Beppe had a dream

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Abstract

The situation which occurred in a dream provides the framework for discussing properties of the theory of viewpoints and in particular the issue of different denotations from different perspectives. We introduce the principle of "referent sharing" in communications and argue that common knowledge resulting from communication should only use constants whose referent is "manifest" to the parties involved.

Keywords: viewpoints, contexts, metalevel theories, reflective theories, natural deduction.

1 Introduction to the theory of viewpoints

The theory of viewpoints was conceived as a general and unified formalism where several varieties of relativized truth could be expressed: beliefs contexts, situations, truth, partition of a knowledge base in microtheories and so on [2, 4, 5]. Differences in the properties of the various notions are captured by additional axioms, thus expanding the set of statements that hold in specialised viewpoints [20].

The theory of viewpoints is a reflective first order logic which amalgamates object and metalanguage by using *names* for each term and statement of the language and which contains an axiomatization of provability in the style of natural deduction. Reflection rules are present which lead to a non conservative but consistent extension of first order logic; these rules are carefully formulated in order to avoid paradoxes arising from self referential sentences, which trickle in by diagonalization [16].

Viewpoints denote *sets of sentences* which represent the assumptions of a theory. A statement of the form $\mathsf{in}('A', vp)$, where vp is a viewpoint expression, is interpreted as "statement A is entailed by the assumptions denoted by vp in the current interpretation" [1].

[1]The notation $'A'$ is an abbreviation for a term denoting sentence A. The naming device is not important in this paper; from now on we also drop quotes inside in without risk of ambiguities.

The motivations and properties of the theory of viewpoints differ in several important respects from existing proposals and formal accounts of contexts [14, 13, 19, 7, 8].

In the formal system by Buvač and Mason for example [8], the semantics of $\mathsf{ist}(c, p)$, to be read as "it is true in context c that p", is essentially (language restrictions apart) entailment: the semantics associates to a context a set of models and p is assessed to be true in such models. As a consequence a context will have complete information of what is true in other contexts:

$$(ND) \qquad k : \mathsf{ist}(k_1, \mathsf{ist}(k_2, \phi)) \vee \mathsf{ist}(k_1, \neg\mathsf{ist}(k_2, \phi))$$

Moreover the context structure, at least in the quantified version [9], is in their terminology *flat*:

$$(Flat) \qquad k : \mathsf{ist}(k_1, \mathsf{ist}(k_2, \phi)) \Leftrightarrow \mathsf{ist}(k_2, \phi)$$

In the theory of viewpoints, since one of the goals is modeling belief contexts, the perspective of observation is important: the fact that something holds in a viewpoint does not imply that "the fact that it holds" is true in any other viewpoint. The property

$$\mathsf{in}(p, vp_1) \Rightarrow \mathsf{in}(\mathsf{in}(p, vp_1), vp_2)$$

is not desired and is not valid. The semantics of in is not entailment but rather *contextual entailment*, meaning that the viewpoint expression should be resolved (interpreted) with respect to the context where it appears, rather than absolutely. In the above formula the viewpoint expression vp_1 in the antecedent might denote a different set of assumptions from the set of assumptions denoted by the vp_1 in the consequent. Note however that:

$$\mathsf{in}(p, vp) \Rightarrow \mathsf{in}(\mathsf{in}(p, vp), vp)$$

is valid, since the *contextuality* of the entailment ensures that viewpoint expressions at different levels of nesting are interpreted coherently: the entailment is in fact restricted to a subclass of all models, those "coherent" with the current context in the interpretation of the viewpoint expressions.

As an additional consequence the rules for entering and exiting a context have to preserve the nesting of contexts, like argumented in [6] and like in the propositional version of the theory of contexts by Buvač and Mason [7, 8] but unlike in their quantified version [9].

The expressivity we require calls for implicitly defined viewpoints and a syntactic treatment of in, so that self referential and mutually referential viewpoints can be defined.

The ability to characterize contexts implicitly through assertions is recognized as important; in many real applications contexts cannot be characterized explicitly by listing a set of sentences. In the terminology of McCarthy contexts are *rich* objects [14].

We allow implicit characterization of viewpoints by giving the user the possibility to express the rules for the derivations of facts of type $\mathsf{in}(p, vp)$. Examples of such assertions are *lifting rules* [14], which relate two different viewpoints; for example it is possible to state that whenever a formula satisfying some condition holds in a viewpoint vp_1 then a related formula holds in viewpoint vp_2; as a special case, we can state that vp_1 subsumes vp_2:

$$\forall x \ \mathsf{in}(x, vp_1) \Rightarrow \mathsf{in}(x, vp_2)$$

This allows for compact statements of problems and leaves to the logic machinery the burden of incrementally specifying viewpoints when the rules for their construction are known. Note that the possibility of quantification over sentences inside the **in** predicate, not provided by modal logics, is essential; hence the choice of a syntactic treatment of **in**.

More important is the following, since it justifies the choice of a reflective theory, as opposed to a stratification of theories as in [21, 12], despite the troubles created by paradoxes: there are facts that simply cannot be asserted without the use of self referential viewpoints. Here are some examples.

"John believes that he has a false belief" [17]

$$\mathsf{in}(\exists x \ \mathsf{in}(x, vp(John)) \land False(x), vp(John))$$

"Agent a believes that whatever he and agent b believe is true, while b does not believe so"

$$\mathsf{in}(\forall x \ \mathsf{in}(x, vp(a)) \lor \mathsf{in}(x, vp(b)) \Rightarrow True(x), vp(a))$$

$$\mathsf{in}(\neg\forall x \ \mathsf{in}(x, vp(a)) \lor \mathsf{in}(x, vp(b)) \Rightarrow True(x), vp(b))$$

"Agent a and agent b have *common knowledge* (or belief) that p"

$$\mathsf{in}(p, CK) \qquad (CK\text{-}1)$$

$$\forall x \ \mathsf{in}(x, CK) \Rightarrow \mathsf{in}(x, vp(a)) \land \mathsf{in}(x, vp(b)) \qquad (CK\text{-}2)$$

$$\forall x \ \mathsf{in}(x, CK) \Rightarrow \mathsf{in}(\mathsf{in}(x, vp(a)) \land \mathsf{in}(x, vp(b)), CK) \qquad (CK\text{-}3)$$

Note that for common knowledge it is not enough that both agents know p but it is also required that they know that they know p, that they know that they know that they know p, ...and so on.

The possibility of dynamic construction of viewpoints (for example through lifting axioms or in particular "auto-lifting" axioms such as *CK-3*) and of expressing self referential viewpoints leads to non finite viewpoints.

2 An example

We will illustrate with an example several features of the theory of viewpoints and the kind of reasoning we would like to be able to perform.

Beppe (Giuseppe Attardi) told Maria one morning about the following dream.

You and me are travelling by train, and you have both our tickets. I go to the toilet. After I come back the ticket inspector passes by and asks for unchecked tickets.

I do nothing, since I know that you have my ticket. You do nothing since you have already shown both tickets to the inspector while I was away. The inspector does not remember seeing my ticket and therefore he asks me for it. At this point I ask you why you do not show the ticket to the inspector.

The following notation will be used in the formalization.

B:	Beppe
M:	Maria
$vp(B,t)$:	Beppe's viewpoint at time t
$vp(M,t)$:	Maria's viewpoint at time t
$vp(C,t)$:	the ticket inspector's viewpoint at time t
$CK(x,t)$:	common knowledge of the set of agents denoted by x at time t; in the example we will use *All* for everybody, *BM* for Beppe and Maria and *MC* for Maria and the inspector;
$Ticket(x)$:	x's ticket;
$Has(x,y)$:	x holds y;
$Shows(x,y,t)$:	x shows ticket y to the inspector at time t;
$Checked(x)$:	ticket x has been checked.

Common knowledge of a set of agents plays a significant role in this example and we express it as one theory for lifting axioms provided for each involved agent to access it. A general formulation of common knowledge which will serve the purpose of the example, is the following.

Axioms for common knowledge:

(1) $\forall x,y,z,t \ \mathsf{in}(x, CK(y,t)) \wedge z \in y \ \Rightarrow \mathsf{in}(x, vp(z,t))$

(2) $\forall x,y,z,t \ \mathsf{in}(x, CK(y,t)) \wedge z \in y \ \Rightarrow \mathsf{in}(\mathsf{in}(x, vp(z,t)), CK(y,t))$

to be used in conjunction with the following:

(3) $All = \{M, B, C\}$

(4) $BM = \{B, M\}$, $MC = \{M, C\}$

Axioms (1) and (2) provide a proper account of common knowledge to a group of agents, allowing to derive the commonly known facts in any viewpoint, no matter how nested. In particular axiom (2) is used to achieve the appropriate level of nesting in CK, and axiom (1) to lift from the CK viewpoint at time t to any other relevant viewpoint at time t.

For example assuming:

(a) $\forall t \ \mathsf{in}(Has(M, Ticket(B)), CK(BM,t))$

it is possible to derive

$$\mathsf{in}(\mathsf{in}(Has(M, Ticket(B)), vp(M, t_3)), vp(B, t_3))$$

which will appear in the proof as line (33), with the following steps:

(b) $\mathsf{in}(Has(M, Ticket(B)), CK(BM, t_3))$ (a)

(c) $\mathsf{in}(\mathsf{in}(Has(M, Ticket(B)), vp(M, t_3)), CK(BM, t_3))$ (2, 4, b)

(d) $\mathsf{in}(\mathsf{in}(Has(M, Ticket(B)), vp(M, t_3)), vp(B, t_3))$ (1, 4, c)

To be more concise, we will skip similar derivation steps from common knowledge from now on. The specific knowledge of the problem can be expressed as follows:

(5) $\forall t\ \mathsf{in}(Has(M, Ticket(B)) \wedge Has(M, Ticket(M)), CK(BM, t))$
Both Beppe and Maria know that Maria has both tickets, at any time;

(6) $\forall t\ \mathsf{in}(Has(x, y) \wedge x \neq z \Rightarrow \neg Has(z, y), CK(All, t))$
Corresponds to the common sense knowledge that only one person can hold a given object.

(7-9) are the axioms which describe Maria's act of showing the tickets to the inspector and its effect. They will be introduced later since they deserve some discussion.

The first request from the inspector, at time t_1, becomes part of common knowledge in the following form:

(10) $\forall t \geq t_1\ \mathsf{in}(\neg Checked(Ticket(y)) \wedge Has(x, Ticket(y)) \Leftrightarrow Shows(x, Ticket(y), t_1), CK(All, t))$

This is the reasoning in Beppe's viewpoint, at time t_1 (i.e. in viewpoint $vp(B, t_1)$):

(11) $Has(M, Ticket(B))$ (5)
Maria has my ticket;

(12) $\neg Has(B, Ticket(B))$ (11, 6)
I do not have my ticket;

(13) $Checked(Ticket(B)) \vee \neg Has(B, Ticket(B)) \Rightarrow \neg Shows(B, Ticket(B), t_1)$ (10)

(14) $\neg Shows(B, Ticket(B), t_1)$ (12, 13)

Therefore Beppe does nothing. Maria reasons as follows at the same time (i.e. viewpoint $vp(M, t_1)$). We assume the following lemma, corresponding to the fact that she knows that her ticket and Beppe's have been checked by the inspector.

Lemma 1 $\forall t > t_0\ \mathsf{in}(Checked(Ticket(B)) \wedge Checked(Ticket(M)), vp(M, t))$

(15) $Checked(Ticket(B)) \vee \neg Has(M, Ticket(B)) \Rightarrow \neg Shows(M, Ticket(B), t_1)$ (10)

(16) $\neg Shows(M, Ticket(B), t_1)$ (lemma 1, 15)

(17) $Checked(Ticket(M)) \vee \neg Has(M, Ticket(M)) \Rightarrow \neg Shows(M, Ticket(M), t_1)$ (10)

(18) $\neg Shows(M, Ticket(M), t_1)$ (lemma 1, 17)

Therefore also Maria has good reasons for doing nothing. After this, at time t_2, the common knowledge is enriched as follows (nobody shows any ticket):

(19) $\forall t \geq t_2\ \mathsf{in}(\forall x, y\ \neg Shows(x, y, t_1), CK(All, t))$

Moreover we have the fact that the inspector doesn't remember seeing Beppe's ticket, which is part of the statement of the problem; this however is not common knowledge but it is asserted outside any viewpoint.

(20) $\neg\mathsf{in}(Checked(Ticket(B)), vp(C, t_2))$

Reasoning about the inspector:

(21) $\mathsf{in}(\forall x\ \neg Shows(x, Ticket(B), t_1), vp(C, t_2))$ (19)
the inspector notices, as anybody, that nobody shows Beppe's ticket at time t_1;

(22) $\mathsf{in}(Checked(Ticket(B)) \vee \forall x\ \neg Has(x, Ticket(B)), vp(C, t_2))$ (10, 21)
he reasons that either the ticket has been checked or nobody has Beppe's ticket.

(23) $\mathsf{in}(\exists x\ Has(x, Ticket(B)), vp(C, t_2))$ (Assumption)
Assume the inspector believes that somebody has Beppe's the ticket;

(24) $\mathsf{in}(Checked(Ticket(B)), vp(C, t_2))$ (22, 23)
he should at least believe that Beppe's ticket has been checked; but he doesn't by (20), therefore:

(25) $\neg\mathsf{in}(\exists x\ Has(x, Ticket(B)), vp(C, t_2))$ (23, 24, 20)
the inspector cannot deduce that somebody has Beppe's ticket.

This same reasoning can be performed by the inspector himself, provided we have the additional and reasonable assumption that he is aware of the fact that he doesn't remember, leading to:

(26) $\mathsf{in}(\neg\mathsf{in}(\exists x\ Has(x, Ticket(B)), vp(C, t_2)), vp(C, t_2))$
a realization by the inspector of the fact that he cannot tell whether somebody has Beppe's ticket.

As a consequence he asks Beppe to show his ticket. It is now common knowledge, at time t_3, that the inspector does not know whether somebody has Beppe's ticket.

(27) $\mathsf{in}(\neg\mathsf{in}(\exists x\ Has(x, Ticket(B)), vp(C, t_3)), CK(All, t_3))$

Beppe now, at time t_3, reasons as follows (viewpoint $vp(B, t_3)$):

(28) $\neg\mathsf{in}(\exists x\ Has(x, Ticket(B)), vp(C, t_3))$ (27)
The inspector does not know whether somebody has my ticket;

(29) $\mathsf{in}(Checked(Ticket(B)), vp(C, t_3))$ (Assumption)
Assume the inspector thinks that he has seen my ticket;

(30) $\mathsf{in}(\exists x\ Has(x, Ticket(B)), vp(C, t_3))$ (10, 29)
He should also believe that somebody has my ticket, but he doesn't by (29);

(31) $\neg\mathsf{in}(Checked(Ticket(B)), vp(C, t_3))$ (29, 30, 28)
therefore he does not believe that my ticket has been checked;

(32) $\mathsf{in}(\neg Shows(M, Ticket(B), t_1), vp(M, t_3))$ (19)
Maria knows that she did not show my ticket when asked;

(33) $\mathsf{in}(Has(M, Ticket(B)), vp(M, t_3))$ (5)
Maria knows that she has the ticket;

(34) $\mathsf{in}(\neg Checked(Ticket(B)) \wedge Has(M, Ticket(B)) \Leftrightarrow Shows(M, Ticket(B), t_1), vp(M, t_3))$ (10)
Maria is aware of what the inspector said;

(35) $\mathsf{in}(Checked(Ticket(B)), vp(M, t_3))$ (32, 33, 34)
Maria believes that my ticket has been checked.

Therefore Beppe notices a difference between the inspector's viewpoint and Maria's viewpoint (lines 31 and 35); that's why he asks Maria why she doesn't show his ticket.

Finally, solicited by the request of the inspector and, further, from Beppe himself, Maria reasons as follows. In Maria's reasoning we assume the following lemma, corresponding to the fact that she knows that the inspector is aware of having checked her ticket and a second one, referred to as *ticket*2.

Lemma 2 $\forall t > t_0\ \mathsf{in}(\mathsf{in}(Checked(Ticket(M)) \wedge Checked(ticket2), vp(C, t)), vp(M, t))$

(36) $\neg\mathsf{in}(Checked(Ticket(B)), vp(C, t_3))$
The inspector is not aware of the fact that he checked Beppe's ticket
(similar to Beppe's reasoning above).

(37) $\mathsf{in}(Checked(ticket2, vp(C, t_3))$ (lemma 2)
but he certainly remembers having checked a second ticket;

(38) $\neg\mathsf{in}(ticket2 = Ticket(B), vp(C, t_3))$ (36, 37)
he doesn't realize that the second ticket is Beppe's one
(otherwise there would be a contradiction).

So she decides that in order to resolve the question she must tell the inspector that the second ticket is Beppe's one. This goes a little beyond Beppe's dream, but it looks like a plausible reason of the fact that the inspector does not remember: Maria, showing both tickets Beppe being absent could not refer to Beppe as the owner of the second ticket.

In the following section we will discuss how knowledge corresponding to the two lemmas used in the proof can be acquired.

3 Sharing of referents in communication

In the reasoning of the inspector and of Maria, they both assume knowing certain information about each other. Each one should have obtained such information through the transaction that happened between them when Maria showed her tickets to the inspector. The transaction should be modeled in such a way that some information flows from one agent to the other and that both agents become aware not only of the transaction but also that the other is aware of the information conveyed in the transaction.

The first problem is about terms and denotations: i.e. which constants should be used in the formalisation of the problem. For instance, in the interaction between Maria and the inspector while Beppe is away, how can Maria refer to Beppe? In our solution we decided that she can't refer to Beppe directly, since she has no way to ensure that her reference to Beppe will be the same as the one of the inspector.

The only things to which she can refer in her interaction with the inspector are what we call "manifest constants", i.e. constants to which she can point directly (for instance because they are objects in the scene), or indirectly through terms built from other manifest constants (for instance *Husband-of(M)*, or *Person-sitting(there)*).

We require that any statement expressing common knowledge between two agents only uses manifest constants: we call this the "principle of referent sharing".

Therefore in the example, the transaction between Maria and the inspector was expressed by means of the manifest constant *ticket2* which refers to the actual ticket that Maria hands over to the inspector.

We discuss later the approach by Guha-McCarthy where the same term is allowed different denotations in different contexts.

Another problem in formalising the information flow is that one must avoid that also unintended information be transferred between viewpoints.

For example, one might express as follows the fact that once Maria has shown her tickets, the inspector checks them and thereafter they both know that they have been checked:

(7) $\forall t > t_0 \; \mathsf{in}(Shows(M, Ticket(M), t_0) \land Shows(M, ticket2, t_0), vp(M, t))$
Maria knows that she has shown both tickets at time t_0;

(8) $\forall t > t_0 \; \mathsf{in}(Ticket(B) = ticket2, vp(M, t))$
Maria knows that the second ticket is Beppe's;

(9) $\mathsf{in}(\forall t'' > t' \; Shows(M, y, t') \Rightarrow \mathsf{in}(Checked(y), CK(MC, t'')), CK(All, t))$
It is common knowledge that after Maria has shown a ticket, she and the inspector know that the ticket has been checked.

However this solution has a flaw: applying the law of substitutivity in Maria's viewpoint, Maria knows that $Shows(M, ticket2, t_0)$ and also $Shows(M, Ticket(B), t_0)$ and therefore she can conclude, for instance, that $\mathsf{in}(Checked(Ticket(B)), vp(C, t_3))$. The effect is therefore that she unduely transfers her information about $ticket2$ to the viewpoint of the inspector.

The solution is then to separate the information flow from the conclusion that each agent is able to draw from the information gathered.

First, we state that showing the ticket is common knowledge of both Maria and the inspector. This is the information that is actually trasmitted.

(7') $\forall t > t_0 \; \mathsf{in}(Shows(M, Ticket(M), t_0) \wedge Shows(M, ticket2, t_0), CK(MC, t))$
Maria and the inspector know that she has shown both tickets at time t_0; while the first ticket can be referred to as Maria's ticket the second one is simply "a second ticket" in the common knowledge viewpoint;

(8) $\forall t > t_0 \; \mathsf{in}(Ticket(B) = ticket2, vp(M, t))$
Maria knows that the second ticket is Beppe's (while the inspector doesn't);

The first statement uses $ticket2$ and cannot use $Ticket(B)$ since this is not a manifest constant and is therefore forbidden by the principle of referent sharing in a statement on common knowledge.

And now we allow each agent to draw his own conclusion about which tickets have been checked:

(9') $\forall t' > t \; \mathsf{in}(\forall y \; Shows(M, y, t) \Rightarrow Checked(y), CK(MC, t'))$
It is common knowledge of Maria and the inspector that after Maria has shown a ticket, the ticket has been checked.

Using these statements, we can easily prove the two lemmas and, in particular, the following facts that Maria used in her reasoning:

$$Checked(Ticket(B)) \wedge Checked(Ticket(M)) \qquad (7', 8, 9')$$

$$\mathsf{in}(Checked(ticket2, vp(C, t_3)) \qquad (7', 9')$$

while it is not the case that the inspector, nor Maria reasoning about the inspector, can deduce:

$$Checked(Ticket(B))$$

Therefore Maria is forced to provide additional information.

4 Important points about this example

This example requires a formal account of what appears a reasonable behaviour for all the agents, still the reasoning is complex enough to offer an interesting case study. In the following we will discuss additional aspects of the proposed formalization.

4.1 Viewpoint functions

We have several agents (Maria, Beppe, the inspector) and several times corresponding to situations with different knowledge involved. We use $vp(a, t)$, a viewpoint function denoting the viewpoint of agent a at time t; the example shows the importance of viewpoint functions with variables quantified in; this way it is possible to express, for example, that a certain fact holds from a certain time on, or, if necessary, that there is a time when a fact holds.

4.2 Propagation of ignorance

The inspector does not remember having checked Beppe's ticket, and as a consequence he makes everybody aware of the fact that he does not know whether Beppe has a ticket, and as a further consequence Maria can realize that the inspector does not know that the second ticket he has seen is Beppe's ticket.

Note that in the example we do not deal with *deduction of ignorance* from the fact somebody cannot deduce something, a property which is in general undecidable in a first order setting (see [1, 10] for examples of this approach). In other words ignorance in not "introduced" but only "propagated".

All we do is to use a much more conservative and sound pattern of reasoning for propagation of ignorance, namely: if we have $\neg\mathsf{in}(p, vp)$ and we want to prove $\neg\mathsf{in}(q, vp)$, we can assume $\mathsf{in}(q, vp)$ and try to prove $\mathsf{in}(p, vp)$; if this succeeds, we have succeeded in proving $\neg\mathsf{in}(q, vp)$.

In other words the following is a sound rule of inference:

$$\frac{\neg\mathsf{in}(p, vp), \mathsf{in}(q \Rightarrow p, vp)}{\neg\mathsf{in}(q, vp)} \qquad \text{(\textit{Ignorance propagation})}$$

4.3 Different denotations for terms

The example shows that limited forms of "different denotations" for terms can be dealt with by different perspectives and having different viewpoints account differently for coreferentiality of terms.

Here are other examples, taken from McCarthy [14] where resorting to context dependent languages with different denotations is not required.

The expression

$$\mathsf{in}(at(jmc, Stanford), c_1)$$

is given the meaning that "John McCarthy is regularly at Stanford University" provided the following also holds:

$$\mathsf{in}(jmc = JohnMcCarthy, c_1)$$

$$\mathsf{in}(Stanford = StanfordUniversity, c_1)$$

$$\mathsf{in}(at(x, y) \Leftrightarrow RegularlyPresentAt(x, y), c_1)$$

but could assume a different meaning, temporary presence, in a different context, say c_2, with the following axiom:

$$\mathsf{in}(at(x, y) \Leftrightarrow PresentAt(x, y, t), c_2(t))$$

As an example of a constant whose denotation should be different in different contexts, the term *Now* is often mentioned in the literature. A general property for *Now* could be the following:

$$\mathsf{in}(Now = t, vp(a, t))$$

Committing to the choice of contexts with different languages is not without problems: most of Guha's thesis [13] is devoted to overcome language restrictions and to write appropriate rules for stating equivalence, at least by default, of equal expressions in different contexts; the quantified version of Buvač formalization of contexts drops the language restriction in favour of a formal treatment. Given these difficulties one would like to see convincing examples where the dependency of language from the context is really needed.

4.4 Viewpoint consistency is an option

In the example we do not deal with nonmonotonic aspects such as changes of mind, assumptions by default, resolving contradictions, and so on. In this problem once a piece of knowledge is acquired in a viewpoint it is also assumed to hold in successive viewpoints in time.

Dealing with nonmonotonic aspects would require writing problem dependent axioms stating what changes and what doesn't from one viewpoint to the next in time.

In this respect some of the ideas presented in connection with the step/active logics by Perlis and co-authors look promising [18, 15]. As an example we could exploit their approach to resolving contradictions, once they manifest explicitly, by failing to inherit to the next step contradicting assumptions.

This can be done since consistency of viewpoints is an option: the axiom $\neg\mathsf{in}(false, vp)$ does not hold in general; a viewpoint can become inconsistent without affecting other viewpoints nor the global reasoning context. In the presence of contradictory viewpoints one can reason about them and adopt the best strategy available to create new consistent viewpoints.

5 Implementation of viewpoints

The implementation of the Omega description logic system [3] provided a viewpoints mechanism which emphasised a hierarchical organization of viewpoints. Viewpoints, as the other descriptions of the Omega system, could be arranged in a lattice, where a viewpoint inherited from another by including all the sentences belonging to it. Thereafter a viewpoint would inherit all the logical consequences from its ancestor viewpoints. This proved useful for instance to create viewpoints describing a basic theory (e.g. natural deduction), from which more specific viewpoints could be created by adding new statements.

A new implementation of the theory of viewpoints is in progress using the Coq Proof Assistant which supports the development of higher order logics [11]. The logical language used by Coq is a variety of type theory, called the *Calculus of Inductive Constructions.*

The λ-calculus notation can be used as a uniform encoding for espressions, assertions and proofs. Type checking rules enforce well-formedness conditions. According to the Curry-Howard isomorphism, assertions are represented as types and proofs are represented as terms whose type is the formula they prove. Proving a formula is therefore seen as exhibiting a term of a given type (proving constructively that the type is populated).

For defining a new logic system one has to provide:

a definition of wff's: by defining a signature for terms and formulas;

a definition of the logical axioms: asserting the existence of constants, whose type is the generic formula corresponding to the axiom schema;

a definition of the inference rules: asserting the existence of constants, of functional type, mapping the premises into the conclusions;

Using the higher order features of the language the naming device used for reifying sentences at the metalevel and for the formulation of the reflection rules is greatly semplified: sentences are represented as terms at any level.

An important task will be to prove that the presentation of the logic (by means of a specific signature) is *adequate* in the sense that the encoding is a compositional bijection between the syntactic entities (terms, formulas, proofs) of the logical system and certain valid λ-terms in the signature. In particular we will have to prove that the higher order does not introduces any additional theorems.

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