Elicitation of the Groups and Group Cognitive Structures: An Application of Ternary Fuzzy Relational Products

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Abstract

The main contribution of this paper is the exploration of ternary relation in fuzzy relational theory and its application in the cognitive science. The ternary fuzzy relational products were defined; data were collected; the *tTriMod* program, which applies the fuzzy relational products, were executed; and evidences, that there are group cognitive structures, were found.

Problems from URBS Project

Dr. W. Bandler, Dr. V. Mancini and VAGOGS (Very Able Group Of Graduate Students) started the URBS(Unbiased Research on Beings in Society) project years ago. By the continuous efforts, the main scheme of the URBS project was designed(Stiller 1989)(Bandler, Mancini, & Stiller 1991); the technique of eliciting the individual cognitive structures was developed(Ogle 1993); the semantic explanation of the cognitive structures and its usage of advice creation in URBS was achieved(Ncube 1993). The comparison of two different structures was explored(Juliano 1993)(Joo 1995). The theoretical ground of the URBS project is the fuzzy relational theory developed by Dr. Bandler and Dr. Kohout(Bandler & Kohout 1978)(Bandler & Kohout 1980), and some modern psychological theories, especially the theory of personal constructs by G. Kelly(Kelly 1955).

The purpose of the URBS project is to develop the URBS system. It is a knowledge based system which will take the results of a survey among city residents as input information and create the advice on the problems about the city development and improvement. In modern psychology, it has been found that human minds are not like a "white board", on which every stimulus from the sense organs will be written; but there are cognitive structures in human minds, by which the sense materials are organized into an understandable, communicable form. Furthermore, as different individuals have different structures in their minds, they will demonstrate different traits of behaviors or express different opinions under the same circumstance. By the means of the calculation derived from the fuzzy relational products invented by Dr. W. Bandler and Dr. L. J. Kohout, it is possible to elicit such internal cognitive structures of the individuals through the analysis of her/his opinions or behavior. From the comparison between the internal structures(aspatial structures) of the city residents and the physical structure(spatial structure) of the city, the URBS system will gives us expert, positive and helpful advice on city problems(Mancini & Bandler 1988).

In order to evaluate the congruency of the aspatial and spatial structure of urban problems, URBS system should be able not only to elicit the cognitive structure of each individual, but also to abstract cognitive structure that shared by some people. Individuals are different and live together, some of them share the same opinions, some do not. It is necessary for URBS system to elicit the cognitive structure that represents the community of the residents in the district. After the technique of eliciting the individual cognitive structure was developed, the problem emerging to the URBS system is how to elicit the cognitive structure of a group of individuals.

By comparing the different individual cognitive structures, it was found that the similarity of individual cognitive structures were not the same(Joo 1995). So, it is possible to put the individuals, whose cognitive structures are highly similar to each other, into one group, and elicit the cognitive structure of this group.

The combination of the concept of personal construct psychology and the triangle product of fuzzy relational calculation, is the foundation of the technique used in URBS system to elicit the cognitive structure. The following is a step by step description of the technique.

Step 1: from the survey of the respondents of the residents, repertory grids have been obtained. Those grids are the personal evaluations of some urban objects in the frame of the constructs.

Step 2: applying the triangle product of relational operation to the grid, the new grid that represents relationship between constructs to constructs has been obtained.

Step 3: choosing an α value to implement α-cut to the construct-construct grid, to get the crispified grid, according to which Hasse diagram is drawn. It can be called cognitive structure.(Ncube 1993)(Ogle 1993)

The author uses the technique previously developed in URBS, and extended Step 2 by using ternary (instead of binary) products of relational calculation to manipulate the multi-person's repertory grids at one time, to reveal the groups within respondents and elicit the cognitive structure for each group.

This paper is a piece in the continuous efforts to achieve the goals of the URBS project, and it is not, of course, the end of it. It mainly focuses on two problems:

1. How to cluster the respondents in groups only according to their similarity of the cognitive structures;
2. How to elicit the group cognitive structures.

The main contributions are two-folds:

1. Extending the fuzzy relational products from binary to ternary;
2. Using ternary fuzzy relational products to resolve the above-mentioned two problems.

Ternary Fuzzy Relational Products

Dr. Bandler and Dr. Kohout defined fuzzy n-ary relation as: "An n-ary relation R is an open sentence with N slots; when these are filled in order by the names of elements from the sets $X_1, ..., X_n$, there results a fuzzy proposition which is judged to hold to a certain degree, or to have a certain degree of truth; this degree is the degree to which the elements put into the slots are related by R."(page 4006)(Bandler & Kohout 1989) It is easy to derive an explicit definition of ternary relation from the above general definition of n-ary relation. A ternary relation R is a set of ordered triplet, written as

$$R \in \mathcal{T}(X, Y, Z)$$

where X, Y, Z are sets. It is a sentence with 3 open slots. It will hold in certain degree from 0 to 1 after those slots have been filled with $x \in X, y \in Y, z \in Z$.

The general definition of the products of r-ary relation and s-ary relation also can be found in (Bandler & Kohout 1989). For ternary relation

$$R \in \mathcal{T}(W, X, Y)$$

and

$$S \in \mathcal{T}(X, Y, Z)$$

the products

$$\mathcal{T}(W, X, Y) \times \mathcal{T}(X, Y, Z) \longrightarrow \mathcal{B}(W \rightarrow Z)$$

can be defined as

$$(R \circ S)_{il} = \max_{jk}(min(R_{ijk}, S_{jkl})); \quad (1)$$

$$(R \triangleleft S)_{il} = \min_{jk}(R_{ijk} \rightarrow S_{jkl}); \quad (2)$$

$$(R \triangleright S)_{il} = \min_{jk}(R_{ijk} \leftarrow S_{jkl}); \quad (3)$$

$$(R \square S)_{il} = \min_{jk}(R_{ijk} \leftrightarrow S_{jkl}). \quad (4)$$

It is more explicit to rewrite the above formula as the following.

$$^{i}(R \circ S)_{l} = \bigvee_{jk} ({^{i}R_{jk}} \wedge {^{jk}S_{l}}); \quad (5)$$

$$^{i}(R \triangleleft S)_{l} = \bigwedge_{jk} ({^{i}R_{jk}} \rightarrow {^{jk}S_{l}}); \quad (6)$$

$$^{i}(R \triangleright S)_{l} = \bigwedge_{jk} ({^{i}R_{jk}} \leftarrow {^{jk}S_{l}}); \quad (7)$$

$$^{i}(R \square S)_{l} = \bigwedge_{jk} ({^{i}R_{jk}} \leftrightarrow {^{jk}S_{l}}). \quad (8)$$

Also, for the relations

$$R \in \mathcal{T}(V, W, X)$$

and

$$S \in \mathcal{T}(X, Y, Z)$$

the product

$$R \times S \longrightarrow \mathcal{Q}(V, W, Y, Z)$$

(Q(V, W, Y, Z) stands for the quaternary relation among sets V, W, Y, Z)
can be defined as

$$^{ij}(R \circ S)_{lm} = \bigvee_{k} ({^{ij}R_{k}} \wedge {^{k}S_{lm}}); \quad (9)$$

$$^{ij}(R \triangleleft S)_{lm} = \bigwedge_{k} ({^{ij}R_{k}} \rightarrow {^{k}S_{lm}}); \quad (10)$$

$$^{ij}(R \triangleright S)_{lm} = \bigwedge_{k} ({^{ij}R_{k}} \leftarrow {^{k}S_{lm}}); \quad (11)$$

$$^{ij}(R \square S)_{lm} = \bigwedge_{jk} ({^{ij}R_{k}} \leftrightarrow {^{k}S_{lm}}). \quad (12)$$

The products of one ternary relation and one binary relation also can be defined in this manner.

The above definitions are *by harsh criterion*(Bandler & Kohout 1978). The *mean value* of binary triangle product could be defined as(Bandler & Kohout 1980)

$$(R \triangleleft S)_{ik} = \frac{1}{N_j} \sum_{j} (R_{ij} \rightarrow S_{jk}). \quad (13)$$

The mean value of ternary triangle product is defined on the same principle:

$$^{i}(R \triangleleft S)_{l} = \frac{1}{N_{jk}} \sum_{jk} ({^{i}R_{jk}} \rightarrow {^{jk}S_{l}}). \quad (14)$$

The ternary square product is also defined in this manner:

$$^{i}(R \square S)_{l} = \frac{1}{N_{jk}} \sum_{jk} ({^{i}R_{jk}} \leftrightarrow {^{jk}S_{l}}). \quad (15)$$

The last two formula will be used frequently later in this paper.

In URBS project, there are three sets:

P: people (all respondents);
C: constructs;
O: objects.
They are interrelated each other.

Specifically, respondents evaluate urban objects by constructs. It is equivalent to stating that constructs are used by the respondents to evaluate the urban objects. This can also be expressed as the statement that objects are evaluated by the respondents in their constructs. It is written as

$$R \in \mathcal{T}(P, C, O); \tag{16}$$

$$S \in \mathcal{T}(C, O, P); \tag{17}$$

$$T \in \mathcal{T}(O, P, C). \tag{18}$$

For example, suppose people P includes *Smith* and *Brown*, construct C consists of *beautiful* and *useful*, and the object O is an *oak tree* between their houses. If *Smith* told us that the oak tree was beautiful but useless, but *Brown* disagreed. Then we recorded it as

R(Smith, {beautiful, not useful}, oak tree)
S({beautiful, not useful}, oak tree, Smith)
T(oak tree, Smith, {beautiful, not useful})

are true; and

R(Brown, {beautiful, not useful}, oak tree)
S({beautiful, not useful}, oak tree, Brown)
T(oak tree, Brown, {beautiful, not useful})

are false.

Having the evaluations of the same urban objects in the same constructs from a survey, application of the formula that define the ternary fuzzy relational products can produce various interesting results.

For $R \in \mathcal{T}(P, C, O)$ of URBS, define $R^{-1} \in \mathcal{T}(C, O, P)$ as the one of the complements of relation R. (The other one is $R^{-2} \in \mathcal{T}(O, P, C)$) By

$$R \,\square\, R^{-1} \tag{19}$$

the respondents will be clustered in groups:

$$P_{group} = \{p \mid p \in R \,\square\, R^{-1}\}. \tag{20}$$

In other words, for all $i, j \in P$,

$$P_{group} = \{i, j \mid {}^{i}(R \,\square\, R^{-1})_j = 1\}. \tag{21}$$

or equivalently, for $j \in P$,

$$j \in P_{group}$$

if and only if

$$\forall i \in P_{group}$$

such that

$${}^{i}(R \,\square\, R^{-1})_j = 1.$$

In the fuzzy case, an α value was preselected in the range from 0 to 1, and the above formula should be re-written as:

For all $i, j \in P$

$$P_{group} = \{i, j \mid {}^{i}(R \,\square\, R^{-1})_j \geq \alpha\}. \tag{22}$$

For $j \in P$,

$$j \in P_{group}$$

if and only if

$$\forall i \in P_{group}$$

such that

$${}^{i}(R \,\square\, R^{-1})_j \geq \alpha.$$

That was an application of so-called α-cut technique, which is an operation that converts a fuzzy relation R into a crispy relation R_α. For any $\alpha \in (0, 1]$,

$$(R_\alpha)_{ij} = \begin{cases} 1 & \text{if } R_{ij} \geq \alpha; \\ 0 & \text{otherwise.} \end{cases}$$

That means R_α holds when R holds at the degree at least as great as α(Bandler & Kohout 1989).

Cognitive structure is a set of relations among the constructs. For each group of respondents, there is a relation:

$$S \in \mathcal{T}(C, O, P_{group}).$$

Its complement is

$$S^{-1} \in \mathcal{T}(O, P_{group}, C)$$

By using

$$S \lhd S^{-1} \longrightarrow \mathcal{B}(C \to C) \tag{23}$$

cognitive structure of the group can be elicited as

$$C_{structure} \in \mathcal{B}(C \to C).$$

URBS Data and Results

URBS Data URBS data were collected by conducting a survey. The respondents were asked to fill a data collecting form, rating each urban object in terms of the constructs. Then, the forms were converted into grids.

The *data collecting form* used here was designed by Dr. V. Mancini. (The original form was written in Italian for an experiment conducted in Italy, and was translated into English by Dr. Bandler.) There are five objects on the form:

1. cypress
2. cultivated fields
3. tree
4. interstate highway
5. woods

And thirteen constructs are listed on the form:

1. young - old
2. beautiful - ugly
3. useful - useless
4. cheerful - sad
5. green - brown
6. big - small
7. likes cartoon - likes fairy tale
8. quiet - agitated
9. friendly - hostile
10. easily noticeable - hidden in background
11. far away to the valley - closed by the valley
12. likable - dislikable
13. modern - ancient

For each construct, there are five rating choices for the respondent to check. For example, the five choices for the first construct "young-old" are

is very young
is a little young
is young or old
is a little old
is very old

And the checked results are converted into the numeric number: 1, 0.75, 0.5, 0.25, 0 for the term "young", and 0, 0.25, 0.5, 0.75, 1 for the term "old" correspondingly. Those numbers fill into a grid.

The data collecting form was originally designed for the investigation in Italy. The cypress is a very typical scenery in Italy. The survey was conducted among the elementary students in Italy several years ago.

The author collected more data among the elementary students in U. S. A. using the translated form. Since the Italian cypress is not easy to find here, a bunch of pictures of the Italian cypress(Busato 1986) was showed to each of the respondent before s/he performed the activity of rating the object.

Groups of Respondents By $R \square R^{-1}$(19) calculation using the data collected both in Italy and in USA, groups can be clustered among those respondents. From the definition by harsh criterion(8),

$$^{i}(R \square R^{-1})_{i'} = \bigwedge_{jk}(^{i}R_{jk} \leftrightarrow {}^{jk}R^{-1}_{i'}) \qquad (24)$$

or by the mean criterion(15),

$$^{i}(R \square R^{-1})_{i'} = \frac{1}{N_{jk}} \sum_{jk}(^{i}R_{jk} \leftrightarrow {}^{jk}R^{-1}_{i'}) \qquad (25)$$

and the definition of

$$a \leftrightarrow b = (a \rightarrow b) \wedge (b \rightarrow a)$$

the square product is calculated as

$$^{i}(R \square R^{-1})_{i'} = \bigwedge_{jk}((^{i}R_{jk} \rightarrow {}^{jk}R^{-1}_{i'}) \wedge (^{jk}R^{-1}_{i'} \rightarrow {}^{i}R_{jk}))$$

by harsh criterion, or

$$^{i}(R \square R^{-1})_{i'} = \frac{1}{N_{jk}} \sum_{jk}((^{i}R_{jk} \rightarrow {}^{jk}R^{-1}_{i'}) \wedge (^{jk}R^{-1}_{i'} \rightarrow {}^{i}R_{jk}))$$

by mean criterion.

According to Dr. Bandler and Dr. Kohout, the most commonly used implication operators are defined as follows(page 227)(Bandler & Kohout 1980):

1. $S^{\#}$ Standard Sharp

$$a \rightarrow_1 b = \begin{cases} 1 & \text{iff } a \neq 1 \text{ or } b = 1; \\ 0 & \text{otherwise.} \end{cases}$$

2. S Standard Strict

$$a \rightarrow_2 b = \begin{cases} 1 & \text{iff } a \leq b; \\ 0 & \text{otherwise.} \end{cases}$$

3. S^{*} Standard Star

$$a \rightarrow_3 b = \begin{cases} 1 & \text{iff } a \leq B; \\ \text{b} & \text{otherwise.} \end{cases}$$

Groups by Lukasiewicz operator in mean criterion Alpha = 0.85

Groups	Who	Age	Sex	Where
1, 3, 4, 5, 6	1iM8	8	M	Italy
1, 3	2iF8	8	F	Italy
2	3iM8	8	M	Italy
1, 4, 6, 7, 8	4iF8	8	F	Italy
3	5iM8	8	M	Italy
4	6iF8	8	F	Italy
5	7aF6	6	F	USA
5, 6, 7, 8	8aF7	7	F	USA
7	9aM10	10	M	USA
8	10aF9	9	F	USA
6, 7	11aM7	7	M	USA
5, 6, 8	12aF8	8	F	USA

Group	Group Members
1	1iM8, 2iF8, 4iF8
2	3iM8
3	1iM8, 2iF8, 5iM8
4	1iM8, 4iF8, 6iF8
5	1iM8, 7aF6, 8aF7, 12aF8
6	1iM8, 4iF8, 8aF7, 11aM7, 12aF8
7	4iF8, 8aF7, 9aM10, 11aM7
8	4iF8, 8aF7, 10aF9, 12aF8

Figure 1: Groups by Łukasiewicz operator at $\alpha = 0.85$

4. G43 Gaines 43

$$a \rightarrow_4 b = min\left(1, \frac{b}{a}\right).$$

4'. G43' Modified Gaines 43

$$a \rightarrow_{4'} b = min\left(1, \frac{b}{a}, \frac{1-a}{1-b}\right)$$

5. Ł Łukasiewicz

$$a \rightarrow_5 b = min(1, 1 - a + b).$$

6. KD Kleene-Dienes

$$a \rightarrow_6 b = (1 - a) \vee b.$$

7. EZ Early Zadeh

$$a \rightarrow_7 b = (a \wedge b) \vee (1 - a)$$

8. W Willmott

$$a \rightarrow_8 b = ((1 - a) \vee b) \wedge (a \vee (1 - b) \vee (b \wedge (1 - a)))$$

Using the harsh criterion, no interesting results were found. That is, the groups, resulted from the previous defined(24) calculations with all implication operators by the harsh critierion, are consisting of only one member.

For each implication operator, the author picked various α values to calculate the groups in mean criterions(25), and different results are obtained. One interesting observation of the groups can be made, namely that the groups overlap each other for all implication operators. (See Figure 1 as an example)

It was demonstrated in (Bandler & Kohout 1980) that a higher α value results more abstract, that is,

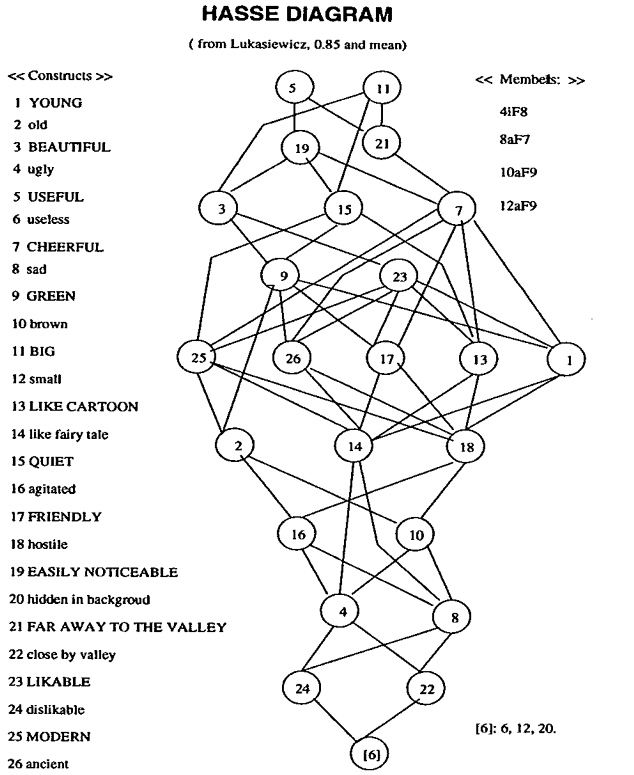

Figure 2: Hasse diagram of Group 8 by Lukasiewicz operator in mean criterion at α=0.85

simpler Hasse diagram. In Ncube's Ph. D. dissertation, it was observed that the higher α value leads to smaller size of equivalence classes(Ncube 1993). The author of this paper has found that as the α value increases, the number of groups increases and the number of the members in each group decreases, and vise versa. The similar phenomenon can be observed for all nine implication operators. The question of the best choice of α values is left for the future study.

Cognitive Structures of Groups Once the groups of respondents were clustered, by triangle calculation, $S \triangleleft S^{-1}$(23), the cognitive structures of groups can be elicited.

$$^{i}(S \triangleleft S^{-1})_{i'} = \bigwedge_{jk} ({}^{i}S_{jk} \rightarrow {}^{jk}S^{-1}_{i'}) \qquad (26)$$

in harsh criterion, and defined as,

$$^{i}(S \triangleleft S^{-1})_{i'} = \frac{1}{N_{jk}} \sum_{jk} ({}^{i}S_{jk} \rightarrow {}^{jk}S^{-1}_{i'}) \qquad (27)$$

in mean criterion.

Here again is the question of implication operator selection. The author selected the same implication operator as the one selected for group clustering calculation.

In order to follow the *conceptual procedure*(Bandler & Kohout 1988) to elicit the cognitive structures of the groups by means of drawing Hasse diagrams, α value selection problem appeared here again. For consistency, it is treated with the same strategy, using the same value as it was used in group clustering calculations.

The procedure of cognitive structure elicitation has five steps:

1. to fill the constructs to constructs grid by $^{i}(S \triangleleft S^{-1})_{i'}$ values;
2. to perform the α-cut for preselected α value;
3. to find the equivalent classes of constructs;
4. to elicit the hierarchical relations among the equivalent classes;
5. to draw the Hasse diagrams.

The examples of the group cognitive structures represented by Hasse diagrams are illustrated in Figure 2.

The author made no efforts on the interpretation of the cognitive structure represented by the Hasse diagram, but would like to recommend Barbara Ogle's thesis *Structure of formation, interpretation and variations of orderings arising from the URBS project*(Ogle 1993) and Cathy Ncube's dessertation *A cognitive hinting structure for deep domain knowledge*(Ncube 1993).

By the observation of the Hasse diagrams, we knew the group cognitive structures are different from each other. Also, the group cognitive structure is different from its members' cognitive structures. Comparing different Hasse diagrams, that represented group cognitive structure, the following common characteristics are discerned.

1. Most of the positive constructs, which are numbered in odd number, are in the equivalence classes together with the other positive constructs; and the same are true for the negative constructs, which are numbered in even.
2. The most of the nodes in the upper part of Hasse diagrams which represent the group cognitive structures are odd-numbered constructs, and those in the lower part are even-numbered constructs.
3. Construct *USEFUL* or/and construct *BIG* are usually on the top of all the Hasse diagrams of the group cognitive structures; and the construct *useless* or/and construct *small* are most likely at the bottom of them.

Based upon the above observations, some heuristic conclusions can be derived for the cognitive science.

1. Most of the individuals are the same in cognition at some abstract level.
2. All the respondents intended to appreciate the urban objects in terms of positive constructs rather than in terms of negative constructs.

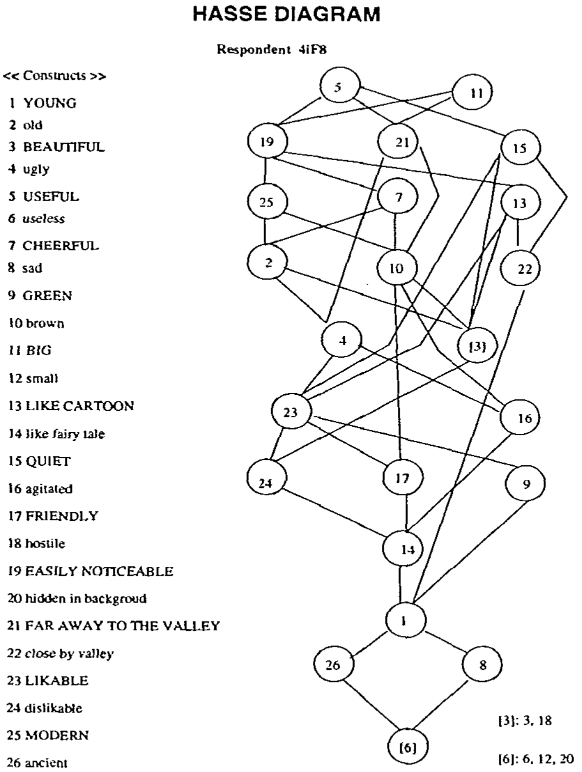

Figure 3: Hasse diagram of respondent 4iF8 by Lukasiewicz operator in mean criterion at α=0.85

Now, we try to compare the group cognitive structure to the individual cognitive structures of its members. By such a comparison, we could verify that the group cognitive structure, obtained by the ternary fuzzy relational product calculations, is the valid abstraction of the cognitive structures of their members.

For example, Group 1 (Figure 2) has four members, (4iF8, 8aF7, 10aF9, 12aF8). All members contributed to the group structure. It is true for all members that the equivalence class of construct number five (*USEFUL*) is at the top of their cognitive structures. Therefore, their group cognitive structure is also topped by the equivalence class of fifth construct. It is the same for the bottom of the cognitive structures: the individual cognitive structures of all members have construct number six (*useless*) at the bottom, and so has their group cognitive structure. Stepping a little further into the details, the constructs at the top of cognitive structures are listed as follows:

4iF8	[5, 11, 19]
8aF7	5
10aF9	[5, 9, 15, 19]
12aF8	[5, 11, 19]
Group	[5, 11, 19]

It is found that if a construct is at the top of two or more cognitive structures of its members, it is also at the top of the group cognitive structure. It is more complicated for comparing the middle part of the cognitive structures, because it appears that they are quite different. It is observed that constructs numbered 1, 14, 18 and 25 are at the central level in the group cognitive structure. They are also in the middle of the cognitive structures of members. Figure 3 shows one member's individual cognitive structure in Group 8.

Generally speaking, a group cognitive structure is a generalization of the individual cognitive structures of its members. It is not a morphism but rather a *reconstruction*.

The theoretical approach to the relationship between the group cognitive structures and the individual cognitive structures of their members is left for the future study.

Discussions We can make following general observations.

1. *Groups are not clustered by the nationality.* People with the same cultural background usually share the similar cognitive structures. Indeed, it is found that some groups consist only of Italian members; and the majority of the members of some groups are living in USA. But not all the groups are divided purely ethnically. There are some groups, whose members are composed of both nationalities. This indicates that the nationality of the respondents is not the criterion for clustering the groups.

2. *Groups are not divided by its gender.* The differences between male and female are less pronounced in the cognitive aspect. There is just one group consisting of only female members. All the other groups, except a singleton group have both male and female members.

3. *Groups are formed by the cognitive structures.* From the visual comparison of the Hasse diagrams presenting the group cognitive structures and those presenting the individual cognitive structures, it can be concluded that the respondents who share the similar cognitive structures are more likely to be in the same group cognitive structure, and the respondents with different cognitive structures are in the different groups.

That is the proof of the validity of the ternary fuzzy relational calculation methods in clustering the respondents based on their cognitive structures.

The proof of the reliability of the fuzzy relational calculation method in clustering respondents into groups and eliciting the cognitive structures of groups in URBS system requires larger population of respondents. More data should be collected in the future.

The further study on the validity and reliability might be helpful in finding the solution to the question of selecting the most proper implication operator and best α value for this technique and URBS data.

The *tTriMod* program that implemented the fuzzy relational calculation to cluster the respondents into groups and elicit the cognitive structures of the groups, has been designed and implemented by the author. It makes it possible to process all the data collected among respondents for the same subject domain of the URBS project. As the result, the URBS system could make advise based on the opinion of the majority of the respondents, as well as the minority of them. The potential further applications of the program, like processing the data from a public poll, is also possible. It is hopeful that it is useful as a tool for the theoretical studies of cognitive science.

Conclusions

The main contributions of the study presented in this paper are (1) extension of fuzzy relational product calculation from binary to ternary relations; (2) application of the ternary fuzzy relational product calculations to clustering the respondents into groups and elicit the group cognitive structures.

The Future of Ternary Fuzzy Relational Products The world can be described as a set of relations. Every person, every event, everything is related to each other. An isolated object, if it exists, is meaningless to this world. The object is defined by the relations it involved. The ternary fuzzy relational products are more powerful. It is applicable to vast research areas. We found more applications of the technique of group cognitive structure elicit that developed in the previous chapters.

In education, for example, by eliciting groups of students with the similar cognitive structures, it may be helpful to put one group of students into the same one classroom so that the teachers can guide the educational activities based on the unique characteristics of the group in that classroom; it may also beneficial to put the students from the different groups into one classroom to promote students learning how to cooperate each other, to make the cooperative learning strategy work better.

Group cognitive structure elicitation can be applied in advertising and market study for commercial products. From the public survey among the perspective customers, the cognitive structures on specific consume domain of customer groups could be obtained by fuzzy relational calculations. That could be helpful for market prediction and advertising strategy.

From the above two examples, it is evident that the further applications of computer supported use of fuzzy relational calculation to discover and explore the cognitive structures, other than those coming from the URBS system application, is worth of further study.

According to Sir Karl Popper, science is progressing by finding problems, trying to solve the problemss and then discovering new problems.(Popper 1968) The question of how to elicit the group cognitive structures in the context of URBS study has been answered here. Also some new problems on fuzzy relational theory and cognitive science have emerged from this work.

There are two questions that remain unanswered.

1. Which implication operator is the best for the definition of fuzzy relational products?
2. How to select an α value for the best structure elicitation?

Furthermore, the study of validity and reliability of the fuzzy relational calculation for elicitation of groups and group cognitive structures requires further efforts.

A Problem for Fuzzy Relational Theory It was found from the results of group clustering calculation that one respondent could appear in two or more groups. According to the definition of the grouping calculation(22)

$$\exists i, j, k \in P,\ \ P_{group1} \subseteq P, P_{group2} \subseteq P,\ \ P_{group1} \neq P_{group2}$$

where

$$i, j \in P_{group1} \wedge i, k \in P_{group2}$$

Therefore, for any $\alpha \in (0, 1]$ it holds that

$$^{j}(R \,\square\, R^{-1})_i \geq \alpha,$$

and

$$^{i}(R \,\square\, R^{-1})_k \geq \alpha,$$

but

$$^{j}(R \,\square\, R^{-1})_k \not\geq \alpha.$$

Using the definition of the ternary fuzzy relational products(12), it yields expressions

$$\bigwedge(\ ^{j}R \leftrightarrow R_i^{-1}\ )$$

and

$$\bigwedge(\ ^{i}R \leftrightarrow R_k^{-1}\ )$$

that hold, but

$$\bigwedge(\ ^{j}R \leftrightarrow R_k^{-1}\ )$$

does not hold.

It is obvious that the *transitivity does not hold* for the square product operation of fuzzy relations.

$$^{j}(R \,\square\, S)_i \ \wedge\ ^{i}(R \,\square\, S)_k \ \not\longrightarrow\ ^{j}(R \,\square\, S)_k \qquad (28)$$

Dr. Bandler and Dr. Kohout proved, "The square product is not associative" (Bandler & Kohout 1986). Here is a problem of the proof of non-transitivity property of the square product.

Problems for Cognitive Science The personal cognitive structure is a set of relationships among the personal constructs. From his clinical practice, G. Kelly discovered that the individuals had systems of personal constructs(Kelly 1955). The cognitive structure of an individual decided the way how this individual would interact with the outside world: how to interpret the sense data, how to respond to the outside stimulation, etc.. From the previous discussions, it was found that some of the individuals shared the similar cognitive structures. How to explain that phenomenon is a question for the cognitive scientists. More questions followed more thinking:

1. Why some persons share similar cognitive structures and some do not?
2. What is the dynamics that makes it possible for some people to form similar cognitive structures even though they live in thousands miles apart and for some people to have different cognitive structures even though they are live in a close proximity of each other?
3. Is it possible for two persons, who possess very different cognitive structures, to communicate each other without misunderstanding?

"Because of the universal applicability of relational structures to the specification of a variety of information-processing systems"(Kohout & Bandler 1985), fuzzy relational product is a methodology which can be used to find the solution to the problems that are beyond the scope of the traditional analytic methods. The problem emerged in the URBS project is a good evidence .

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