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Formalising Temporal Constraints on Part-Whole Relations

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Abstract

Representing part-whole relations and effectively using them in domain ontologies and conceptual data models poses multiple challenges. In this paper we face the issue of imposing temporal constraints on part-whole relationships, introducing a way to account for *essential* and *immutable* parts (and wholes) in addition to the usual *mandatory* parts (and wholes). Our approach is based on i) an explicit temporalization of the part-whole relation, which allows us to introduce a novel notion of *status* for part-whole relationships; ii) an explicit account of the ontological nature of the classes involved in a part-whole relationships, which distinguishes between *rigid* and *anti-rigid* classes. The main novelty in this paper is to resort to a temporal logic approach to capture the above mentioned notions. The formalization proposed here is grounded on the temporal description logic $\mathcal{DLR}_{\mathcal{US}}$ and is based on previous successful efforts to formalize temporal conceptual models.

Introduction

The proper account of parthood relations in knowledge representation languages and formal conceptual data models has received much attention in the literature recently. For instance, issues such as the transitivity of part-whole relations and the specific behavior of functional parts can be considered as relatively well clarified nowadays (Varzi 2006; Vieu 2006; Keet & Artale 2008). However, some subtle issues concerning the various ways two classes may be related by a formula containing modal constraints on parthood relations are still a matter of discussion in the conceptual modelling community. Moreover, a proper formalization for such modal constraints in terms of representation languages with well-understood computational properties, such as description logics, is still lacking.

Consider, for instance, the UML class diagram shown in Fig. 1, which constrains the part-whole relations in the following way: every human has exactly one brain, one heart and at most two hands, while every boxer (a subclass of human) has exactly two hands. As discussed by Guizzardi (Guizzardi 2005; 2007), the intended interpretation of these part-whole relationships is typically different, since we normally assume that each human has *necessarily* a specific brain, while *not necessarily* a specific heart (thanks to heart transplantation). In addition, if a human is also a boxer, he must *necessarily* have exactly his own two hands. Standard discussions on parthood relationships distinguish between *essential* (the brain) and *mandatory* (the heart) parts of a whole (the human). In UML modeling, a further case of parthood relationship is introduced: that of so-called *immutable* parts (Barbier *et al.* 2003; Guizzardi 2005; 2007). This is indeed the case of the boxer's hands, which must exist as long as a human is a boxer (assuming hand transplantation is forbidden for boxers), but are neither essential nor mandatory for the human's life.

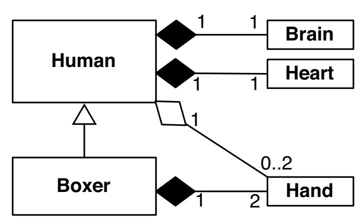

Figure 1: UML diagram with part-whole relations.

Clearly, cardinality constraints alone are not enough to distinguish between these interpretations: some kinds of *modal constraints* have to be taken into account. A representation language with "full" modal operators would be the obvious choice, but it would certainly present computational and semantic problems. For this reason, we adopt in this paper a temporal logic approach, assuming that a temporal modality is enough to capture most practical cases, and reducing therefore the constraints concerning the so-called "alethic modality"—pure possibility/necessity, independently of time—to those expressible in terms of temporal modality only. In particular, we shall concentrate in this paper on what we may call the *life cycle semantics* of part-whole relationships, focusing on the constraints concerning the temporal existence of *both* their participants—the part and the whole—*and* the part-whole relationship itself. The explicit recognition given to such distinct temporal behaviors is one of the main contributions of this paper.

Currently, no commonly used conceptual data modelling language, such as UML and EER (Extended Entity-Relationship), is explicitly tailored to account for these distinctions, although efforts in this direction pointed out useful modeling patterns and clarified some expressivity requirements (Artale *et al.* 1996; Barbier *et al.* 2003; Guizzardi 2005; 2007; Keet 2006; Motschnig-Pitrik & Kaasboll 1999). In this paper we focus in particular on the formal clarification of the notions of mandatory, essential and immutable parts when used in conceptual models. The formalization proposed here is grounded on the temporal description logic $\mathcal{DLR}_{\mathcal{US}}$ and is based on previous efforts to formalize temporal conceptual models. Namely, we rely on a previous work to define the temporal EER model $\mathcal{ER}_{VT}$ that can be fully captured with $\mathcal{DLR}_{\mathcal{US}}$ (Artale *et al.* 2002; Artale, Franconi, & Mandreoli 2003; Artale, Parent, & Spaccapietra 2007). The choice of using $\mathcal{DLR}_{\mathcal{US}}$ is motivated by its ability to logically reconstruct and extend representational tools such as object-oriented and conceptual data models, frame-based and web ontology languages (Berardi, Calvanese, & De Giacomo 2005; Calvanese, Lenzerini, & Nardi 1999; Horrocks, Patel-Schneider, & van Harmelen 2003). In particular, we will show that $\mathcal{DLR}_{\mathcal{US}}$ has the expressivity required to formalize the above mentioned distinctions, while being able, at the same time, to capture two relevant notions like the rigidity properties of a class (Guarino & Welty 2000; Welty & Andersen 2005) and the (newly introduced) notion of status for relations. Concerning the use of automated reasoning services to check quality properties of a conceptual model we have to remember here that $\mathcal{DLR}_{\mathcal{US}}$ is an undecidable language (Artale *et al.* 2002). A promising direction is to resort to the a weaker but decidable temporal DL, *TDL-Lite* (first results appeared in (Artale *et al.* 2007)), that temporally extends the computationally simpler *DL-Lite* with both temporal roles and concepts. It will be part of our further work to investigate the expressive power of *TDL-Lite* to capture temporal conceptual models.

In the remainder of this paper, we first discuss the notions of essential, immutable and mandatory parts exemplified by means of the running example. We then analyse related works on temporal part-whole relations and provide an overview of the description logic $\mathcal{DLR}_{\mathcal{US}}$. We proceed by introducing and formalising the notion of status for relations, and then we show how axioms in $\mathcal{DLR}_{\mathcal{US}}$ capture the distinction between rigid and anti-rigid classes. The main section of the paper deals with the constraints on parts, wholes, and part-whole relations, proving that our formalism easily captures mandatory, essential, and immutable parts (and wholes). We close with conclusions and future work.

Preliminaries on essential, immutable and mandatory parts

Considering the conceptual model of Fig. 1, we would like to distinguish between the brain, which we assume to be an essential part of a human, the heart, which we assume to be a mandatory part for humans, and the two hands, which we assume to be immutable parts of every boxer.

The distinction between essential and mandatory parts can be explained in terms of a *specific* vs. *generic* dependence relationship between the class that describes the whole and the one that describes the part—denoted in the following as `Whole` and `Part`, respectively. Mandatory parts express a *generic* dependence relationship in the sense that, although a part of a certain kind must be always present when the whole exists, the particular part can be different at different moments of time (namely the part can be replaced, as in the human's heart example). On the other hand, essential parts express a *specific* dependence relationship, that is, the whole must be always associated with the very same part—i.e., the part must be the same along the entire lifetime of the whole.

If we assume that `Whole` is *rigid* (Guarino & Welty 2000; Welty & Andersen 2005) (that is, if something is a `Whole`, then it is a `Whole` forever), then the distinction between essential and mandatory parts—corresponding to the distinction between specific and generic dependence—is enough to capture all the useful parthood behaviors.

If `Whole` is *not rigid*, however, it is useful to introduce a further notion, a kind of weaker version of 'essential part': the notion of *immutable part*. Immutable parts, like essential parts, are bound to the instances of `Whole` by a *specific* dependence relationship. However, this relationship does not hold necessarily through the whole life of such individuals, but only as long as they are instances of the class `Whole`. Immutable parts are therefore only *conditionally* essential *for being a Whole*, but not essential *tout court*. For this reason, immutable parts might also be called *conditionally essential parts*. Consider again the class `Boxer` in Fig. 1, which we assume to be not rigid (indeed, anti-rigid). Every boxer must not only have exactly two hands, but he must have *his own* hands: if he does not, then he ceases to be a boxer, while still being a human. In this case, having two (specific) hands as parts is conditionally essential for being a boxer. If, on the other hand, boxing regulations allow for substitution of hands (be it by hand transplantation or protheses), then having *some* hands is just mandatory for the boxer. In sum, what we need to account for is another kind of constraint for the parthood relationship that only holds for the time the `Whole` property holds. As we shall see, this will be formalized by introducing a "guard" condition in the definition for specific dependence which will lead to the notion of *conditionally essential* parts (and, in analogy, conditionally essential wholes).

Temporal part-whole relations: Related works

Given that our focus is on temporal modality, let us briefly discuss related works on temporal part-whole relations. The most straightforward, yet also limited, way to temporalize part-whole relations is to turn a part-of predicate from a binary into a ternary relation, such that we have p part of w at time t: $part\_of(p, w, t)$. To the best of our knowledge, almost all current temporalizations of parthood take this approach (Bittner & Donnelly 2007; Masolo *et al.* 2003; Smith *et al.* 2005)[1], but do not go further to take advantage of a temporal knowledge representation language. An ex-

[1]We focus here on temporal parts of objects (i.e., *endurants* in DOLCE). Neither parts of events nor parts of 4-dimensional entities (Hawley 2004) are considered here.

ception is (Barbier *et al.* 2003), where life-time dependencies are represented in UML class diagrams by introducing an oclUndefined observer function to "assert that all parts of result do not exist before (@pre)" the creation of the whole instance w. Barbier and colleagues also tried a representation of immutability, which, however, remained an open problem due to the lack of a full implementation of temporal UML. In addition, (Barbier *et al.* 2003) discussed nine basic life-cycle cases, obtained by fixing the lifespan of the `Whole` and varying the lifespan of the `Part`. Clearly, one can consider further cases corresponding to the opposite perspective, obtained by fixing the part's lifespan, and assessing its temporal relationships with the whole's lifespan (see also Fig. 5). This may sound as a simple inverse, but we shall see in the following sections that these two views require distinct constrains.

Regarding the ternary temporal part-whole relation, we have, for instance, Bittner and Donnelly's "temporal mereology" (Bittner & Donnelly 2007), which was developed to deal with "portions of stuff", i.e., to deal with amounts of matter, such as gold, and mixtures, such as lemonade, varying in time.

Formalizing temporal aspects by limiting oneself to ad hoc ternary part-whole relations runs into rather complicated formalizations. On the contrary, well-defined temporal logics—and those applied to temporal conceptual data modeling in particular—can hide at least some of the details, therefore enhancing understandability and (re)usability of conceptual models both for the modeler and the domain expert. This is the reason why we adopt in this paper a well-studied temporal description logic, $\mathcal{DLR}_{\mathcal{US}}$, which is already used to model temporal conceptual models (see the $\mathcal{ER}_{VT}$ language in (Artale, Franconi, & Mandreoli 2003; Artale, Parent, & Spaccapietra 2007)), and we will show how it enables us to model essential, immutable, and mandatory parts and wholes in a precise and clear way.

The temporal Description Logic $\mathcal{DLR}_{\mathcal{US}}$

The temporal description logic $\mathcal{DLR}_{\mathcal{US}}$ (Artale *et al.* 2002) combines the propositional temporal logic with *Since* and *Until* operators with the (non-temporal) description logic $\mathcal{DLR}$ (Calvanese & De Giacomo 2003), which serves as common foundational language for various conceptual data modeling languages (Calvanese, Lenzerini, & Nardi 1999). $\mathcal{DLR}_{\mathcal{US}}$ can be regarded as an expressive fragment of the first-order temporal logic $L^{\{\textbf{since, until}\}}$ (Chomicki & Toman 1998; Hodgkinson, Wolter, & Zakharyaschev 1999; Gabbay *et al.* 2003).

The basic syntactical types of $\mathcal{DLR}_{\mathcal{US}}$ are *classes* and n-ary *relations* ($n \geq 2$). Starting from a set of *atomic classes* (denoted by CN), a set of *atomic relations* (denoted by RN), and a set of *role symbols* (denoted by U), we can define inductively (complex) class and relation expressions (see upper part of Fig. 2), where the binary constructors ($\sqcap, \sqcup, \mathcal{U}, \mathcal{S}$) are applied to relations of the same arity, i, j, k, n are natural numbers, $i \leq n$, j does not exceed the arity of R, and all the Boolean constructors are available for both class and relation expressions. The selection expression $U_i/n : C$ denotes an n-ary relation whose i-th argument ($i \leq n$), named U_i, is of type C. If it is clear from the context, we omit n and write $(U_i : C)$. The projection expression $\exists^{\lessgtr k}[U_j]R$ is a generalisation with cardinalities of the projection operator over argument U_j of relation R; the classical projection is $\exists^{\geq 1}[U_j]R$.

The model-theoretic semantics of $\mathcal{DLR}_{\mathcal{US}}$ assumes a flow of time $\mathcal{T} = \langle \mathcal{T}_p, < \rangle$, where $\mathcal{T}_p$ is a set of time points and $<$ a binary precedence relation on $\mathcal{T}_p$, assumed to be isomorphic to $\langle \mathbb{Z}, < \rangle$. The language of $\mathcal{DLR}_{\mathcal{US}}$ is interpreted in *temporal models* over $\mathcal{T}$, which are triples of the form $\mathcal{I} \doteq \langle \mathcal{T}, \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}(t)} \rangle$, where $\Delta^{\mathcal{I}}$ is non-empty set of objects (the *domain* of $\mathcal{I}$) and $\cdot^{\mathcal{I}(t)}$ an *interpretation function*. Since the domain, $\Delta^{\mathcal{I}}$, is time independent, we assume here the so called *constant domain assumption* with *rigid designator*—i.e., an instance is *always* present in the interpretation domain and it identifies the same instance at different points in time. The interpretation function is such that, for every $t \in \mathcal{T}$ (a shortcut for $t \in \mathcal{T}_p$), every class C, and every n-ary relation R, we have $C^{\mathcal{I}(t)} \subseteq \Delta^{\mathcal{I}}$ and $R^{\mathcal{I}(t)} \subseteq (\Delta^{\mathcal{I}})^n$. The semantics of class and relation expressions is defined in the lower part of Fig. 2, where $(u, v) = \{w \in \mathcal{T} \mid u < w < v\}$. For classes, the temporal operators $\Diamond^+$ (some time in the future), $\oplus$ (at the next moment), and their past counterparts can be defined via $\mathcal{U}$ and $\mathcal{S}$: $\Diamond^+ C \equiv \top \mathcal{U} C$, $\oplus C \equiv \bot \mathcal{U} C$, etc. The operators $\Box^+$ (always in the future) and $\Box^-$ (always in the past) are the duals of $\Diamond^+$ (some time in the future) and $\Diamond^-$ (some time in the past), respectively, i.e., $\Box^+ C \equiv \neg \Diamond^+ \neg C$ and $\Box^- C \equiv \neg \Diamond^- \neg C$, for both classes and relations. The operators $\Diamond^*$ (at some moment) and its dual $\Box^*$ (at all moments) can be defined for both classes and relations as $\Diamond^* C \equiv C \sqcup \Diamond^+ C \sqcup \Diamond^- C$ and $\Box^* C \equiv C \sqcap \Box^+ C \sqcap \Box^- C$, respectively.

A *knowledge base* is a finite set Σ of $\mathcal{DLR}_{\mathcal{US}}$ axioms of the form $C_1 \sqsubseteq C_2$ and $R_1 \sqsubseteq R_2$, with R_1 and R_2 being relations of the same arity. An interpretation $\mathcal{I}$ satisfies $C_1 \sqsubseteq C_2$ ($R_1 \sqsubseteq R_2$) if and only if the interpretation of C_1 (R_1) is included in the interpretation of C_2 (R_2) at *all time*, i.e., $C_1^{\mathcal{I}(t)} \subseteq C_2^{\mathcal{I}(t)}$ ($R_1^{\mathcal{I}(t)} \subseteq R_2^{\mathcal{I}(t)}$), for all $t \in \mathcal{T}$. Thus, $\mathcal{DLR}_{\mathcal{US}}$ axioms have a global reading. To see examples on how a $\mathcal{DLR}_{\mathcal{US}}$ knowledge base looks like we refer to the following sections where examples are provided.

Status Relations and Rigid Classes

The formalization of essential and immutable parts is based on two preliminary notions that will be introduced in this section: the original contribution of *status relations* and the distinction between *rigid* and *anti-rigid* classes.

Status relations

Status relations extend the notion of *status classes* (Spaccapietra, Parent, & Zimanyi 1998; Etzion, Gal, & Segev 1998) to statuses for relations. Status classes—already formalized in $\mathcal{DLR}_{\mathcal{US}}$ in (Artale, Parent, & Spaccapietra 2007)—constrain the evolution of an instance's membership in a class along its lifespan. According to (Spaccapietra, Parent, & Zimanyi 1998; Artale, Parent, & Spaccapietra 2007),

$$
\begin{array}{rcl}
C & \rightarrow & \top \mid \bot \mid CN \mid \neg C \mid C_1 \sqcap C_2 \mid C_1 \sqcup C_2 \mid \exists^{\leqslant k}[U_j]R \mid \\
 & & \Diamond^+ C \mid \Diamond^- C \mid \Box^+ C \mid \Box^- C \mid \oplus C \mid \ominus C \mid C_1 \,\mathcal{U}\, C_2 \mid C_1 \,\mathcal{S}\, C_2 \\
R & \rightarrow & \top_n \mid RN \mid \neg R \mid R_1 \sqcap R_2 \mid R_1 \sqcup R_2 \mid U_i/n : C \mid \\
 & & \Diamond^+ R \mid \Diamond^- R \mid \Box^+ R \mid \Box^- R \mid \oplus R \mid \ominus R \mid R_1 \,\mathcal{U}\, R_2 \mid R_1 \,\mathcal{S}\, R_2
\end{array}
$$

$$
\begin{array}{rcl}
\top^{\mathcal{I}(t)} & = & \Delta^{\mathcal{I}}; \\
\bot^{\mathcal{I}(t)} & = & \emptyset; \\
CN^{\mathcal{I}(t)} & \subseteq & \top^{\mathcal{I}(t)}; \\
(\neg C)^{\mathcal{I}(t)} & = & \top^{\mathcal{I}(t)} \setminus C^{\mathcal{I}(t)}; \\
(C_1 \sqcap C_2)^{\mathcal{I}(t)} & = & C_1^{\mathcal{I}(t)} \cap C_2^{\mathcal{I}(t)}; \\
(C_1 \sqcup C_2)^{\mathcal{I}(t)} & = & C_1^{\mathcal{I}(t)} \cup C_2^{\mathcal{I}(t)}; \\
(\exists^{\leqslant k}[U_j]R)^{\mathcal{I}(t)} & = & \{\, d \in \top^{\mathcal{I}(t)} \mid \sharp\{\langle d_1, \ldots, d_n\rangle \in R^{\mathcal{I}(t)} \mid d_j = d\} \leqslant k\}; \\
(C_1 \,\mathcal{U}\, C_2)^{\mathcal{I}(t)} & = & \{\, d \in \top^{\mathcal{I}(t)} \mid \exists v > t. (d \in C_2^{\mathcal{I}(v)} \wedge \forall w \in (t, v). d \in C_1^{\mathcal{I}(w)})\}; \\
(C_1 \,\mathcal{S}\, C_2)^{\mathcal{I}(t)} & = & \{\, d \in \top^{\mathcal{I}(t)} \mid \exists v < t. (d \in C_2^{\mathcal{I}(v)} \wedge \forall w \in (v, t). d \in C_1^{\mathcal{I}(w)})\}; \\
 & & \\
(\top_n)^{\mathcal{I}(t)} & \subseteq & (\Delta^{\mathcal{I}})^n; \\
RN^{\mathcal{I}(t)} & \subseteq & (\top_n)^{\mathcal{I}(t)}; \\
(\neg R)^{\mathcal{I}(t)} & = & (\top_n)^{\mathcal{I}(t)} \setminus R^{\mathcal{I}(t)}; \\
(R_1 \sqcap R_2)^{\mathcal{I}(t)} & = & R_1^{\mathcal{I}(t)} \cap R_2^{\mathcal{I}(t)}; \\
(R_1 \sqcup R_2)^{\mathcal{I}(t)} & = & R_1^{\mathcal{I}(t)} \cup R_2^{\mathcal{I}(t)}; \\
(U_i/n : C)^{\mathcal{I}(t)} & = & \{\langle d_1, \ldots, d_n\rangle \in (\top_n)^{\mathcal{I}(t)} \mid d_i \in C^{\mathcal{I}(t)}\}; \\
(R_1 \,\mathcal{U}\, R_2)^{\mathcal{I}(t)} & = & \{\langle d_1, \ldots, d_n\rangle \in (\top_n)^{\mathcal{I}(t)} \mid \exists v > t. (\langle d_1, \ldots, d_n\rangle \in R_2^{\mathcal{I}(v)} \wedge \forall w \in (t, v). \langle d_1, \ldots, d_n\rangle \in R_1^{\mathcal{I}(w)})\}; \\
(R_1 \,\mathcal{S}\, R_2)^{\mathcal{I}(t)} & = & \{\, \langle d_1, \ldots, d_n\rangle \in (\top_n)^{\mathcal{I}(t)} \mid \exists v < t. (\langle d_1, \ldots, d_n\rangle \in R_2^{\mathcal{I}(v)} \wedge \forall w \in (v, t). \langle d_1, \ldots, d_n\rangle \in R_1^{\mathcal{I}(w)})\}; \\
(\Diamond^+ R)^{\mathcal{I}(t)} & = & \{\langle d_1, \ldots, d_n\rangle \in (\top_n)^{\mathcal{I}(t)} \mid \exists v > t. \langle d_1, \ldots, d_n\rangle \in R^{\mathcal{I}(v)}\}; \\
(\oplus R)^{\mathcal{I}(t)} & = & \{\langle d_1, \ldots, d_n\rangle \in (\top_n)^{\mathcal{I}(t)} \mid \langle d_1, \ldots, d_n\rangle \in R^{\mathcal{I}(t+1)}\}; \\
(\Diamond^- R)^{\mathcal{I}(t)} & = & \{\langle d_1, \ldots, d_n\rangle \in (\top_n)^{\mathcal{I}(t)} \mid \exists v < t. \langle d_1, \ldots, d_n\rangle \in R^{\mathcal{I}(v)}\}; \\
(\ominus R)^{\mathcal{I}(t)} & = & \{\langle d_1, \ldots, d_n\rangle \in (\top_n)^{\mathcal{I}(t)} \mid \langle d_1, \ldots, d_n\rangle \in R^{\mathcal{I}(t-1)}\}.
\end{array}
$$

Figure 2: Syntax and semantics of $\mathcal{DLR}_{\mathcal{US}}$.

status modeling includes up to four different statuses: *scheduled, active, suspended, disabled*, where each one entails different constraints.

Concerning status relations there are two options: (1) to derive a relation's status from the status of the classes participating in the relation, or (2) to explicitly define it on the relation itself, where the latter, in turn, puts constraints on the statuses of the classes. Since we are interested in modeling relations as first-class citizens, we choose to have a means to explicitly model the status of a relation. Therefore, as for classes, we have four different statuses for relations, too—*scheduled, active, suspended, disabled*—each illustrated with an example before we proceed to the formal characterization.

- Scheduled: a relation is scheduled if its instantiation is known but its membership will only become effective some time later. Objects in its participating classes must be either scheduled, too, be active, or suspended. For instance, a pillar for finishing the interior of the Sagrada Familia in Barcelona is scheduled to become part of that church, i.e., this $part\_of$ relation between the pillar and the church is scheduled.
- Active: the status of a relation is active if the particular relation fully instantiates the type-level relation: the part is currently part of the whole. For instance, the Mont Blanc mountain is part of the Alps mountain range, and the country Republic of Ireland is part of the European Union. Only active classes can participate into an active relation.
- Suspended: to capture a temporarily inactive relation. For example, an instance of a CarEngine is removed from the instance of a Car it is part of, for purpose of maintenance at the car mechanic. Note that at the moment of suspension, both the part and the whole must be active, but can upon suspension of the relation be either active or become suspended, too, but neither scheduled (see below constraints on scheduled) nor disabled.
- Disabled: to model expired relations that never again can be used. For instance, to represent the donor of an organ who has donated that organ and one wants to keep track of who donated what to whom: say, the heart p_1 of donor w_1 used to be a structural part of w_1 but it will never be again a part of it. The heart, p_1, then may have become participant in a new part-of relation with a new whole, w_2 where $w_1 \neq w_2$, but the original part-of between p_1 and w_1 remains disabled. Observe that participating objects can be member of the active, suspended or disabled class.

We assume that active relations involve only active classes and, by default, the name of a relation denotes already its ac-

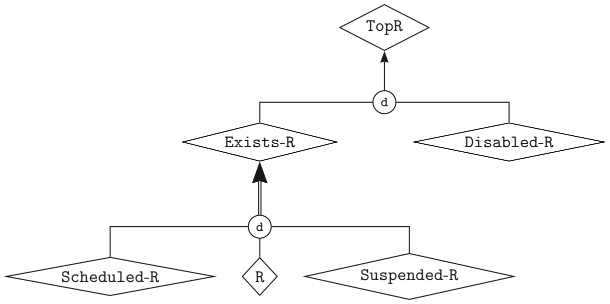

Figure 3: Status relations.

tive status—i.e., $\mathtt{Active\text{-}R} \equiv \mathtt{R}$. Disjointness and ISA constraints among the four status relations are analogous to the one for status classes and can be represented with an EER diagram as illustrated in Fig. 3 (double arrows indicate covering and an encircled d means disjointness). In addition to hierarchical constraints, the following constraints hold (we present both the model-theoretic semantics and the correspondent $\mathcal{DLR}_{\mathcal{US}}$ axioms considering, without loss of generality, binary relations only):

(ACT) *Active relations involve only active classes.*
$\langle o_1, o_2\rangle \in \mathtt{R}^{\mathcal{I}(t)} \rightarrow o_1 \in C_1^{\mathcal{I}(t)} \wedge o_2 \in C_2^{\mathcal{I}(t)}$
$\mathtt{R} \sqsubseteq \mathtt{U_1}:\mathtt{C_1} \sqcap \mathtt{U_2}:\mathtt{C_2}$

(REXISTS) *Existence persists until Disabled.*
$\langle o_1, o_2\rangle \in \mathtt{Exists\text{-}R}^{\mathcal{I}(t)} \rightarrow \forall t' > t.(\langle o_1, o_2\rangle \in \mathtt{Exists\text{-}R}^{\mathcal{I}(t')} \vee \langle o_1, o_2\rangle \in \mathtt{Disabled\text{-}R}^{\mathcal{I}(t')})$
$\mathtt{Exists\text{-}R} \sqsubseteq \Box^+(\mathtt{Exists\text{-}R} \sqcup \mathtt{Disabled\text{-}R})$

(RDISAB1) *Disabled persists.*
$\langle o_1, o_2\rangle \in \mathtt{Disabled\text{-}R}^{\mathcal{I}(t)} \rightarrow \forall t' > t.\langle o_1, o_2\rangle \in \mathtt{Disabled\text{-}R}^{\mathcal{I}(t')}$
$\mathtt{Disabled\text{-}R} \sqsubseteq \Box^+\mathtt{Disabled\text{-}R}$

(RDISAB2) *Disabled was Active in the past.*
$\langle o_1, o_2\rangle \in \mathtt{Disabled\text{-}R}^{\mathcal{I}(t)} \rightarrow \exists t' < t.\langle o_1, o_2\rangle \in \mathtt{R}^{\mathcal{I}(t')}$
$\mathtt{Disabled\text{-}R} \sqsubseteq \Diamond^-\mathtt{R}$

(RSUSP1) *Suspended was Active in the past.*
$\langle o_1, o_2\rangle \in \mathtt{Suspended\text{-}R}^{\mathcal{I}(t)} \rightarrow \exists t' < t.\langle o_1, o_2\rangle \in \mathtt{R}^{\mathcal{I}(t')}$
$\mathtt{Suspended\text{-}R} \sqsubseteq \Diamond^-\mathtt{R}$

(RSUSP2) *Suspended involve Active or Suspended Classes.*
$\langle o_1, o_2\rangle \in \mathtt{Suspended\text{-}R}^{\mathcal{I}(t)} \rightarrow o_i \in \mathtt{C_i}^{\mathcal{I}(t)} \vee o_i \in \mathtt{Suspended\text{-}C_i}^{\mathcal{I}(t)},\ i = 1, 2$
$\mathtt{Suspended\text{-}R} \sqsubseteq \mathtt{U_i}:(\mathtt{C_i} \sqcup \mathtt{Suspended\text{-}C_i}),\ i = 1, 2$

(RSCH1) *Scheduled will eventually become Active.*
$\langle o_1, o_2\rangle \in \mathtt{Scheduled\text{-}R}^{\mathcal{I}(t)} \rightarrow \exists t' > t.\langle o_1, o_2\rangle \in \mathtt{R}^{\mathcal{I}(t')}$
$\mathtt{Scheduled\text{-}R} \sqsubseteq \Diamond^+\mathtt{R}$

(RSCH2) *Scheduled can never follow Active.*
$\langle o_1, o_2\rangle \in \mathtt{R}^{\mathcal{I}(t)} \rightarrow \forall t' > t.\langle o_1, o_2\rangle \notin \mathtt{Scheduled\text{-}R}^{\mathcal{I}(t')}$
$\mathtt{R} \sqsubseteq \Box^+\neg\mathtt{Scheduled\text{-}R}$

In the following we denote with Σ_{st} the above set of $\mathcal{DLR}_{\mathcal{US}}$ axioms that formalize status relations. Given Σ_{st}, the following logical implications are relevant to model immutable parts.

PROPOSITION 1 (Status Relations: Logical Implications). *Given the set of axioms Σ_{st} and an n-ary relation, $R \sqsubseteq U_1 : C_1 \sqcap \ldots \sqcap U_n : C_n$, the following logical implications hold:*

(RACT) *Active will possibly evolve into Suspended or Disabled.*
$\Sigma_{st} \models \mathtt{R} \sqsubseteq \Diamond^+(\mathtt{R} \sqcup \mathtt{Suspended\text{-}R} \sqcup \mathtt{Disabled\text{-}R})$

(RDISAB3) *Disabled will never become active anymore.*
$\Sigma_{st} \models \mathtt{Disabled\text{-}R} \sqsubseteq \Box^+\neg\mathtt{R}$

(RDISAB4) *Disabled classes can participate only in disabled relations.*
$\Sigma_{st} \models \mathtt{Disabled\text{-}C_i} \sqcap \Diamond^-\exists[\mathtt{U_i}]\mathtt{R} \sqsubseteq \exists[\mathtt{U_i}]\mathtt{Disabled\text{-}R}$

(RDISAB5) *Disabled relations involve active, suspended, or disabled classes.*
$\mathtt{Disabled\text{-}R} \sqsubseteq \mathtt{U_i} : (\mathtt{C_i} \sqcup \mathtt{Suspended\text{-}C_i} \sqcup \mathtt{Disabled\text{-}C_i})$, *for all* $i = 1, \ldots, n$.

(RSCH3) *Scheduled persists until active.*
$\Sigma_{st} \models \mathtt{Scheduled\text{-}R} \sqsubseteq \mathtt{Scheduled\text{-}R}\,\mathcal{U}\,\mathtt{R}$

(RSCH4) *Scheduled cannot evolve directly to Disabled.*
$\Sigma_{st} \models \mathtt{Scheduled\text{-}R} \sqsubseteq \oplus\neg\mathtt{Disabled\text{-}R}$

(RSCH5) *Scheduled relations do not involve disabled classes.*
$\mathtt{Scheduled\text{-}R} \sqsubseteq \mathtt{U_i}:\neg\mathtt{Dibabled\text{-}C_i}$, *for all* $i = 1, \ldots, n$.

Proof. See Appendix. □

Lifespan and related notions. The lifespan of an object with respect to a class describes the set of temporal instants where the object can be considered a member of that class. We can distinguish between the following notions: EXISTENCESPAN$_C$, LIFESPAN$_C$, ACTIVESPAN$_C$, BEGIN$_C$, BIRTH$_C$, and DEATH$_C$ depending on the status of the class, C, the object is member of. We briefly report here their definition as presented in (Artale, Parent, & Spaccapietra 2007).

$\textsc{ExistenceSpan}_C(o) = \{t \in \mathcal{T} \mid o \in \mathtt{Exists\text{-}C}^{\mathcal{I}(t)}\}$
$\textsc{LifeSpan}_C(o) = \{t \in \mathcal{T} \mid o \in \mathtt{C}^{\mathcal{I}(t)} \cup \mathtt{Suspended\text{-}C}^{\mathcal{I}(t)}\}$
$\textsc{ActiveSpan}_C(o) = \{t \in \mathcal{T} \mid o \in \mathtt{C}^{\mathcal{I}(t)}\}$
$\textsc{Begin}_C(o) = \mathtt{min}(\textsc{ExistenceSpan}_C(o))$
$\textsc{Birth}_C(o) = \mathtt{min}(\textsc{ActiveSpan}_C(o)) \equiv \mathtt{min}(\textsc{LifeSpan}_C(o))$
$\textsc{Death}_C(o) = \mathtt{max}(\textsc{LifeSpan}_C(o))$

Representing Rigidity

As the running example in Fig. 1 shows, there is a peculiar difference between human and boxer, the latter being a "role" that a human can play, while a human is not necessarily a boxer and if a human ceases to be a boxer, then he is still a human. Such differences have been investigated in detail in (Guarino & Welty 2000), which forms the basis of the OntoClean methodology (Guarino & Welty 2004). For the purpose of this paper, we only require the distinctions between *rigid* properties vs. *non-rigid* and *anti-rigid* properties. We repeat these three definitions here, comment on them and provide examples in Fig. 4.

Definition 1 (+R). A *rigid* property ϕ is a property that is essential to *all* its instances, i.e., $\forall x \phi(x) \rightarrow \Box\phi(x)$.

Definition 2 (-R). A *non-rigid* property ϕ is a property that is not essential to *some* of its instances, i.e., $\exists x\phi(x) \wedge \neg\Box\phi(x)$.

Definition 3 (∼R). An *anti-rigid* property ϕ is a property that is not essential to *all* its instances, i.e., $\forall x\phi(x) \rightarrow \neg\Box\phi(x)$.

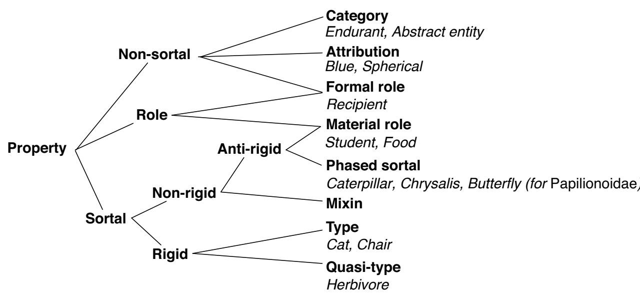

Figure 4: Taxonomy of properties with several examples (Source: based on (Guarino & Welty 2000)).

Practically, for conceptual data modeling and ontology development the anti-rigid properties are highly useful to represent entity types such as boxer and, more generally, those object types for which object migration may apply. For example, a human, h_1, may be recorded in a university database as an instance of PostDoc during some time and be promoted to Professor at a later time. A common example for bio-ontologies is the transformation from caterpillar to butterfly where the same entity changes membership from one class to another.

Furthermore, as pointed out in (Welty & Andersen 2005), in the literature different kinds of rigidity have been introduced. Here we also consider the so called *existential rigidity* that limits rigidity to the actual existence of the instance (captured by the *exist predicate* "$E(x)$").

Definition 4 (Existential Rigidity). An *existentially rigid* property ϕ is a property that is essential to *all* its instances as long as they exist, i.e., $\forall x\phi(x) \rightarrow \Box(E(x) \rightarrow \phi(x))$.

To capture the distinction between *immutable* and *essential* parts and wholes, we need to capture the distinction between *rigid* and *anti-rigid* classes. Indeed, while essential parts are properties of a whole that cannot change without destroying the whole, the fact that a part-whole relation is immutable is *conditional* to the whole belonging contingently to a particular class in a given interval of time. Thus, while essential parts are properties of a whole as being member of a rigid class, immutable parts describe properties of a whole as being member of an anti-rigid class. Furthermore, since for those instances that belong to the same non-rigid class at all points in time it is more appropriate to speak of essential parts, we will formally speak of immutable parts only in the stricter case of an anti-rigid class.

Starting from Definition 1 and Definition 3, we can enforce a class C to be either rigid or anti-rigid with the following $\mathcal{DLR_{US}}$ axioms, respectively (remember that: $\mathcal{DLR_{US}}$ axioms hold globally, $\Box^*$ means "*at all moments*", and $\Diamond^*$ means "*at some moment*").

(RIGID) *Rigid Class*
$\mathtt{C} \sqsubseteq \Box^*\mathtt{C}$

(A-RIGID) *Anti-Rigid Class*
$\mathtt{C} \sqsubseteq \Diamond^*\neg\mathtt{C}$

In addition, we have the constraint that for each anti-rigid class $\mathtt{C_A}$ we must have a rigid class $\mathtt{C_R}$ that subsumes $\mathtt{C_A}$ (Guarino & Welty 2000; Guizzardi 2005):

(A-SUB-R) *Each Anti-Rigid Class is a subclass of a Rigid Class*
$\mathtt{C_A} \sqsubseteq \mathtt{C_R}$

Finally, due to the constant domain assumption in $\mathcal{DLR_{US}}$, we can capture the notion of *existential rigidity* by introducing an *exist class*, E, collecting, at each point in time, the objects that actually exist.

(EXIST-RIGID) *Existentially Rigid Class*
$\mathtt{C} \sqsubseteq \Box^*(\mathtt{E} \rightarrow \mathtt{C})$

Given the above formalization for anti-rigid classes, then for each object, o, member of an anti-rigid class, $\mathtt{C_A}$, we have that:

$$\text{ACTIVESPAN}_{\mathtt{C_A}}(o) \subset \text{ACTIVESPAN}_{\mathtt{C_R}}(o)$$

On the other hand, for rigid and existentially rigid classes the notions of $\text{EXISTENCESPAN}_C(o)$, $\text{LIFESPAN}_C(o)$ and $\text{ACTIVESPAN}_C(o)$ all denote the full set of time points, $\mathcal{T}$, or the set $\text{ACTIVESPAN}_E(o)$, respectively.

We now have sufficient ingredients to formally characterize the different kinds of dependence between a whole (part) and its part (whole).

Representing mandatory, immutable, and essential parts and wholes

The distinction between *essential*, *immutable* and *mandatory* parts (Guizzardi 2007) can be formalized in terms of two criteria: *i)* the nature of the dependence relationship between the class that describes the whole and the one that describes the part (which can be either *specific* or *generic*), and *ii)* the strength of the membership between the whole and the class that describes it (which, depending whether the class is rigid or not, will result in an *unconditional* or *conditional* dependence). We thus distinguish between the following different cases.

1. *Generic Dependence – Mandatory Part*. The whole must have a part at each instant of its lifetime. Thus, the presence of the part is mandatory, but it can be replaced over time (e.g., the human heart example).
2. *Unconditional Specific Dependence – Essential Part*. The part is mandatory, but it cannot be replaced without destroying the whole (e.g., the human brain example).
3. *Conditional Specific Dependence – Immutable Part* (also called *conditionally essential part*). The part is mandatory and cannot be replaced, but only as long as the whole belongs to the class that describes it (e.g., the boxer's hand example).

In a symmetric way we can define the notions of *mandatory, immutable*, and *essential* wholes. Furthermore, we say that a part is *exclusive* if it can be part of at most one whole (similarly for *exclusive wholes*).

In the following we provide a formalization using an axiomatization expressed with the temporal description logic $\mathcal{DLR}_{\mathcal{US}}$ of such mandatory, immutable, essential and exclusive parts and wholes.

Mandatory and exclusive parts and wholes

Let $\mathtt{PartWhole} \sqsubseteq \mathtt{part}:\mathtt{P} \sqcap \mathtt{whole}:\mathtt{W}$ be a generic part-whole relation between a whole, W, and a part, P. The following $\mathcal{DLR}_{\mathcal{US}}$ axioms give a formalization of *mandatory* and *exclusive* parts and wholes:

(MANP) *Mandatory Part*
$\mathtt{W} \sqsubseteq \exists[\mathtt{whole}]\mathtt{PartWhole}$

(MANW) *Mandatory Whole*
$\mathtt{P} \sqsubseteq \exists[\mathtt{part}]\mathtt{PartWhole}$

(EXLP) *Exclusive Part*
$\mathtt{P} \sqsubseteq \exists^{\leq 1}[\mathtt{part}]\mathtt{PartWhole}$

(EXLW) *Exclusive Whole*
$\mathtt{W} \sqsubseteq \exists^{\leq 1}[\mathtt{whole}]\mathtt{PartWhole}$

The human's heart example can thus be represented by introducing the part-whole relation HumanHeartPW as a subrelation of the generic PartWhole relation:

$\mathtt{HumanHeartPW} \sqsubseteq \mathtt{PartWhole}$
$\mathtt{HumanHeartPW} \sqsubseteq \mathtt{part}:\mathtt{Heart} \sqcap \mathtt{whole}:\mathtt{Human}$

and then constraining every human to participate at least once in the HumanHeartPW relation by adding the following (MANP) axiom:

$\mathtt{Human} \sqsubseteq \exists[\mathtt{whole}]\mathtt{HumanHeartPW}$

Given the semantics of $\mathcal{DLR}_{\mathcal{US}}$ axioms, a *generic* heart must be always associated to a human, i.e., the heart is possibly a different heart at different times but each human must have a heart along its lifetime. While this example specifies a mandatory part (heart) for a rigid class (human), in the general case mandatory parts can be specified regardless of the meta-properties concerning rigidity of the whole.

Finally, if we want to stress the *exclusiveness* of the heart, i.e., that a given heart can be part of at most one human, then we add the (EXLP) axiom:

$\mathtt{Heart} \sqsubseteq \exists^{\leq 1}[\mathtt{part}]\mathtt{HumanHeartPW}$

Note that to represent mandatory and exclusive parts (wholes) we do not need to use the temporal operators of $\mathcal{DLR}_{\mathcal{US}}$.

Essential parts and wholes

Essential parts express a *specific* dependence between the whole and the correspondent part as opposed to the *generic* dependence of a mandatory part. Furthermore, the very same part (i.e., the *specific* part) must actively exist during the entire existence of the whole, *unconditionally* whether the whole belongs to different classes during its existence. Given an instance, o_w, of the rigid class, W, then, o_p is an *essential part* of o_w if $\textsc{ActiveSpan}_{\mathtt{W}}(o_w) = \textsc{ActiveSpan}_{\mathtt{P}}(o_p) = \mathcal{T}$ and o_p is (an active) part of o_w at each instant $t \in \mathcal{T}$. Thus, *essential parts* are global properties of rigid wholes that can be formalized in $\mathcal{DLR}_{\mathcal{US}}$ with the following axioms:

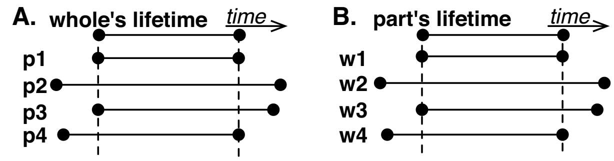

Figure 5: Lifelines of immutable parts ($p1 \ldots p4$) w.r.t. the lifeline of the whole (A)—and vice versa (B).

(RIGID) *Rigid Whole*
$\mathtt{W} \sqsubseteq \Box^{*}\mathtt{W}$

(ESSP) *Essential Part*
$\mathtt{W} \sqsubseteq \exists[\mathtt{whole}]\Box^{*}\mathtt{PartWhole}$

While *essential wholes* can be captured as:

(RIGID) *Rigid Part*
$\mathtt{P} \sqsubseteq \Box^{*}\mathtt{P}$

(ESSW) *Essential Whole*
$\mathtt{P} \sqsubseteq \exists[\mathtt{part}]\Box^{*}\mathtt{PartWhole}$

By substituting the above rigidity axioms with the existential rigidity axiom we can capture essential parts and wholes just along the existence lifespan of an object, i.e., it must be that $\textsc{ActiveSpan}_{\mathtt{W}}(o_w) = \textsc{ActiveSpan}_{\mathtt{P}}(o_p) = \textsc{ActiveSpan}_{\mathtt{E}}(o_w)$, where E is the exist class.

The human's brain example can be represented by introducing the part-whole relation HumanBrainPW as a subrelation of the generic PartWhole relation:

$\mathtt{HumanBrainPW} \sqsubseteq \mathtt{PartWhole}$
$\mathtt{HumanBrainPW} \sqsubseteq \mathtt{part}:\mathtt{Brain} \sqcap \mathtt{whole}:\mathtt{Human}$

and then adding both the (RIGID) and (ESSP) axioms:

$\mathtt{Human} \sqsubseteq \Box^{*}\mathtt{Human}$
$\mathtt{Human} \sqsubseteq \exists[\mathtt{whole}]\Box^{*}\mathtt{HumanBrainPW}$

This asserts that the very same brain is a part of the very same human in all possible time instants. Obviously, exclusiveness axioms can be added to further constrain essential parts and wholes.

One can also conceive examples of exclusive essential parts or wholes for artifacts, such as that each DisposableLighter has as an essential part at most one Firestone, or that each USBStick has exactly one MemoryCard as essential part.

Immutable parts and wholes

As we said above, immutable parts are a form of *conditionally essential parts*—i.e., the part is mandatory and cannot be replaced, but only as long as the whole belongs to the class that describes it. Thus, given an anti-rigid class, $\mathtt{W_A}$, together with its rigid super-class, $\mathtt{W_R}$, and an individual whole, o_w, that is member of $\mathtt{W_A}$, with $\textsc{ActiveSpan}_{\mathtt{W_A}}(o_w) \subset \textsc{ActiveSpan}_{\mathtt{W_R}}(o_w)$, then, o_p is an *immutable part* of o_w if

$\textsc{ActiveSpan}_{\mathtt{W_A}}(o_w) \subseteq \textsc{ActiveSpan}_{\mathtt{P}}(o_p)$ and o_p is (an active) part of o_w at each instant $t \in \textsc{ActiveSpan}_{\mathtt{W_A}}(o_w)$. The temporal relations $p1, \ldots, p4$ of Fig. 5-A illustrate exactly all the possible different temporal relationships between the activespan of the given whole (top line of the figure) and the activespan of its immutable part.

A similar reading can be associated to Fig. 5-B, where $w1, \ldots, w4$ now denote all the possible different temporal relations between a fixed part and its immutable whole. Note that the temporal relation $p2$ ($w2$) can be regarded as the most general notion of immutable part (whole).

To formalize the notion of *immutable parts* (wholes) in $\mathcal{DLR}_{\mathcal{US}}$, we need to resort to the notion of *status* for both classes and relations as illustrated previously. In addition, the whole (part) must belong to an *anti-rigid* class[2].

Let us show now that $\mathcal{DLR}_{\mathcal{US}}$ has enough expressivity to account for the various cases of immutable parts and wholes introduced above. To see that, we shall first introduce a set of useful $\mathcal{DLR}_{\mathcal{US}}$ axioms expressing some basic constraints and then we shall prove that each of the possibilities described by Fig. 5-A (Fig. 5-B) can be expressed by suitable combination of these axioms. The $\mathcal{DLR}_{\mathcal{US}}$ axioms are as follows:

(SUSW) *If part-whole is suspended then the whole is suspended.*
$\mathtt{Suspended\text{-}PartWhole} \sqsubseteq \mathtt{whole} : \mathtt{Suspended\text{-}W}$

(DISP) *If part-whole is disabled then the part is disabled.*
$\mathtt{Disabled\text{-}PartWhole} \sqsubseteq \mathtt{part} : \mathtt{Disabled\text{-}P}$

(DISW) *If part-whole is disabled then the whole is disabled.*
$\mathtt{Disabled\text{-}PartWhole} \sqsubseteq \mathtt{whole} : \mathtt{Disabled\text{-}W}$

(SCHPW) *The part-whole was scheduled sometime in the past.*
$\mathtt{PartWhole} \sqsubseteq \Diamond^{-}\mathtt{Scheduled\text{-}PartWhole}$

(SCHP) *If part-whole is scheduled then the part is scheduled.*
$\mathtt{Scheduled\text{-}PartWhole} \sqsubseteq \mathtt{part} : \mathtt{Scheduled\text{-}P}$

(SCHW) *If part-whole is scheduled then the whole is scheduled.*
$\mathtt{Scheduled\text{-}PartWhole} \sqsubseteq \mathtt{whole} : \mathtt{Scheduled\text{-}W}$

The following Theorem shows the $\mathcal{DLR}_{\mathcal{US}}$ axioms needed to represent the various forms of immutable parts as illustrated in Fig. 5-A.

THEOREM 2 (Immutable Parts). *Let* $\mathtt{W_R}$ *be a rigid class (i.e.,* $\mathtt{W_R} \sqsubseteq \Box^{*}\mathtt{W_R}$*),* $\mathtt{W}$ *be an anti-rigid class (i.e.,* $\mathtt{W} \sqsubseteq \Diamond^{*}\neg\mathtt{W}$*) s.t.* $\mathtt{W} \sqsubseteq \mathtt{W_R}$*, and* $\mathtt{PartWhole} \sqsubseteq \mathtt{part} : \mathtt{P} \sqcap \mathtt{whole} : \mathtt{W}$ *be a generic part-whole relation satisfying* Σ_{st}*. Then, for each whole,* o_w*, of type* $\mathtt{W}$ *there exists an immutable part,* o_p*, of type* $\mathtt{P}$ *that is temporally related to* o_w *with the relation:*

1. $\mathtt{p2}$ *if* (MANP), (SUSW), (DISW) *hold.*
2. $\mathtt{p4}$ *if* (MANP), (SUSW), (DISW), (DISP) *hold.*
3. $\mathtt{p3}$ *if* (MANP), (SUSW), (DISW), (SCHPW), (SCHP) *hold.*
4. $\mathtt{p1}$ *if* (MANP), (SUSW), (DISW), (DISP), (SCHPW), (SCHP) *hold.*

Proof. See Appendix. □

A similar result can be proved for immutable wholes. In analogy with the (SUSW) axiom, we need to introduce an axiom that governs the case in which the part is suspended:

(SUSP) *If part-whole is suspended then the part is suspended.*
$\mathtt{Suspended\text{-}PartWhole} \sqsubseteq \mathtt{part} : \mathtt{Suspended\text{-}P}$

THEOREM 3 (Immutable Wholes). *Let* $\mathtt{P_R}$ *be a rigid class (i.e.,* $\mathtt{P_R} \sqsubseteq \Box^{*}\mathtt{P_R}$*),* $\mathtt{P}$ *be an anti-rigid class (i.e.,* $\mathtt{P} \sqsubseteq \Diamond^{*}\neg\mathtt{P}$*) s.t.* $\mathtt{P} \sqsubseteq \mathtt{P_R}$*, and* $\mathtt{PartWhole} \sqsubseteq \mathtt{part} : \mathtt{P} \sqcap \mathtt{whole} : \mathtt{W}$ *be a generic part-whole relation satisfying* Σ_{st}*. Then, for each part,* o_p*, of type* $\mathtt{P}$ *there exists an immutable whole,* o_w*, of type* $\mathtt{W}$ *that is temporally related to* o_p *with the relation:*

1. $\mathtt{w2}$ *if* (MANW), (SUSP), (DISP) *hold.*
2. $\mathtt{w4}$ *if* (MANW), (SUSP), (DISP), (DISW) *hold.*
3. $\mathtt{w3}$ *if* (MANW), (SUSP), (DISP), (SCHPW), (SCHW) *hold.*
4. $\mathtt{w1}$ *if* (MANW), (SUSP), (DISP), (DISW), (SCHPW), (SCHW) *hold.*

Proof. Similar to Theorem 2. □

Observe that our formalization of immutability allows for suspension of parts, wholes and their part-whole relations, thanks to the introduction of status for both classes and relations. Since status is not considered in the literature for immutability of part-whole relations (Barbier *et al.* 2003; Guizzardi 2005; 2007), we can enforce immutability to hold just for continuously active classes. To achieve this, it is enough to substitute the (SUSP) and (SUSW) axioms with the following axiom that forbids suspension of the part-whole relation:

(CONPW) *Continuous active part-whole relation.*
$\mathtt{Suspended\text{-}PartWhole} \sqsubseteq \bot$

Using the (CONPW) axiom, the immutable parts and wholes are always active while participating in the part-whole relation and cannot be suspended. Furthermore, when the stricter case is allowed (i.e. either p1 or w1 holds), then they are either *both* member of their respective Scheduled class, or both Active, or both member of their respective Disabled classes. Hence, a change of status from one of the two objects implies an *instantaneous* change of the other in the same type of status class.

The boxer's hands example can be represented with the following axioms—enforcing e.g. case $p2$. First, we introduce the part-whole relation HumanHandPW as a sub-relation of the generic PartWhole relation:

$\mathtt{HumanHandPW} \sqsubseteq \mathtt{PartWhole}$
$\mathtt{HumanHandPW} \sqsubseteq \mathtt{part} : \mathtt{Hand} \sqcap \mathtt{whole} : \mathtt{Human}$

then we assert that boxer is anti-rigid and it is a sub-class of the rigid class human:

[2] To capture immutable parts for the more general case of *non-rigid* classes, our approach forces the introduction of an anti-rigid sub-class typing the instances having a limited lifetime for which we can speak of immutable parts.

$\texttt{Boxer} \sqsubseteq \Diamond^* \neg \texttt{Boxer}$
$\texttt{Boxer} \sqsubseteq \texttt{Human}$

and finally we add the (MANP), (SUSW), (DISW) axioms rephrased in terms of the HuamnHandPW part-whole relation:

$\texttt{Boxer} \sqsubseteq \exists^{=2}[\texttt{whole}]\texttt{HumanHandPW}$
$\texttt{Suspended-HumanHandPW} \sqsubseteq \texttt{whole} : \texttt{Suspended-Boxer}$
$\texttt{Disabled-HumanHandPW} \sqsubseteq \texttt{whole} : \texttt{Disabled-Boxer}$

The use of the (SUSW) axiom instead of (CONPW) captures the scenario where it is permitted for a boxer to be temporarily member of the Suspended-Boxer class during which the part-whole relation is suspended as well (e.g., the boxer has a broken hand that is healing). To be compatible with extant literature on immutability, we can enforce a continuously active boxer by replacing the (SUSW) axiom with the (CONPW) axiom:

$\texttt{Suspended-HumanHandPW} \sqsubseteq \bot$

Thinking of examples of immutable parts for artifacts, we may consider, e.g., an EcoFarm (anti-rigid class) as a sub-class of RealEstate (rigid class), where an area of Farmland is an immutable part of the EcoFarm: if, say, the ecofarm's farmland is confiscated, then the residual real estate becomes member of either the Suspended-EcoFarm or Disabled-EcoFarm class (while continuing to be an active member of RealEstate).

Finally, observe that exclusiveness and immutability are orthogonal notions in our modeling framework. The formalization of immutable parts and wholes can be easily extended by adding to the axiomatization of Theorems 2-3 either (EXLP) or (EXLW), depending whether we want to capture *exclusive immutable* parts or wholes.

Conclusions

In this paper we proposed a formalization of mandatory, immutable, and essential parts and wholes adopting a temporal logic approach. For each different parthood property we presented its formalization with the aim to clarify their meaning when used in a conceptual modelling language. The temporal description logic $\mathcal{DLR}_{\mathcal{US}}$ has been adopted thanks to its already established ability to give a logical reconstruction of (temporal) conceptual modelling languages. In addition to represent the different interpretations of a part-whole relation, we introduced the novel notion of status for relations that temporally constrains the evolution of a relation. The notion of status relations has been showed crucial to capture immutable parts and has been formalized in $\mathcal{DLR}_{\mathcal{US}}$.

Concerning the computational properties, we are aware of the undecidability of $\mathcal{DLR}_{\mathcal{US}}$. A promising direction is to resort to the a weaker but decidable temporal DL, *TDL-Lite* (Artale *et al.* 2007) that temporally extends *DL-Lite* with both temporal roles and concepts. We are currently working on both extending the framework of *TDL-Lite* with sub-roles and showing the ability of *TDL-Lite* to represent temporal conceptual models.

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Appendix

PROPOSITION 1 (Status Relations: Logical Implications)**.** *Given the set of axioms Σ_{st} and an n-ary relation, $R \sqsubseteq U_1 : C_1 \sqcap \ldots \sqcap U_n : C_n$, the following logical implications hold:*

(RACT) *Active will possibly evolve into Suspended or Disabled.*
$\Sigma_{st} \models \mathtt{R} \sqsubseteq \Box^+(\mathtt{R} \sqcup \mathtt{Suspended\text{-}R} \sqcup \mathtt{Disabled\text{-}R})$

(RDISAB3) *Disabled will never become active anymore.*
$\Sigma_{st} \models \mathtt{Disabled\text{-}R} \sqsubseteq \Box^+\neg\mathtt{R}$

(RDISAB4) *Disabled classes can participate only in disabled relations.*
$\Sigma_{st} \models \mathtt{Disabled\text{-}C_i} \sqcap \Diamond^-\exists[\mathtt{U_i}]\mathtt{R} \sqsubseteq \exists[\mathtt{U_i}]\mathtt{Disabled\text{-}R}$

(RDISAB5) *Disabled relations involve active, suspended, or disabled classes.*
$\mathtt{Disabled\text{-}R} \sqsubseteq \mathtt{U}_i\mathtt{:}(\mathtt{C}_i \sqcup \mathtt{Suspended\text{-}C}_i \sqcup \mathtt{Disabled\text{-}C}_i)$*, for all* $i = 1, \ldots, n$.

(RSCH3) *Scheduled persists until active.*
$\Sigma_{st} \models \mathtt{Scheduled\text{-}R} \sqsubseteq \mathtt{Scheduled\text{-}R}\,\mathcal{U}\,\mathtt{R}$

(RSCH4) *Scheduled cannot evolve directly to Disabled.*
$\Sigma_{st} \models \mathtt{Scheduled\text{-}R} \sqsubseteq \oplus\neg\mathtt{Disabled\text{-}R}$

(RSCH5) *Scheduled relations do not involve disabled classes.*
$\mathtt{Scheduled\text{-}R} \sqsubseteq \mathtt{U_i} : \neg\mathtt{Dibabled\text{-}C_i}$*, for all* $i = 1, \ldots, n$.

Proof. We prove here just (RDISAB4). The others cases are similar to what was already proved in (Artale, Parent, & Spaccapietra 2007).

(RDISAB4). Let $o_i \in \mathtt{Disabled\text{-}C_i}^{\mathcal{I}(t)}$ and $r = \langle o_1, \ldots, o_i, \ldots, o_n \rangle \in R^{\mathcal{I}(t')}$ for some $t' < t$. Then, by (RACT), $r \in (\mathtt{R} \sqcup \mathtt{Suspended\text{-}R} \sqcup \mathtt{Disabled\text{-}R})^{\mathcal{I}(t)}$. Since active relations, by (ACT), can involve just active classes, then, $r \notin R^{\mathcal{I}(t)}$. On the other hand, by (RSUSP2), $r \notin \mathtt{Suspended\text{-}R}^{\mathcal{I}(t)}$. Thus, $r \in \mathtt{Disabled\text{-}R}^{\mathcal{I}(t)}$. □

THEOREM 2 (Immutable Parts)**.** *Let $\mathtt{W_R}$ be a rigid class (i.e., $\mathtt{W_R} \sqsubseteq \Box^*\mathtt{W_R}$), $\mathtt{W}$ be an anti-rigid class (i.e., $\mathtt{W} \sqsubseteq \Diamond^*\neg\mathtt{W}$) s.t. $\mathtt{W} \sqsubseteq \mathtt{W_R}$, and $\mathtt{PartWhole} \sqsubseteq \mathtt{part} : \mathtt{P} \sqcap \mathtt{whole} : \mathtt{W}$ be a generic part-whole relation satisfying Σ_{st}. Then, for each whole, o_w, of type $\mathtt{W}$ there exists an immutable part, o_p, of type $\mathtt{P}$ that is temporally related to o_w with the relation:*

1. p2 *if* (MANP), (SUSW), (DISW) *hold.*
2. p4 *if* (MANP), (SUSW), (DISW), (DISP) *hold.*
3. p3 *if* (MANP), (SUSW), (DISW), (SCHPW), (SCHP) *hold.*
4. p1 *if* (MANP), (SUSW), (DISW), (DISP), (SCHPW), (SCHP) *hold.*

Proof. Let $o_w \in \mathtt{W}^{\mathcal{I}(\mathtt{t_0})}$ with $t_0 = \text{BIRTH}_{\mathtt{W}}(o_w)$, then by (MANP), $\exists o_p \in \mathtt{P}^{\mathcal{I}(t_0)}$ and $\langle o_p, o_w \rangle \in \mathtt{PartWhole}^{\mathcal{I}(t_0)}$. Since $\mathtt{W} \sqsubseteq \mathtt{W_R}$, then, $o_w \in \mathtt{W_R}^{\mathcal{I}(\mathtt{t_0})}$, while since $\mathtt{W_R}$ is rigid and $\mathtt{W}$ is anti-rigid, then, $\text{ACTIVESPAN}_{\mathtt{W_R}}(o_w) = \mathcal{T}$ and $\text{ACTIVESPAN}_{\mathtt{W}}(o_w) \subset \mathcal{T}$. Thus, $\text{ACTIVESPAN}_{\mathtt{W}}(o_w) \subset \text{ACTIVESPAN}_{\mathtt{W_R}}(o_w)$.

CASE p2. To prove that p2 of Fig. 5 holds we shall prove that $\text{ACTIVESPAN}_{\mathtt{W}}(o_w) \subseteq \text{ACTIVESPAN}_{\mathtt{P}}(o_p)$—i.e., $\text{BIRTH}_{\mathtt{W}}(o_w) \geq \text{BIRTH}_{\mathtt{P}}(o_p)$ and $\text{DEATH}_{\mathtt{W}}(o_w) \leq \text{DEATH}_{\mathtt{P}}(o_p)$—and o_p is an active part of o_w at each instant $t \in \text{ACTIVESPAN}_{\mathtt{W}}(o_w)$. Now, let $t_0 < t' < \text{DEATH}_{\mathtt{W}}(o_w)$ and $o_w \in \mathtt{W}^{\mathcal{I}(\mathtt{t'})}$. Then, by (RACT), either (i) $\langle o_p, o_w \rangle \in \mathtt{PartWhole}^{\mathcal{I}(\mathtt{t'})}$, or (ii) $\langle o_p, o_w \rangle \in \mathtt{Suspended\text{-}PartWhole}^{\mathcal{I}(\mathtt{t'})}$ or (iii) $\langle o_p, o_w \rangle \in \mathtt{Disabled\text{-}PartWhole}^{\mathcal{I}(\mathtt{t'})}$. The last two cases cannot happen since, by assumption $o_w \in \mathtt{W}^{\mathcal{I}(\mathtt{t'})}$, while: (iii) by (DISW), $o_w \in \mathtt{Disabled\text{-}W}^{\mathcal{I}(\mathtt{t'})}$; (ii) by (SUSW), $o_w \in \mathtt{Suspended\text{-}W}^{\mathcal{I}(\mathtt{t'})}$. Thus, o_p is an active part of o_w and, since by assumption active relations involve only active classes, $o_p \in \mathtt{P}^{\mathcal{I}(\mathtt{t}')}$, for all t' s.t. $t_0 < t' < \textsc{Death}_{\mathtt{W}}(o_w)$. For $t_1 \geq \textsc{Death}_{\mathtt{W}}(o_w)$ and $t_2 \leq \textsc{Birth}_{\mathtt{W}}(o_w)$ none of the axioms constraints the lifespan of o_p. Thus, $\textsc{Death}_{\mathtt{W}}(o_w) \leq \textsc{Death}_{\mathtt{P}}(o_p)$, and $\textsc{Birth}_{\mathtt{W}}(o_w) \geq \textsc{Birth}_{\mathtt{P}}(o_p)$.

CASE p1. To prove that the case p1 of Fig. 5 holds we should prove that $\textsc{ActiveSpan}_{\mathtt{W}}(o_w) = \textsc{ActiveSpan}_{\mathtt{P}}(o_p)$—i.e., $\textsc{Birth}_{\mathtt{W}}(o_w) = \textsc{Birth}_{\mathtt{P}}(o_p)$ and $\textsc{Death}_{\mathtt{W}}(o_w) = \textsc{Death}_{\mathtt{P}}(o_p)$—and o_p is an active part of o_w at each instant $t \in \textsc{ActiveSpan}_{\mathtt{W}}(o_w)$. As for case p2, since (SUSW) and (DISW) hold, then, o_p is an active part of o_w, and $o_p \in \mathtt{P}^{\mathcal{I}(\mathtt{t}')}$, for all t' s.t. $t_0 < t' < \textsc{Death}_{\mathtt{W}}(o_w)$. Now, let $t_1 = \textsc{Death}_{\mathtt{W}}(o_w)$, then, by (RDISAB4), $\langle o_p, o_w \rangle \in \mathtt{Disabled\text{-}PartWhole}^{\mathcal{I}(t_1)}$, and, by (DISP), $o_p \in \mathtt{Disabled\text{-}P}^{\mathcal{I}(t_1)}$. Thus, $\textsc{Death}_{\mathtt{W}}(o_w) = \textsc{Death}_{\mathtt{P}}(o_p)$. Now, by absurd, let's assume that $o_p \in P^{\mathcal{I}(t_2)}$, with $t_2 < t_0$. By (SCHPW), there is a $t' < t_0$ s.t. $\langle o_p, o_w \rangle \in \mathtt{Scheduled\text{-}PartWhole}^{\mathcal{I}(t')}$ and, by (SCHP), $o_p \in \mathtt{Scheduled\text{-}P}^{\mathcal{I}(t')}$. Then, $t_2 \neq t'$. Also, $t_2 \not< t'$ since, by (SCH2), an active class cannot evolve into its scheduled status. Finally, $t_0 > t_2 \not> t'$ since, by (RSCH3) $\langle o_p, o_w \rangle \in \mathtt{Scheduled\text{-}PartWhole}^{\mathcal{I}(t_2)}$ and, by (SCHP), $o_p \in \mathtt{Scheduled\text{-}P}^{\mathcal{I}(t_2)}$. Thus, $\textsc{Birth}_{\mathtt{W}}(o_w) = \textsc{Birth}_{\mathtt{P}}(o_p)$.

Cases p4, p3 can be easily obtained from the above cases. □