Decision Schema and Game Theory

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Abstract

This paper suggests a practical model of the game player's decision schema. Decision schema can be characterized by the way a person processes information. Because cellular automata can be constructed with the same information processing characteristics as people's decision schema, cellular automata can be used for modeling the way people play games. The model will be demonstrated here by applying it to a simple Nim game. There were three outcomes: efficient players, quasi-efficient players and inefficient players.

Introduction

Game theorists have few practical models for determining how game players make decisions. The efficient player concept is often used, but a common criticism is that it doesn't actually represent how people make decisions. Decision schema (the way a person makes decisions) influence the way users interact with computer systems. These decision schema should also influence the way people play games. This paper suggests a practical model of the player's decision making process. The model will be demonstrated here by applying it to a simple Nim game.

Decision Schema

Decision schema can be characterized by the way a person processes information. These information processing characteristics include the mappings and relations used in set theory. Decision element mappings are: one-to-one, one-to-many, many-to-one, into and onto. Decision element relations are: reflexive, symmetrical, associative and transitive.

Previous research has shown that people's information processing characteristics are scalable (see below) and the consequences of their decisions can be predicted (McGrew, 1998). Information processing characteristics correlated $0.64, p < 0.10$ with the types of decision making techniques people used (such as fads, authority, politics, checklist, Likert scales, statistics, multidimensional scaling, clustering techniques and rule-based systems.) Decision schema based on information processing characteristics could also be used to predict a user's interaction with computers (McGrew, 1999).

Idiosyncratic decision schema: At one end of the scale are idiosyncratic decision schema (which have one-to-many, many-to-many and into mappings, and are irreflexive, asymmetrical, unassociative and intransitive). The decisions based on idiosyncratic decision techniques such as fads, authority or politics are unstable and either die out or cycle endlessly. 50% of the people in the study had idiosyncratic decision schema.

Data analytic decision schema: At the other end of the scale are data analytic decision schema (which have one-to-one, many-to-one and onto mappings, and are reflexive, symmetrical, anti-symmetrical, associative and transitive). The decisions based on data analytic techniques such as statistics, multidimensional scaling, clustering techniques and rule-based systems explore the decision space and are stable. 25% of the people in the study had data analytic decision schema.

Cellular Automata

Cellular automata are used for modeling a variety of dynamic systems (Chaudhuri, 1997; Gaylord and Wellin, 1995; Gaylord and Nishidate, 1996; Wolfram, 1994; and Nowak and May, 1992). Because cellular automata can be constructed with the same information processing characteristics as people's decision schema, cellular automata can be used for modeling the way people play games.

Method

The method followed three steps. Step 1: Form neighborhood rules for the cellular automata with the same information processing characteristics as decision schema. Step 2: Construct a lattice for a simple Nim game. Step 3: Evolve the cellular automata over the Nim game lattices.

Step 1. Neighborhood Rules for the Cellular Automata

In order to provide a simple, single-choice situation, an asymmetrical cellular automaton with a single neighbor was used. (With a neighborhood of two cells, a total of 16 possible cellular automata can be formed.) One cell represents the person's state of mind, and its neighbor cell represents a decision element. The first cell (representing the person's state of mind) contains a 1 for accepting state of mind and an 0 for a rejecting state of mind. The neighbor cell (representing the decision element) contains a 1 for present and a 0 for absent. A cell's state is updated after each iteration, becoming either a 1 or a 0, depending on the state of its neighbor and the update rules.

Two of the 16 possible neighborhood rules are illustrated below. The rules for determining a person's state of mind in single-choice behavior can be characterized by the mapping of X (the person's state of mind) and Y (the decision element) to X, shown below. In this mapping, the interaction of X and Y is updated according to the rules. If X is updated to 1, it indicates that the person has accepted the decision element. If the X is updated to 0, it indicates that the person has rejected the decision element. The updated lattice represents a historical view of the decision space after the person has acted on it.

One rule that can characterize idiosyncratic decision schema (asymmetrical, intransitive and unassociative) is:

X	Y→	X
1	1	0
1	0	0
0	1	1
0	0	0

One rule that can characterize data analytic decision schema (symmetrical, intransitive and associative) is:

X	Y→	X
1	1	1
1	0	1
0	1	1
0	0	0

Step 2. Construct a Lattice for a Simple Nim Game

An adjacency matrix was constructed to represent the tree (extensive form) of a simple Nim game (taken after Jones, 2000). This matrix was used as the decision space (lattice) of the cellular automata. (See Figures 1, 2, and 3.)

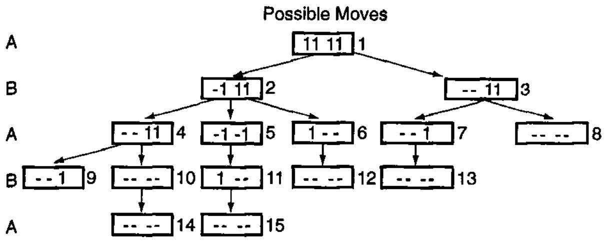

Figure 1. Extensive form game tree for the Nim game, with two players and two piles of two sticks each.

		Player	A	B	B	A	A	A	A	A	B	B	B	B	B	A	A
		Mover #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
		Pile State	11 11	1- 11	-- 11	-- 11	1-1-	1- --	- 1-	-- --	-- 1-	-- --	1- --	-- --	-- --	-- --	-- --
A	1	11 11		1	1												
B	2	1- 11				1	1	1									
B	3	-- 11							1	1							
A	4	-- 11									1	1					
A	5	1- 1-											1				
A	6	1- --												1			
A	7	-- 1-													1		
A	8	-- --								1							
B	9	-- 1-														1	
B	10	-- --										1					
B	11	1- --															1
B	12	-- --												1			
B	13	-- --													1		
A	14	-- --														1	
A	15	-- --															1

Figure 2. The adjacency matrix for an extensive form tree for the Nim game.

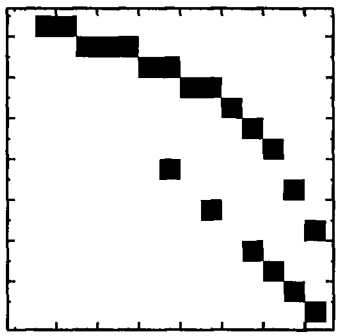

Figure 3. Density plot for the game tree adjacency matrix of the Nim game.

Step 3. Evolve the Cellular Automata over the Nim Game Lattices

The cellular automata were iterated 1, 2, 10 and 20 times.

Results

By the twentieth iteration, the output of the cellular automata had stopped changing. The interpretation of the results depends on whether or not the matrix structure of the game tree is maintained during the iterations. There were three outcomes, interpreted as follows: Efficient player: The game tree pattern stayed the same. Quasi-efficient player: The game tree pattern changed, but the original pattern was still contained within. Inefficient player: The game tree pattern was completely different.

In Figure 4, the top row shows an efficient player. the cellular automaton (rules for "while X") maintains the game tree pattern throughout the iterations. The bottom row shows a quasi-efficient player. The cellular automaton (rules for disjunction) contains the game tree pattern within the pattern it generated. Both patterns are consistent with data analytic decision schema.

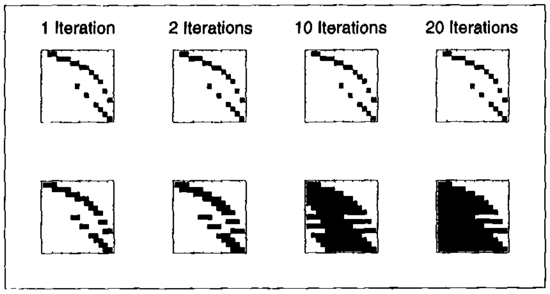

Figure 4. The cellular automata in the top row (efficient players) matched the game tree pattern throughout the iterations. The cellular automata in the bottom row (quasi-efficient players) contained the game tree pattern within the patterns it generated.

In Figure 5, all three rows are examples of inefficient players. The cellular automata here did not match the game tree pattern throughout the iterations. All three patterns are consistent with idiosyncratic decision schema.

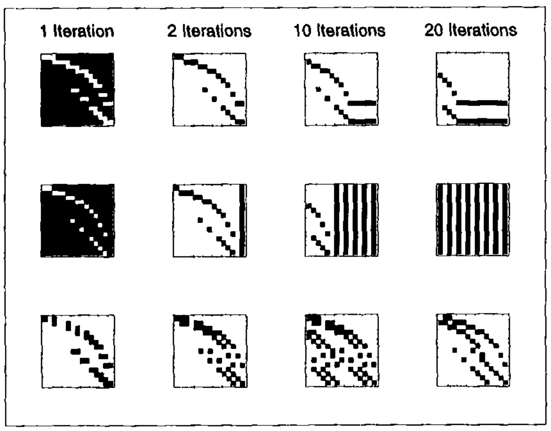

Figure 5. The cellular automata in all three rows (inefficient players) did not match the game tree pattern.

Summary and Conclusion

A person's decision schema can be characterized by set theoretic concepts. Set theoretic concepts can also be used to construct cellular automata rules. Cellular automata were used here to model a simple Nim game. The results appear to be consistent with the way people may play games. This paper introduces the concept of quasi-efficient and inefficient players, and suggests a practical game theory model that extends beyond the efficient player concept.

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