Front Propagation: A Framework for Topology Independent Shape Modeling and Recovery

R. Malladi,[1] J. A. Sethian,[2] and B. C. Vemuri[1]

[1]Department of Computer & Information Sciences
University of Florida, Gainesville, FL 32611
[2]Department of Mathematics
University of California, Berkeley, CA 94720

Introduction

.mportant goal of computational vision is to recover ;hapes of objects in 2D and 3D from various types of al data. One way to achieve this goal is via model- :d techniques. Broadly speaking, these techniques, ıe name suggests, involve the use of a model whose ndary representation is matched to the image to re- r the object of interest. These models can either be l or nonrigid. In the former case, we have what are ılarly known as correlation-based template match- .echniques, while the later involves a dynamic model ıg process to the data. In this paper we present a :l framework for shape modeling and shape recov- based on ideas developed by Osher & Sethian for t (solid-liquid boundary or boundary between burnt unburnt regions) propagation. In this framework, ›es are modeled by level sets of implicit functions are evolving over time. The dynamics of evolution ›verned by front propagation equations. This dy- ic shape modeling scheme retains the most attrac- features of existing methods, and overcomes some eir prominent limitations. For the rest of this paper /ill use the word shape model to mean a boundary 'ace) representation of an object shape.

ıape recovery typically precedes the symbolic rep- ıtation of surfaces and shape models are expected id the recovery of detailed structure from noisy using only the weakest of the possible assumptions ıt the observed shape. To this end, several varia- ıl shape recovery methods have been proposed and ; is abundant literature on the same [2]. General- spline models with continuity constraints are well ·d for fulfilling the goals of shape recovery. Gen- zed splines are the key ingredient of the dynamic e modeling paradigm introduced by Terzopoulos ., [13]. Following the advent of the dynamic shape eling paradigm, there was a flurry of research ac- y in the area, with numerous application specific ifications to the modeling primitives, and external ·s derived from data constraints [4, 14].

›wever, the aforementioned schemes for shape modeling have two noticeable limitations – the dependence of the recovered surface shape on the initial guess used to start the numerical reconstruction procedure, and an assumption that the object is of a known topology. The first of these limitations stems from the fact that the nonconvex energy functionals used in the variational formulations have multiple local minima. As a consequence of this feature, the numerical procedures, for convergence to a satisfactory solution require an initial guess which is "reasonably" close to the desired solution (shape). Also, existing shape representation schemes lack the ability to dynamically sense the topological changes that may occur in the shape of interest (e.g., cell division) over time. In this paper, we present a modeling scheme that makes no assumption about the object's topology, and leads to a numerical algorithm whose convergence to the desired shape is independent of the shape initialization as long as the initialization is either completely enclosed by the shape of interest or vice-versa. Our method can also recover shapes whose topology changes over time, e.g., the cell boundary in cell division.

The framework of energy minimization ("snakes") has also been used successfully in the past for extracting salient image contours such as edges and lines by Kass *et al.* [4]. In automatic segmentation of images, snakes may perform poorly unless they are placed close to the preferred shapes. In a move to make the final result relatively insensitive to the initial conditions, Cohen [3] defines an inflation force on the active contour which makes it behave like an inflating balloon. The modified contour model will be "stopped" by a strong edge and will simply pass through a spurious edge since the force exerted by it is too weak relative to the ambient inflation force. Also, the smoothing property of the snake namely, the curvature minimizing component of the stabilizer energy does not allow formation of corners thus causing the snake to snap out from the grips of the spurious edge caused by noise.

Although the inflation force prevents the curve from getting "trapped" by isolated spurious edges, the active contour model has difficulty in segmenting complex shapes like the one shown in figure 2(a). Moreover, despite a good initialization, the active contour model, due to its arc-length and curvature minimization properties, cannot be forced to extrude through any significant protrusions that a shape may possess. So the problem is one of *accurately modeling bifurcations and protrusions in complex structures.* We propose a dynamic shape modeling method that will start with a single instance of the model and will automatically "*sprout*" branches during the evolutionary process. Thus, in our framework of modeling, objects are perceived as being made of a single primitive (front) which can evolve into complicated shapes rather than the view of different parts being glued together to form a complicated object.

Most existing surface modeling techniques require that the topology of the object be known before the shape recovery can commence. However, it is not always possible to specify the topology of an object prior to its recovery. As a result, most existing shape recovery schemes in vision literature make strong assumptions about the object topology. Unknown topology is an important concern in object tracking and motion detection applications where the positions of object boundaries are tracked in a time sequence of images. During their evolution, these closed contours may change connectivity and split, thereby undergoing a topological transformation (eg., in cell division). A heuristic criterion for splitting and merging of curves in 2D which is based on monitoring deformation energies of points on the elastic curve has been discussed in [9].

In the context of static problems, more recently, particle systems have been used to model surfaces of arbitrary topology [12]. Smoothness and continuity constraints are imposed by subjecting the particle system to interaction potentials which locally prefer planar or spherical arrangement. Particles can be added and deleted dynamically to enlarge and trim the surface respectively, while the system dynamics strive continually to organize the particles into smooth shapes. The result is a versatile method with applications in surface fitting to sparse data and 3D medical image segmentation.

The framework described in this paper can be applied to static as well as dynamic situations, where no *a priori* assumption about the object's topology is made. A single instance of our model, when presented with an image having more than one shape of interest, has the ability to split freely to represent each shape [6, 7]. We have shown that by using our approach, it is also possible to extract the bounding contours of *shapes with holes* in a seamless fashion [7]. Shape recovery from 3D images is also possible as shown in [7].

Outline of the Solution

In this subsection we briefly outline our shape modeling and recovery scheme. This method is inspired by ideas first introduced in Osher & Sethian [10, 8, 11] to model propagating fronts with curvature-dependent speeds. Two such examples are flame propagation and crystal growth, in which the speed of the moving interface normal to itself depends on transport terms modified by the local curvature. The challenge in these problems is to devise numerical schemes for the equations of the propagating front which will accurately approximate these highly unstable physical phenomena. In [10] it was shown that direct parameterization of the moving front may be unstable since it relies on local properties of the solution. In contrast, a method which preserves the global properties of the motion is sought. Osher & Sethian [8] achieve this by viewing the propagating surface as a specific level set of a higher-dimensional function. The equation of motion for this function is reminiscent of an initial valued "Hamilton-Jacobi" equation with a parabolic right-hand side and is closely related to a viscous hyperbolic conservation law.

In our work, we adopt these level set techniques to the problem of shape recovery. To isolate a shape from its background, we first consider a closed, nonintersecting, initial hypersurface placed inside (or outside) it. Following the above level set approach, this hypersurface is then made to *flow* along its gradient field with a speed $F(K)$, where K is the curvature of the hypersurface. As in [8, 10, 11], we adopt a global approach and view the $(N-1)$ dimensional moving surface *as a level set of a time-dependent function* ψ of N space dimensions. The equations of motion written for this higher dimensional function are then amenable to stable *entropy-satisfying* numerical schemes designed to approximate hyperbolic conservation laws. Topological changes can be handled naturally in this approach, since a particular level set $\{\psi = 0\}$ of the function ψ need not be simply connected. However, there are two problems that need to be surmounted before we can use this design for shape recovery. First, it is required that we stop the hypersurface in the neighborhood of the desired shape. We do this by synthesizing a *speed term* from the image data. Secondly, we have to construct an *extension* of this speed function to other level sets $\{\psi = C\}$ in order to give a consistent meaning to the image-based speed term at all points in the image. In the following sections we outline a possible solution to these problems.

We note that this work on interface motion and hyperbolic conservation laws as discussed in [8], has been applied in the area of computer vision for shape characterization by Kimia *et al.* [5], who unify many diverse aspects of shape by defining a continuum of shapes (reaction/diffusion space), which places shapes within a neighborhood of other similar shapes. This leads to a hierarchical description of a shape which is suitable for its recognition. The key distinguishing feature of our work from that of Kimia *et al.*, is that they assume the boundary of the object shape to be known, while we recover it from image data. In other words, they show that by *evolving a known shape boundary*, explicit clues can be derived towards the goal of developing a hierarchical shape description. In contrast, *we start with an arbitrary function* ψ *and recover complex shapes* by

pagating it along its gradient field.
[he dynamics of the front is given by,

$$\psi_t + (F_A + \hat{F}_I) \mid \nabla\psi \mid = 0. \quad (1)$$

iere, F_A is referred to as the advection term and is a ed of the front expansion or contraction analogous the inflation force on snakes in [3]. F_I is the speed n synthesized from image data and is made to equal he negative of F_A near large image gradients thereby ing the propagating front to stop near edges in an ge. $\hat{F}_I$ is called an *extension* of F_I to points away n the boundary $\gamma(t)$, i.e. at points $(x, y) \in (\Omega -$)), and is equal to F_I on $\gamma(t)$. We introduce a fast nerical technique for solving this governing dynamic ation of the propagating front. Our new algorithm loits the fact that the front, which is a particular l set $\{\psi = 0\}$ of a higher dimensional function, can dvanced by updating the function ψ at a small set of its. This scheme is an alternative to updating the ψ tion at all the points in the computational domain The set of points at which the update procedure pplied belong to a narrow band lying on either side he level set $\{\psi = 0\}$. Since the *narrow-band* update tegy involves only a fraction of the total number of its, a significant saving in time is realized, making method a very attractive alternative to other shape very schemes. A complete discussion of the narrow-d techniques for interface propagation may be found l].

summary, we present a novel scheme for shape leling which can be used in both computer vision pe recovery from images) and computer graphics pe synthesis) applications. Given the reconstructed e, our approach can also be extended to decipher constituent part structure for high-level processing We now present two example applications of shape very with the propagating front in medical image . In figure 1, we depict the front initialization in- the shape of interest. Note that the front must be alized either completely inside or outside the shape terest. In this example, the choice of inside initial- on is obvious since the outside contains other mean- ul boundaries belonging to shapes that are not of iary interest. Narrow-band computation was done 128 × 128 grid – the front was made to propagate speed $F = \hat{k}_I(-1.0 - 0.025K)$ and the time step vas set to 0.0005. ψ was recomputed once every ime steps. Note that the final recovered shape in vas obtained after the front overshot all the spuri- edges present inside the shape of interest (see (b)). feature is a consequence of the ϵK component in speed which diffuses regions of high curvature on front and forces it to attain a smooth shape.

the second experiment we recover the complicated cture of the branch of an arterial tree. The real ge has been obtained by clipping a portion of a al subtraction angiogram. This is an example of ape with extended branches or significant protrusions. In this experiment we compare the performance of our scheme with the active contour model ('snakes') to bring the limitations of the later into focus. In an attempt to recover the arterial tree, we tried three distinct 'snake' initializations, one closer to the final shape than the preceding. In figure 2(a), we show one such snake initialization. In all three cases, despite good initializations, the snake model is unable to recover arterial shape (see figure 2(b)). This is due to the existence of multiple local minima in the (nonconvex) energy functional which the numerical procedure explicitly minimizes. Also, the snake prefers regular shapes because shapes with protrusions have very high deformation energies. We now apply our level set algorithm to reconstruct the same shape. After initialization in figure 2(d), the front is made to propagate in the normal direction. We employ the narrow-band updation scheme with a band width of $\delta = 0.075$ to move the front. It can be seen that the front literally "flows" into the branches and 2(i) completely recovers the complex shape. The advantages of our scheme over the 'snake' model are quite apparent from this example. Since our front advancement process does not involve optimization of any quantity, the shape recovery results we obtain do not require an initialization close to the final desired shape. In addition, a *single instance of our shape model* "sprouts" branches and recovers all the connected components of a given shape.

Concluding remarks

In this paper we presented a new shape modeling scheme. Our approach while retaining the desirable features of existing methods for shape modeling, overcomes some of their deficiencies. We adopt the level set based techniques – of Osher & Sethian [8] for front propagation – to the problem of shape recovery. In this approach, unlike in the 'snakes' method, a good shape initialization is not needed for shape recovery so long as the initialization is either enclosed by the shape of interest or vice-versa. Moreover, our scheme makes no *a priori* assumption about the object's topology. Other salient features of our shape modeling scheme include its ability to split and merge freely without any additional bookkeeping during the evolutionary process, and its easy extensibility to higher dimensions. Our future research efforts will be focussed on extending this method to other application domains.

References

[1] D. Adalsteinsson and J. A. Sethian, "A narrow-band approach to level set techniques for propagating interfaces," LBL Report, Oct. 1993.

[2] R. M. Bolle and B. C. Vemuri, "On three-dimensional surface reconstruction methods," *IEEE Trans. on Pattern Analysis and Machine Intelligence*, vol. PAMI 13, No. 1, pp. 1–13, 1991.

[3] L. D. Cohen, "On Active Contour Models and Balloons," *Computer Vision, Graphics, and Image Processing*, Vol. 53, No. 2, pp. 211–218, March 1991.

[4] M. Kass, A. Witkin, and D. Terzopoulos, "Snakes: Active Contour Models," *International Journal of Computer Vision*, pp. 321-331, 1988.

[5] B. B. Kimia, A. R. Tannenbaum, and S. W. Zucker, "Shapes, shocks, and deformations I: The components of shape and reaction-diffusion space," to appear in *International Journal of Computer Vision*, 1992.

[6] R. Malladi, J. A. Sethian, and B. C. Vemuri, "A topology-independent shape modeling scheme," in *Proceedings of SPIE Conference on Geometric Methods in Computer Vision II*, San Diego, California, pp. 246–258, July 1993.

[7] R. Malladi, J. A. Sethian, and B. C. Vemuri, "Shape modeling with front propagation: A level set approach," Center for Pure and Applied Mathematics, Report PAM-589, Univ. of California, Berkeley, August 1993; also to appear in *IEEE Trans. on Pattern Analysis and Machine Intel.*.

[8] S. Osher and J. A. Sethian, "Fronts propagating with curvature dependent speed: Algorithms based on Hamilton-Jacobi formulation," *Journal of Computational Physics*, Vol. 79, pp. 12-49, 1988.

[9] R. Samadani, "Changes in connectivity in active contour models," *Proceedings of the Workshop on Visual Motion*, pp. 337–343, Irvine California, March 1989.

[10] J. A. Sethian, "Curvature and the evolution of fronts," *Commun. in Mathematical Physics*, Vol. 101, pp. 487–499, 1985.

[11] J. A. Sethian, "Numerical algorithms for propagating interfaces: Hamilton-Jacobi equations and conservation laws," *Journal of Differential Geometry*, Vol. 31, pp. 131-161, 1990.

[12] R. Szeliski and D. Tonnesen, "Surface modeling with oriented particle systems," *Computer Graphics SIGGRAPH*, Vol. 26, No. 2, pp. 185–194, July 1992.

[13] D. Terzopoulos, A. Witkin, and M. Kass, "Constraints on deformable models: Recovering 3D shape and nonrigid motion," *Artificial Intelligence*, 36, pp. 91–123, 1988.

[14] B. C. Vemuri and R. Malladi, "Constructing intrinsic parameters with active models for invariant surface reconstruction," *IEEE Trans. on Pattern Analysis and Machine Intelligence*, Vol. 15, No. 7, pp. 668-681, July 1993.

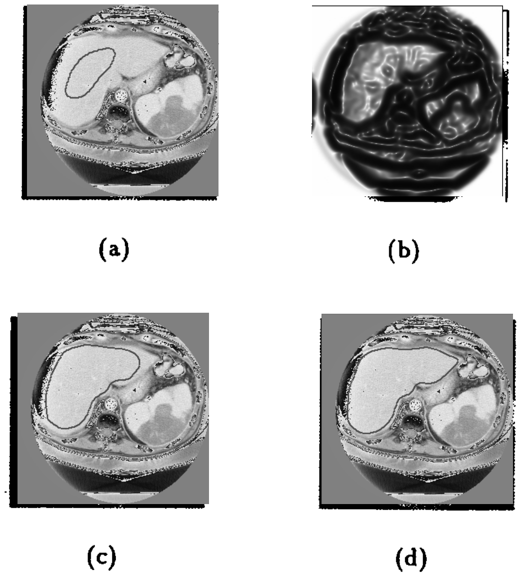

Figure 1: Recovery of the stomach shape from a CT image of an abdominal section. (a) Initialization, (b) image-based speed term, (c) shape after 450 iterations and (d) after 575 iterations.

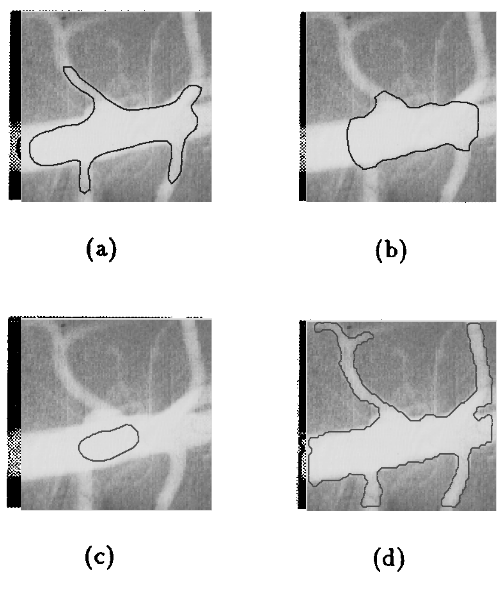

Figure 2: Reconstruction of a shape with "significant" protrusions: an arterial tree structure. An unsuccessful attempt to recover the shape using the 'snake' model initialized in (a) is shown in (b). (c) Initialized front and (d) recovered shape after 391 iterations.