} = \langle Args, Atts, R\rangle$ where $Args = \{IncTaxGood, IncTaxBad, GoodUse, GoodIncome\}$. Three kinds of attack may be modeled in this example: from argument conclusion to argument conclusion ($\rightarrow_{cc}$), from argument conclusion to argument assumption ($\rightarrow_{ca}$) and subjective contradiction ($\rightarrow_{sc}$). Thus, the set of attacks is $Atts = \{\rightarrow_{cc}, \rightarrow_{ca}, \rightarrow_{sc}\}$ where $\rightarrow_{cc} = \{(IncTaxBad, IncTaxGood), (IncTaxGood, IncTaxBad)\}$, $\rightarrow_{ca} = \{(GoodUse, IncTaxBad)\}$ and $\rightarrow_{sc} = \{(GoodIncome, IncTaxBad)\}$. Note that $\rightarrow_{cc}$ is symmetric. A possible order of strength attacks is $R = \{\rightarrow_{ca} \gg \rightarrow_{cc}, \rightarrow_{sc} \ll \rightarrow_{cc}, \rightarrow_{sc} \ll \rightarrow_{ca}\}$. In this order, the subjective comparison is the weakest form of attacks, while an attack from conclusion to premise is the strongest one.*

When an argument has a number of attackers, adversarial pieces of knowledge of different forces are taking place. This scenario is suitable for quality considerations on potential argument extensions. For example, an argument may be required to be defended by an attack involved in a stronger attack relation. In this case, attacks are overruled only be the use of a stronger force. On the other hand, perhaps it is desirable that weaker forms of attack be completely ignored. In legal argumentation doubtful sources of information can lead to the construction of attacking arguments, but these attacks can be considered irrelevant (or overruled) in most contexts.

Observation 1 *If $\rightarrow_i \approx \rightarrow_j$ for all $\rightarrow_i, \rightarrow_j \in Atts$, then the result is the Dung's classical abstract framework (Dung 1995) $AF = \langle Args, At\rangle$ where*

$$At = \rightarrow_1 \cup \rightarrow_2 \cup \ldots \cup \rightarrow_n$$

We will focus mainly on argument defense. We depict argumentation frameworks using graphs, where arguments are represented as black triangles and a labeled arc ($\leftarrow$) is used to denote attacks. An arc with label i denotes the attack $\rightarrow_i$. Consider the argumentation framework depicted in Figure 1. Arguments $\mathcal{C}$ and $\mathcal{D}$ are attacking $\mathcal{B}$, which in turn attacks $\mathcal{A}$. Thus, it is said that $\mathcal{C}$ and $\mathcal{D}$ are *defenders* of $\mathcal{A}$ against $\mathcal{B}$. Regarding argument $\mathcal{A}$, the attack $\rightarrow_i$ is an *offensive* attack, while attacks $\rightarrow_j$ and $\rightarrow_k$ are *defensive* attacks. Two kinds of attack comparison can be made. First, the defensive attack can be compared to the offensive attack, leading to a measure of strength of one particular defense. We call this an offense-defense comparison. In Figure 1, $\rightarrow_i$ can be compared to $\rightarrow_j$. Second, all the defensive attacks on a single argument can be compared to each other. We call this a defense-defense comparison. In Figure 1, $\rightarrow_j$ can be compared to $\rightarrow_k$.

The following definition classifies defenders according to a offense-defense comparison.

Definition 3 (Defense strength) *Let $\langle Args, Atts, R\rangle$ be an AFV. Let $\mathcal{A}, \mathcal{B}, \mathcal{C} \in Args$ such that $\mathcal{B} \rightarrow_i \mathcal{A}$ and $\mathcal{C} \rightarrow_j \mathcal{B}$. Then*

[1] For simplicity, we will omit reflexive cases when describing frameworks

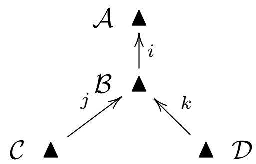

Figure 1: $\mathcal{C}$ and $\mathcal{D}$ are defending $\mathcal{A}$ against $\mathcal{B}$.

- *$\mathcal{C}$ is a strong defender of $\mathcal{A}$ against $\mathcal{B}$ if $\rightarrow_j \gg \rightarrow_i$.*
- *$\mathcal{C}$ is a weak defender of $\mathcal{A}$ against $\mathcal{B}$ if $\rightarrow_j \ll \rightarrow_i$.*
- *$\mathcal{C}$ is a normal defender of $\mathcal{A}$ against $\mathcal{B}$ if $\rightarrow_j \approx \rightarrow_i$.*
- *$\mathcal{C}$ is an unqualified defender of $\mathcal{A}$ against $\mathcal{B}$ if $\rightarrow_j ? \rightarrow_i$.*

As attacks are ordered by its force, strong defenders are considered better than normal defenders. In the same manner, normal defenders are considered better than unqualified defenders. The framework Φ_{tax} is shown in Figure 2. Argument $GoodUse$ is a strong defender of $IncTaxGood$, while $GoodIncome$ is a weak defender of $IncTaxGood$. As $IncTaxGood$ and $IncTaxBad$ are mutual attackers, each argument is a normal defender of itself.

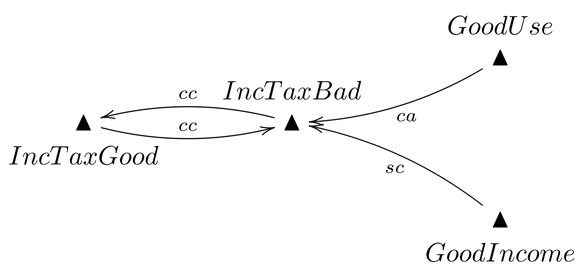

Figure 2: Framework Φ_{tax}

When applying a defense-defense comparison it is important to observe that a defense achieved by the stronger force is naturally predominant. In the context of framework Φ_{tax} the best defender of $IncTaxGood$ is $GoodUse$. We said then that some defenders *dominate* other in the task of defense.

Definition 4 (Dominant defender) *Let $\langle Args, Atts, R\rangle$ be an AFV. Let $\mathcal{A}, \mathcal{B}, \mathcal{C}$ and $\mathcal{D}$ be arguments in $Args$ such that $\mathcal{B} \rightarrow_i \mathcal{A}$, $\mathcal{C} \rightarrow_j \mathcal{B}$ and $\mathcal{D} \rightarrow_k \mathcal{B}$. Argument $\mathcal{C}$ is said to dominate $\mathcal{D}$ as a defender if $\rightarrow_j \gg \rightarrow_k$.*

For an attacked argument, the ideal defense against an attacker $\mathcal{X}$ is achieved by a strong defender $\mathcal{Y}$ such that $\mathcal{Y}$ is not dominated by any other defender on the same argument $\mathcal{X}$. However, this is not always possible. For example, there may not be strong defenders, but there is a normal defender which dominates all other defenders. What is important is to distinguish the relative difference of strength.

Example 2 *Consider the AFV of Figure 3, where $\rightarrow_1 \gg \rightarrow_2$, $\rightarrow_2 \gg \rightarrow_3$, $\rightarrow_4 ? \rightarrow_2$ and $\rightarrow_4 \gg \rightarrow_3$. Every argument achieve its defense with different strength. Argument $\mathcal{E}$ is a strong defender of $\mathcal{A}$. Argument $\mathcal{C}$ is a weak defender of $\mathcal{A}$ while $\mathcal{F}$ is an unqualified defender of $\mathcal{A}$. In this case, $\mathcal{F}$ dominates $\mathcal{C}$ as $\rightarrow_4$ is a stronger attack than $\rightarrow_3$.*

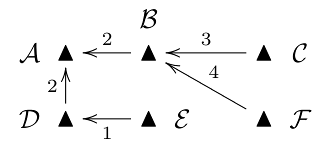

Figure 3: Defenses of varied strength

In the following section, the classic notions of acceptability of arguments and admissible sets are applied to the abstract framework with varied-strength attacks.

Admissibility semantics

Argumentation semantics is about argument classification through several rational positions of acceptance. A central notion in most argument extensions is *acceptability*.

Definition 5 (Classic Acceptability) *(Dung 1995) An argument $\mathcal{A}$ is acceptable with respect to a set of arguments S if and only if every attacker $\mathcal{B}$ of $\mathcal{A}$ has an attacker in S.*

Acceptability is the basis of many argumentation semantics and leads to the notion of *admissibility*, which is applied to conflict-free sets.

Definition 6 (Conflict-free) *Let $\langle Args, Atts, R\rangle$ be an AFV. A set of arguments $S \subseteq Args$ is said to be conflict-free if for all $\mathcal{A}, \mathcal{B} \in S$ it is not the case that $\mathcal{A} \rightarrow_i \mathcal{B}$ for any $\rightarrow_i \in Atts$.*

The classical notion of admissible sets defined by Dung in (Dung 1995) is as follows.

Definition 7 (Classic Admissibility) *(Dung 1995) A set of arguments S is said to be admissible if it is conflict-free and every argument in S is acceptable with respect to S.*

An admissible set is able to defend any argument included in that set. These widely accepted definitions are suitable for an AFV, where an attack is interpreted as any $\rightarrow_i \in Atts$ However, when a specific strength constraint is desired in argument defense, this global notion of acceptability is no longer sufficient. In that case, there is a need to capture defense under certain conditions, as shown in the following definitions.

Definition 8 (Attack scenario) *Let $\Phi = \langle Args, Atts, R\rangle$ be an AFV, $\mathcal{A} \in Args$, $S \subseteq Args$ and $P \subseteq \{\gg, \ll, \approx, ?\}$. The pair $p = [S, P]$ is called an (attack) scenario of Φ. The set P is called the defense condition of p.*

An attack scenario is formed by a set of arguments and a set of defense conditions. These conditions are capturing the difference of strength in defense that is take into consideration for acceptability purposes.

Definition 9 (Constrained Acceptability.) *Let Φ be an AFV, and let $[S, P]$ be an attack scenario in Φ. An argument $\mathcal{A}$ is acceptable with respect to $[S, P]$ if, and only if, for any argument $\mathcal{X}$ such that $\mathcal{X} \rightarrow_i \mathcal{A}$, there is an argument $\mathcal{Y} \in S$ such that $\mathcal{Y} \rightarrow_j \mathcal{X}$ and $\rightarrow_j \rho \rightarrow_i$ where $\rho \in P$.*

The set $P \subseteq \{\gg, \ll, \approx, ?\}$ is a form of defense condition that must be satisfied for every defender of an argument. In this way we are modelling conditions regarding difference of strength between offensive and defensive attacks.

Example 3 *Consider the framework of Figure 3. Argument $\mathcal{A}$ is acceptable with respect to $[\{\mathcal{E}, \mathcal{F}\}, \{\gg, ?\}]$ but not with respect to $[\{\mathcal{C}, \mathcal{E}\}, \{\gg, ?\}]$.*

Having Definition 9, the derived notion of constrained admissibility is straightforward.

Definition 10 (Constrained Admisibility) *Let Φ be an AFV, and let $[S, P]$ be an attack scenario in Φ. The pair $[S, P]$ is called an admissible scenario if, and only if, S is conflict free and every argument $\mathcal{X} \in S$ is acceptable with respect to $[S, P]$.*

In the abstract framework depicted in Figure 3, the pair $[\{\mathcal{A}, \mathcal{E}, \mathcal{F}\}, \{\gg, ?\}]$ is an admissible scenario.

Proposition 1 *If $[S, P]$ is an admissible scenario then*

1. *$[S, Q]$ is also an admissible scenario, for any $Q \supseteq P$.*
2. *S is (classical) admissible.*
3. *If $[T, P]$, $T \subseteq Args$, is an admissible scenario and $S \cup T$ is conflict-free then $[S \cup T, P]$ is also an admissible scenario.*

Proof:

1. *If $\mathcal{A} \in S$ is acceptable with respect to $[S, P]$ then all its defenders are present in S and the defense condition P is satisfied. Adding new forms of defense constraints to an already admissible scenario can not disrupt these facts and therefore the admissibility property is kept.*
2. *From previous proof, $[S, \{\ll, \gg, ?, \approx\}]$ is an admissible scenario and thus every argument in S is defended under all conditions. Then S is admissible.*
3. *The set T includes defenders for every argument in T, and therefore every argument in $S \cup T$ has its defenders in $S \cup T$. As in both cases all the defense conditions are met, then $[S \cup T, P]$ is an admissible scenario.*

□

In the following section the comparison between defenders is presented in order to characterize the inner strength of an admissible set.

Defense-defense comparison

Constrained acceptability focuses on the use of defenders by imposing restrictions on its attacks. Thus, some attacks will not be eligible for defense. However, as stated before, for a given valid defense it is possible to evaluate the quality of a defender among its partners.

Definition 11 (Stronger Defense) *Let $\langle Args, Atts, R\rangle$ be an AFV, $\mathcal{A} \in Args$, $S_1, S_2 \subseteq Args$ and $P, Q \subseteq \{\gg, \ll, \approx, ?\}$ such that $\mathcal{A}$ is acceptable with respect to $[S_1, P]$. The pair $[S_2, Q]$ is said to be a stronger collective defense of $\mathcal{A}$ if $\mathcal{A}$ is acceptable with respect to $[S_2, Q]$, and*

1. *There are no two arguments $\mathcal{X} \in S_1$ and $\mathcal{Y} \in S_2$ such that $\mathcal{X}$ dominates $\mathcal{Y}$*
2. *For at least one defender $\mathcal{X} \in S_1$ of $\mathcal{A}$, there exists an argument $\mathcal{Y} \in S_2 - S_1$ such that $\mathcal{Y}$ dominates $\mathcal{X}$.*

A pair $[S_2, P]$ is a stronger collective defense of $\mathcal{A}$ if at least one defender in S_1 is dominated by a defender in S_2 and no argument in S_1 dominates an argument in S_2.

Example 4 *Consider the AFV of Figure 4 where $\rightarrow_2 ? \rightarrow_i$ with $i \in \{1, 3, 4, 5\}$, $\rightarrow_3 \gg \rightarrow_1$, $\rightarrow_1 \approx \rightarrow_4$ and $\rightarrow_5 \gg \rightarrow_1$, $\rightarrow_3 \gg \rightarrow_4$. Argument $\mathcal{A}$ is acceptable, for example, with respect to*

- $p_1 = [\{\mathcal{D}, \mathcal{F}\}, \{\gg\}]$
- $p_2 = [\{\mathcal{E}, \mathcal{F}\}, \{\gg, \approx\}]$
- $p_3 = [\{\mathcal{G}\}, \{?\}]$

The pair p_1 constitutes a stronger collective defense than p_2 because $\mathcal{D}$ dominates $\mathcal{E}$. No pair constitutes a stronger collective defense than p_3, nor p_3 can be stronger defense than any other, as $\rightarrow_2$ is incomparable to every other notion of attack.

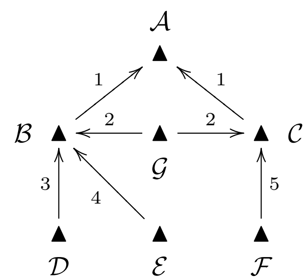

Figure 4: Stronger defenses

Example 5 *Consider the AFV of Figure 5 where $\rightarrow_1 \ll \rightarrow_i$, $2 \leq i \leq 4$, $\rightarrow_3 \gg \rightarrow_2$ and $\rightarrow_4 \gg \rightarrow_3$. Argument $\mathcal{A}$ is acceptable with respect to $[\{\mathcal{C}\}, \{\gg\}]$, $[\{\mathcal{D}\}, \{\gg\}]$ and $[\{\mathcal{E}\}, \{\gg\}]$. Given the difference of strength between attacks, $[\{\mathcal{E}\}, \{\gg\}]$ is a stronger collective defense than $[\{\mathcal{D}\}, \{\gg\}]$, which is in turn a stronger collective defense than $[\{\mathcal{C}\}, \{\gg\}]$.*

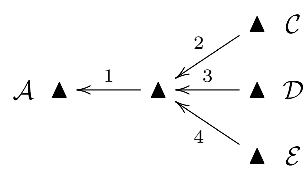

Figure 5: Several defenses for $\mathcal{A}$.

In the following section we formalize a notion of strength equilibrium between arguments. This is a property that can be exhibited by admissible sets under certain conditions.

Looking for the best defenders

When several admissible sets are present as an alternative for acceptance, the strength of defenses can be observed towards a concrete acceptance decision. What seems to be relevant here is to choose those extensions providing best defenses for its elements.

Example 6 *Consider the AFV of Figure 6 where $\rightarrow_2 \gg \rightarrow_1$. Argument $\mathcal{A}$ is acceptable with respect to $p_1 = [\{\mathcal{D}\}, \{\approx\}]$. It is also acceptable with respect to $p_2 = [\{\mathcal{C}\}, \{\gg\}]$. Note that p_2 is a stronger defense than p_1 as $\mathcal{C}$ dominates $\mathcal{D}$. The only (and therefore the stronger one) defense for $\mathcal{F}$ is p_1 and for $\mathcal{H}$ it is p_2. The pairs $adm_1 = [\{\mathcal{A}, \mathcal{D}, \mathcal{F}\}, \{\gg, ?\}]$ and $adm_2 = [\{\mathcal{A}, \mathcal{C}, \mathcal{H}\}, \{\approx, ?\}]$ are both admissible scenarios. However, adm_1 exhibits more strength, as every argument is defended by its stronger available defense.*

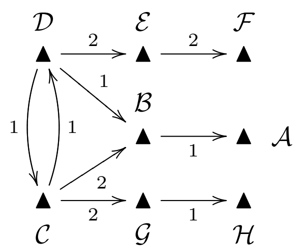

Figure 6: $[\{\mathcal{A}, \mathcal{D}, \mathcal{F}\}, \{\gg, ?\}]$ is a strong admissible set

Definition 12 (Top-admissibility) *Let $\langle Args, Atts, R\rangle$ be an AFV, $S \subseteq Args$ and $P \subseteq \{\gg, \ll, \approx, ?\}$. A pair $[S, P]$ is said to be a top-admissible set if,*

- *$[S, P]$ is an admissible scenario and*
- *For any argument $\mathcal{A} \in S$, no other admissible scenario includes a stronger defense of $\mathcal{A}$.*

A top-admissible scenario is such that every argument in the scenario is defended by a strongest defense. Clearly, it is a very restrictive requirement as a strongest defense may not always be available.

Example 7 *Consider the AFV of Figure 7 where $\rightarrow_2 \gg \rightarrow_1$ The scenario $p_1 = [\{\mathcal{A}, \mathcal{C}, \mathcal{F}\}, \{\gg, \approx\}]$ is top-admissible. Also $p_2 = [\{\mathcal{A}, \mathcal{D}, \mathcal{F}\}, \{\gg, \approx\}]$ is top admissible as $\mathcal{A}$ has no dominant defenders. The scenario $p_3 = [\{\mathcal{A}, \mathcal{C}, \mathcal{D}, \mathcal{G}\}, \{\gg, \approx\}]$ is not top-admissible as $\mathcal{F}$ is a stronger defender of $\mathcal{C}$ and $\mathcal{D}$.*

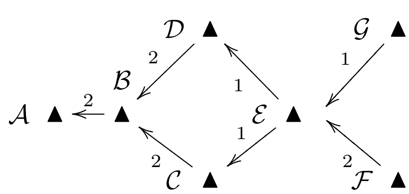

Figure 7: $[\{\mathcal{A}, \mathcal{D}, \mathcal{F}\}, \{\gg, \approx\}]$ top-admissible

As some arguments are defended in several ways with different strengths, it may be possible to *choose*, for a given argument, the best set of defenders. However, in some scenarios the best defense is not necessarily the strongest one. The following definition captures a notion of simple defense upgrade. This upgrade is specifically applied to *direct defenders* of a single argument in an admissible scenario. A *direct defender* of an argument $\mathcal{A}$ is an attacker of an attacker of $\mathcal{A}$. An *indirect defender* is defending a direct or indirect defender of $\mathcal{A}$.

Definition 13 (Upgraded defense) *Let $\langle Args, Atts, R\rangle$ be an AFV, $S_1, S_2 \subseteq Args$ and $P \subseteq \{\gg, \ll, \approx, ?\}$. Let $p_1 = [S, P]$ be an admissible scenario and let $\mathcal{A}$ be an argument in S such that $p_2 = [S_2, P]$ is a stronger defense of $\mathcal{A}$. The scenario obtained by upgrading the defense of $\mathcal{A}$ by p_2 in p_1 is $p_1{\uparrow}_{\mathcal{A}}^{p_2} = [(S - def_{p_1}(\mathcal{A})) \cup def_{p_2}(\mathcal{A}), P]$ where $def_{p_i}(\mathcal{A})$ is the set of direct defenders of $\mathcal{A}$ in p_i, $i \in \{1, 2\}$.*

The upgrade of defense of an argument $\mathcal{A}$ in a set S is simply the removal of its defenders in S and a subsequent addition of other defenders. Note that the structure of S is being disrupted: probably a part of the defense will remain the same, but others defenders were replaced. Moreover, being this operation centered on the defense of a single argument, the obtained set may not be conflict-free, as the new defenders may attack or be attacked by other arguments in the set. However, in some cases, the result of this replacement may lead to an admissible set.

Example 8 *Consider the AFV of Figure 7. The scenario $p_1 = [\{\mathcal{A}, \mathcal{C}, \mathcal{F}\}, \{\gg, \approx\}]$ is top-admissible and no upgrade defense is possible. The scenario $p_3 = [\{\mathcal{A}, \mathcal{C}, \mathcal{D}, \mathcal{G}\}, \{\gg, \approx\}]$ is not top-admissible. Argument $\mathcal{C}$ is acceptable with respect to $q = [\{\mathcal{F}\}, \{\gg, \approx\}]$ which is a stronger defense than that provided by p_3. The scenario $p_3{\uparrow}_{\mathcal{C}}^{q}$ is admissible.*

Definition 14 (Safe upgrade) *Let $\langle Args, Atts, R\rangle$ be an AFV, $p_1 = [S, P]$ be an admissible scenario and let $\mathcal{A}$ be an argument in S such that $p_2 = [S_2, P]$ is a stronger defense of $\mathcal{A}$. The upgrade of $\mathcal{A}$'s defense by p_2 in p_1 is said to be safe if $p_1{\uparrow}_{\mathcal{A}}^{p_2}$ is an admissible scenario.*

If the upgrade is safe, then perhaps this upgrade favors some arguments with stronger defenses. In Example 8, the upgrade $p_3{\uparrow}_{\mathcal{C}}^{q}$ also provides a stronger defense for $\mathcal{D}$. It is also possible that an upgrade replaces some of the defenders of an argument $\mathcal{X}$ by weaker defenders. It is said then that the defense of $\mathcal{X}$ is *weakened* by the upgrade. This leads to the notion of an equilibrated-admissible scenario.

Definition 15 (Equilibrated-admissible scenario) *Let $\Phi = \langle Args, Atts, R\rangle$ be an AFV, $S \subseteq Args$ and $P \subseteq \{\gg, \ll, \approx, ?\}$. A pair $[S, P]$ is said to be an equilibrated-admissible scenario if,*

- *$[S, P]$ is an admissible scenario and*
- *For every argument $\mathcal{A} \in S$, any safe upgrade of $\mathcal{A}$'s defense will weaken the defense of some of the rest of the arguments.*

An admissible scenario can be viewed as a kind of contract among the participants. Every argument is defended by arguments in the set. An argument $\mathcal{A}$ in an equilibrated scenario p cannot change its defenders $def_p(\mathcal{A})$ by stronger ones and provide, by that change, a stronger defense for some of the arguments in $S - def_p(\mathcal{A})$. Thus, $\mathcal{A}$ is somehow tied up with arguments in $S - \{\mathcal{A}\}$, as its defense cannot be freely changed without harming other arguments. Clearly some arguments are dropped out, $i.e.$ the old defenders, but the new defenders leave other arguments with lower defenses or, at worst, without defense at all or in conflict with other arguments ($i.e.$, the upgrade is not even safe).

Example 9 *Consider the AFV of Figure 8 where $\rightarrow_1 \gg \rightarrow_2$. The pairs $p_1 = [\{\mathcal{A}, \mathcal{C}, \mathcal{F}\}, \{\gg, \approx\}]$ and $p_2 = [\{\mathcal{A}, \mathcal{D}, \mathcal{F}\}, \{\gg, \approx\}]$ are two admissible scenarios. Note that p_2 is a stronger collective defense for $\mathcal{A}$ than p_1, as $\mathcal{C}$ dominates $\mathcal{D}$ when defending $\mathcal{A}$. In a similar way, p_1 is a stronger collective defense for $\mathcal{F}$ than p_2, as $\mathcal{D}$ dominates $\mathcal{C}$ when defending $\mathcal{F}$. Therefore, p_1 and p_2 are not top-admissible scenarios as they do not provide the strongest defense for one of its arguments. Even more, no argument in such scenarios can "upgrade" its defense whitout affecting the defense of other arguments. For example, $\mathcal{A}$ cannot prefer $\mathcal{D}$ as a defeater without lowering the defense of $\mathcal{F}$ (which is not a defender of $\mathcal{A}$).*

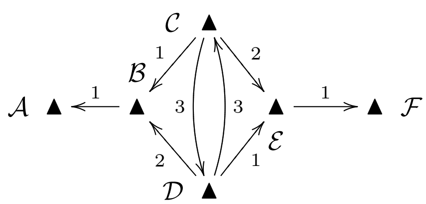

Figure 8: Several defenses for $\mathcal{A}$.

There is an inner equilibrium in the admissible scenarios p_1 and p_2 of Example 9. All defenses are as good as possible given the set of arguments in every scenario. An upgrade of defense of any participant will weakens the defense of other members. This can be viewed as a rational position of a dialoguing agent which examines its own knowledge base and does not choose arguments with the strongest defense in order to maintain a good defense in other arguments.

Top admissibility and equilibrated admissibility are dealing with inner strength of admissible extensions. The first one requires a strongest defense in the framework, while the second one requires a strongest defense in a given scenario shared with other arguments.

Proposition 2 *Any top-admissible scenario p is equilibrated.*

Proof: If p is a top-admissible scenario, then every defense is as strong as it is possible in the whole framework. Then no defense upgrade can be applied and thus p is an equilibrated admissible scenario. □

An equilibrated scenario is not necessarily top-admissible, as shown in Example 9. This situation arises when two or more arguments are sharing defenders in the same scenario.

Proposition 3 *Let Φ be an AFV with $p = [S, P]$ an equilibrated admissible scenario which is not top-admissible. Then there are two arguments $\mathcal{A}, \mathcal{B} \in S$ such that $def_p(\mathcal{A}) \cap def_p(\mathcal{B}) \neq \emptyset$,*

Proof: If p is not top-admissible then at least one argument, say $\mathcal{A}$, is not benefited by its strongest defense. However, the available defense of $\mathcal{A}$ in p cannot be upgraded because p is equilibrated. This upgrade would affect other argument $\mathcal{B}$, either by weakening its defense or by removal of defenders. Both cases are caused by sharing at least a common defender in the scenario. Then $def_p(\mathcal{A}) \cap def_p(\mathcal{B}) \neq \emptyset$, □

In Example 9, arguments $\mathcal{A}$ and $\mathcal{F}$ are sharing arguments $\mathcal{C}$ and $\mathcal{D}$ as defenders, but these defenders are considered of different quality. Argument $\mathcal{C}$, for example, is a normal defender of $\mathcal{A}$ and a strong defender of $\mathcal{B}$.

As admissible sets are good candidates for argument acceptance, it is interesting to define a form of expansion of a given set. In the following section we introduce an expansion method which takes defense conditions into account.

Expanding admissible scenarios

Defense conditions are specially interesting in dialectical processes. Deciding on the acceptance of an argument requires the analysis of its direct and indirect attackers and defenders. The bigger the set of arguments, the harder the process of acceptance. Defense conditions are naturally bounding this analysis, as shown in the following proposition.

Proposition 4 *Let $\Phi = \langle Args, Atts, R \rangle$ be an AFV where $|Atts| = n$. Let $p = [S, \{\gg\}]$ be an admissible scenario. Then no argument in S has more than n defenders in a sequence of attacks*

Proof: In p every argument is a strong defender. A defender in S can eventually be attacked with the same strength than the attack it provides as defense, but it must be defended with a stronger attack. Thus, a sequence of attacks involving defenses of arguments in S has the form $[\mathcal{D}_1, \mathcal{A}_1, \mathcal{D}_2, \mathcal{A}_2, \mathcal{D}_3, \ldots, \mathcal{D}_k]$ where every $\mathcal{D}_i \in S$. Every defensive attack provided by $\mathcal{D}_i$ is necessarily stronger than the defensive attack provided by $\mathcal{D}_{i-1}$. Having n attacks, this sequence cannot include more than n defenders. □

An admissible set $p = [S, \{\gg\}]$ includes only strongly defended arguments. An admissible scenario with defense condition P can be safely expanded into another admissible scenario using a different defense condition Q. This is achieved by the identification of acceptable arguments according to Q

Definition 16 (Defense upgrade) *Let $p = [S, P]$ be an attack scenario, and let Q be a set of defense conditions. The expansion of p according to Q is defined as $p \oplus Q = [S' \cup S, P \cup Q]$, where*

$$S' = \{\mathcal{A} \in Args : \mathcal{A} \text{ is acceptable with respect to } [S, Q]\}$$

The admissible scenario $p \oplus Q$ is constructed over p by the inclusion of acceptable arguments under defense condition Q.

Example 10 *Consider the AFV of Figure 9 where $\rightarrow_2 \gg \rightarrow_1$, $\rightarrow_3 \gg \rightarrow_2$ and $\rightarrow_4 \approx \rightarrow_3$ The scenario $p_1 = [\{\mathcal{A}, \mathcal{E}, \mathcal{I}\}, \{\gg\}]$ is admissible. Then*

$p_2 = p_1 \oplus \{\approx\} = [\{\mathcal{A}, \mathcal{E}, \mathcal{I}, \mathcal{H}, \mathcal{C}, \mathcal{D}\}, \{\gg, \approx\}].$
$p_3 = p_1 \oplus \{\gg\} = [\{\mathcal{A}, \mathcal{E}, \mathcal{I}, \mathcal{C}, \mathcal{D}\}, \{\gg\}].$

Note that arguments $\mathcal{C}$ and $\mathcal{D}$ are included in any expansion, because these arguments are acceptable with respect to the empty set, and therefore no defense condition is needed.

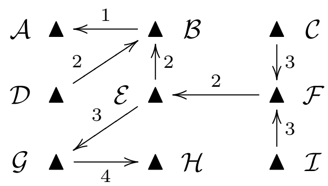

Figure 9: Example 10

As it is based on acceptability, the expansion operator preserves admissibility, as stated in the following proposition.

Proposition 5 *Let $p = [S, P]$ be an admissible scenario, Q a set of defense conditions. The set $p \oplus Q = [S' \cup S, P \cup Q]$ is also an admissible scenario.*

Proof: Suppose $[S' \cup S, P \cup Q]$ is not an admissible scenario. Then either exists an argument in $S' \cup S$ which is not acceptable with respect to $[S' \cup S, P \cup Q]$ or $S' \cup S$ is not conflict-free.

- *Suppose there exists $\mathcal{A} \in S' \cup S$ is not acceptable with respect to $[S' \cup S, P \cup Q]$. Clearly, $\mathcal{A} \notin S$ as p is admissible. Then $\mathcal{A} \in S'$ and it is acceptable with respect to $[S, Q]$. As new defense conditions cannot change this fact, $\mathcal{A}$ is acceptable with respect to $[S, P \cup Q]$. As a defense of $\mathcal{A}$ is included in S, then it is also included in $S' \cup S$. But then $\mathcal{A}$ is acceptable with respect to $[S' \cup S, P \cup Q]$ which is a contradiction.*
- *Suppose $S' \cup S$ is not conflict-free. Then there are two arguments $\mathcal{X}$ and $\mathcal{Y}$ in $S' \cup S$ such that $\mathcal{X}$ attacks $\mathcal{Y}$. As p is admissible, clearly both arguments cannot belong to S. (a) suppose $\mathcal{Y}$ belongs to S'. Then $\mathcal{X}$ must be attacked by an argument in S because $\mathcal{Y}$ is acceptable with respect to $[S, Q]$. Then $\mathcal{X} \notin S'$. However, also $\mathcal{X} \notin S$ as it is not defended by p, which is a contradiction. (b) Suppose $\mathcal{X} \in S'$. Then $\mathcal{Y} \notin S'$ because the attack from $\mathcal{X}$ cannot be defended by $[S, Q]$. Then $\mathcal{Y} \in S$ and being p an admissible scenario, S includes an attacker of $\mathcal{X}$. But then $\mathcal{X}$ is not acceptable with respect to p and therefore $\mathcal{X} \notin S$, which is a contradiction.* □

The expansion operator $\oplus$ adds acceptable arguments (satisfying certain conditions) to an already admissible set, leading to an admissible scenario. As this addition is based on acceptability on a given set, the order of defense conditions subsequently applied for expansions is determinant on the final scenario. In other words, for an attack scenario $p = [S, P]$ and two sets of defense conditions Q and T, probably $(p \oplus Q) \oplus T \neq (p \oplus T) \oplus Q$.

Example 11 *Consider the AFV of Figure 10 where $\rightarrow_2 \gg \rightarrow_1$ and $\rightarrow_2 ? \rightarrow_3$. The scenario $p_1 = [\{\mathcal{A}, \mathcal{C}\}, \{\gg\}]$ is admissible. Expanding by condition $\{\approx\}$ leads to*

$p_2 = p_1 \oplus \{\approx\} = [\{\mathcal{A}, \mathcal{C}, \mathcal{D}\}, \{\gg, \approx\}].$

Expanding p_2 by $\{?\}$ leads to

$p_3 = (p_1 \oplus \{\approx\}) \oplus \{?\} = [\{\mathcal{A}, \mathcal{C}, \mathcal{D}, \mathcal{F}\}, \{\gg, \approx, ?\}].$

Note that $(p_1 \oplus \{?\}) \oplus \{\approx\} = [\{\mathcal{A}, \mathcal{C}, \mathcal{D}\}, \{\gg, \approx, ?, \}]$

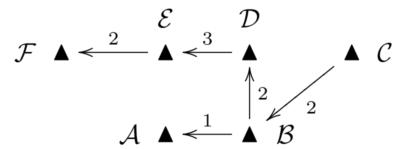

Figure 10: Example 11

Although simple, this form of expansion allows the construction of a bigger set of admissible scenarios. Classical notions as preferred and grounded extensions can be captured by the use of the $\oplus$ operator. Note that when no arguments are acceptable with respect to an admissible scenario $p = [S, P]$, then $p \oplus P = p$. In fact, when any additional defense condition does not expand an admissible scenario $[S, P]$, then the set S is maximal admissible.

Proposition 6 *Let $p = [S, P]$ be an admissible scenario. If $p \oplus Q = [S, P \cup Q]$ for any condition Q then S is a (classic) preferred extension.*

Proof: If $[S, P] \oplus Q = [S, P \cup Q]$ for any defense condition Q, then no arguments other than those in S are acceptable with respect to S, despite the condition of defense. Thus, S is maximal admissible. □

The characterization of skeptical acceptance under a defense condition P can be achieved by gradually expanding admissible scenarios. Naturally, an argument without attackers does not need defense as it is acceptable with respect to the empty set.

Proposition 7 *Let $\Phi = \langle Args, Atts, R \rangle$ be an AFV. For any defense condition P, the scenario $[\emptyset, P] \oplus P = [ND, P]$ where ND is the set of arguments free of attackers in $Args$.*

Proof: An argument free of attackers does not need defense, and thus it is acceptable with respect to any set. In particular, it is acceptable with respect to the empty set, and thus $[\emptyset, P] \oplus P$ includes only arguments without attackers. □

Thus, for any defense condition P, the set $[\emptyset, P] \oplus P$ is the set of all attackers-free arguments. The repeated application of operator $\oplus$ with defense condition P will subsequently add new arguments to an already admissible scenario, where every defense fulfills P.

Definition 17 (Grounded scenario) *Let Φ be an AFV and let $P \subseteq \{\gg, \ll, \approx, ?\}$. The scenario $\Phi \uparrow^+ P$, called the grounded scenario according to P, is defined as the least fixpoint of $\oplus$ using defense condition P.*

Example 12 *Consider the AFV Φ of Figure 11 where $\rightarrow_2 \gg \rightarrow_1$ and $\rightarrow_2 \approx \rightarrow_3$. The following are the grounded scenarios obtained by considering different defense conditions.*

Under defense condition $\{\gg\}$
$p_0 = [\emptyset, \{\gg\}] \oplus \{\gg\} = [\{\mathcal{A}, \mathcal{B}, \mathcal{D}, \mathcal{H}\}, \{\gg\}]$
$p_1 = p_0 \oplus \{\gg\} = [\{\mathcal{A}, \mathcal{B}, \mathcal{D}, \mathcal{H}, \mathcal{I}\}, \{\gg\}]$
$p_2 = p_1 \oplus \{\gg\} = [\{\mathcal{A}, \mathcal{B}, \mathcal{D}, \mathcal{H}, \mathcal{I}\}, \{\gg\}] = p_1$
Then $\Phi\uparrow^+\{\gg\} = [\{\mathcal{A}, \mathcal{B}, \mathcal{D}, \mathcal{H}, \mathcal{I}\}, \{\gg\}]$

Under defense condition $\{\approx\}$
$p_0 = [\emptyset, \{\approx\}] \oplus \{\approx\} = [\{\mathcal{A}, \mathcal{B}, \mathcal{D}, \mathcal{H}\}, \{\approx\}]$
$p_1 = p_0 \oplus \{\approx\} = [\{\mathcal{A}, \mathcal{B}, \mathcal{D}, \mathcal{H}\}, \{\gg\}] = p_0$
Then $\Phi\uparrow^+\{\approx\} = [\{\mathcal{A}, \mathcal{B}, \mathcal{D}, \mathcal{H}\}, \{\gg\}]$

Under defense condition $\{\gg, \approx\}$
$p_0 = [\emptyset, \{\gg, \approx\}] \oplus \{\gg, \approx\} = [\{\mathcal{A}, \mathcal{B}, \mathcal{D}, \mathcal{H}\}, \{\gg, \approx\}]$
$p_1 = p_0 \oplus \{\gg, \approx\} = [\{\mathcal{A}, \mathcal{B}, \mathcal{D}, \mathcal{H}, \mathcal{I}\}, \{\gg, \approx\}]$
$p_2 = p_1 \oplus \{\gg, \approx\} = [\{\mathcal{A}, \mathcal{B}, \mathcal{D}, \mathcal{H}, \mathcal{I}, \mathcal{K}\}, \{\gg, \approx\}]$
$p_3 = p_2 \oplus \{\gg, \approx\} = [\{\mathcal{A}, \mathcal{B}, \mathcal{D}, \mathcal{H}, \mathcal{I}, \mathcal{K}\}, \{\gg, \approx\}] = p_2$
Then $\Phi\uparrow^+\{\gg, \approx\} = [\{\mathcal{A}, \mathcal{B}, \mathcal{D}, \mathcal{H}, \mathcal{I}, \mathcal{K}\}, \{\gg, \approx\}]$

Note that $\mathcal{G}$ and $\mathcal{C}$ are not included in any set, as they are mutual attackers.

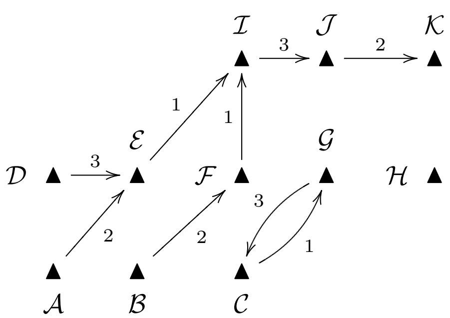

Figure 11: Example 12

The grounded scenario $\Phi \uparrow^+ P$ is admissible. The term *grounded* is used because it captures the skeptical acceptance modeled by the classical grounded extension (Dung 1995), but limited to certain defense conditions. In fact, if all defense conditions are taken into account, then the result is an attack scenario equivalent to the classical grounded extension.

Proposition 8 *Let $\Phi = \langle Args, Atts, R\rangle$ be an AFV, P_{all} be the set of all defense conditions. Then $\Phi\uparrow^+ P = [GE_\Phi, P]$, where GE_Φ is the (classical) grounded extension of Φ.*

Proof: Classical grounded extension is the least fixpoint of characteristic function

$$F_{AF}(S) = \{\mathcal{A} : \mathcal{A} \text{ is acceptable with respect to } S\}$$

under classical acceptability (Definition 5). If $P = \{\gg, \ll, \approx, ?\}$ (i.e., every defense condition is considered), then an argument $\mathcal{A}$ is acceptable with respect to an attack scenario $[S, P]$ if and only if $\mathcal{A}$ is (classical) acceptable with respect to S, as every defense satisfies P. Thus $\mathcal{A} \in [S, P] \oplus P$ if and only if $\mathcal{A} \in F_{AF}(S)$. Both formalisms capture the same notion, and then the least fixpoint captures the same final set of arguments. $\square$

An argumentation framework with varied-strength attacks allows several skeptical sets of acceptance, depending on the permitted defense conditions. In Example 12, the framework has three grounded scenarios. Note that $\Phi\uparrow^+\{\approx\}$ only includes arguments without attackers, as the defense condition is very restrictive for Φ. In that framework, argument $\mathcal{I}$ cannot be defended by normal defenders.

Conclusions and future work

In this work we defined an argumentation framework equipped with a set of abstract attack relations of varied strength. This improvement of Dung's classical framework is a pathway to new elaborations about arguments and preferences, specially the definition of new semantics extensions. In order to achieve this, the notion of defense condition was introduced as a set of requirements in the relative difference of strength between offensive and defensive attacks. Under this defense condition an admissible structure was defined. An *admissible scenario* is formed by a set of arguments S fulfilling a set of defense conditions P. These conditions are basic requirements on the relative difference of strength between defensive and offensive attacks. A *top-admissible scenario* is an admissible scenario which includes the strongest defense for every participant An *equilibrated scenario* is an admissible scenario in which no argument can strengthen its own defense without lowering the defense of other remaining arguments in the same scenario.

A simple form of expansion of admissible scenarios taking new defense conditions into account was finally introduced and some properties about these notions were discussed. This works is a generalization of (Martínez, García, & Simari 2008), where blocked and proper defeaters (in the sense of (García & Simari 2004)) are analyzed in the context of extended abstract frameworks This kind of framework can be formalized as a framework with varied-strength attacks

$$\langle Args, \{\rightarrow_{bl}, \rightarrow_{pr}\}, \mathbf{R}\rangle$$

where $\rightarrow_{bl}$ is the blocking defeat relation and $\rightarrow_{pr}$ is the proper defeat relation and $\rightarrow_{pr} \gg \rightarrow_{bl}$. This, in turn, is an abstract formalization of Defeasible Logic Programming (García & Simari 2004).

Future work has several directions. It is interesting to formalize new notions of defense upgrade. In this work only direct defenders are replaced by stronger ones. An upgrade to a stronger defense including indirect defenders is motivating by its complexity, as several levels of defense convenience must be considered. The framework of varied-strength attacks may be combined with other abstract proposals where the preference is applied to arguments as in (Amgoud & Cayrol 1998; Amgoud & Perrussel 2000; Bench-Capon 2002) to mention a few. This combination is an interesting contribution to the study of the use of preferences in argumentation as ordered attacks were not previously used in this abstract level. We are also interested in the formalization of equilibrated scenarios in dialogue contexts, where rational agent exchange arguments for and against some proposition. The search of well defended arguments is an essential heuristic in this context, and the modelization of varied-strength attacks is suitable to be applied to inter-agent dialogues.

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