Game Theoretic Strategies in Decision Making Under Uncertainty

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Abstract

Two important classes of decision making problems, decision making under uncertainty and competitive decision making, game theory, are described and shown to be closely related. We use this relationship to draw upon a key concept used in game theory, the use of mixed strategies, and apply this idea to decision making under uncertainty.

Decision Making Framework

Decision making permeates all aspects of human activities. As our efforts grow in the use of intelligent agents to perform many of our functions on the internet the ability to provide agents with effective rational decision making capabilities become a paramount issue. The rich body of ideas on decision making emanating from the ideas described in [1] provides a rich source for the development of such capabilities. A useful framework for discussing decision making is captured by the matrix shown in figure #1.

$$\begin{array}{c} \\ A_1 \\ A_2 \\ \\ A_m \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & \quad S_n \end{array} \\ \left[\begin{array}{ccc} & & \\ & & \\ & C_{ij} & \\ & & \end{array} \right] \end{array}$$

Figure 1. Decision matrix

The A_i are a collection of alternative actions open to a decision maker, the $S_j \in \mathbf{S}$ are the possible values for some variable, denoted V, whose value affects the payoff received by the decision maker. Here C_{ij} is the payoff to the decision maker if he selects alternative A_i and $V = S_j$. The decision maker's goal is to select the alternative which gives him the highest payoff. In many situations the attainment of this goal is made difficult by the fact that the decision maker does not know the value of V at the time he must select his preferred alternative.

Two important special cases of the above can be differentiated by the process assumed to underlie the determination of the variable V. In the first case, called decision making under **uncertainty** (DMUU), it is assumed that the value of V, unknown at the time the decision maker must select his action, is ultimately generated by some *capricious* mechanism, normally called nature. In this case the variable V is often called the state of nature. An extreme case of decision making under uncertainty, is one in which the decision maker has no knowledge about the state of nature other than that lies in the set S, has been the given the name decision making under **ignorance** (DMUI). Here the decision maker, in order to make a decision, must act as if he knows the mechanism used by this capricious nature, he must assume a mechanism. In this situation the assumed mechanism can be seen to be a *reflection of the attitude of the decision maker regarding their view of nature.* One scale which can be used to express a decision maker's attitude regarding the mechanism used by nature is along a dimension of a benevolent and malevolent nature, with an indifferent nature being in the middle. This scale can be seen to be related to whether a decision maker is optimistic or pessimistic. Closely related to this is a reflection of the aggressiveness or conservativeness of the decision maker's nature. The notable observation here is that the "selection" mechanism attributed to this value generating capricious nature is a reflection of the decision maker's own attitude to the world. As religion was in part developed to help man deal with the unknown it appears that religion may play a strong role in mediating one's view of nature.

A second class of problems falling within the framework shown in figure #1 is competitive decision making, game theory [1]. In this environment the determination of V rather than being made by a capricious nature is made by another sentient agent, the competitor. In this environment, the values in the set

S_j, are considered as alternative actions open to this sentient competitor. Here, also the decision maker is unaware of the action chosen by the competitor, however, the motivation used by the competitor is assumed known, it is the same motivation as the decision maker is using, it wants to maximize the payoff it gets. In this competitive environment, two extreme interpretations can be considered regarding the meaning of the payoffs in the matrix in figure #1. In the first interpretation it is assumed that when A_i and S_j are selected the decision maker gets C_{ij} and the competitor loses C_{ij}. We call this the *pure adversarial* environment, it corresponds to the zero sum game. Here the competitors goal is to obtain a solution that minimizes C_{ij}. In the second interpretation, it is assumed that when A_i and S_j are selected, **both** the decision maker and the competitor get C_{ij}. This is called the pure *allied* environment. Here the players goal are to get a solution that maximizes C_{ij}.

One distinction between DMUU and competitive decision making is the mechanism used to supply the variable values. In DMUU this determination is assumed made by some capricious (irrational) agent called nature who we know very little about other then our empirical observations of its manifestations. In the second case, competitive decision making, this determination is being made by some sentient agent, assumed rational like ourselves, whose motivations the decision maker feels he knows or can reason intelligently about. A competitor uses the payoff matrix as a measure while nature assigns no intrinsic value to the payoff matrix.

Decision Making Under Ignorance

One commonly used approach in DMUI is the Max-Min approach, the decision maker calculates $\mathrm{Min}_j[C_{ij}]$ and then selects the alternative with the largest of these values. This approach is a very pessimistic approach, nature is viewed as being malevolent, it is assumed that given any selection of alternative by the decision maker the worst possible payoff will occur. Another approach is the Max-Max approach, the decision maker calculates $\mathrm{Max}_j[C_{ij}]$ and then selects the alternative with the largest of these. This is an optimistic approach, nature is viewed as being benevolent, it will select the best possibility.

In [2], Yager provided a unifying framework using the OWA operator for modeling approaches to alternative selection under ignorance.

Definition: An OWA operator of dimension n is a mapping $F_W(a_1, a_2, \ldots, a_n)$ that has an associated weighting vector W such that $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$ and where $F_W(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j$, with b_j being the jth largest of the a_i.

Using this we associate with each alternative A_i a value $U(i) = F_W(C_{i1}, C_{i2}, \ldots, C_{in})$, an OWA aggregation of the payoffs for that alternative. We then select the alternative which has the largest U(i) value.

In this formulation the parameter W, the weighting vector, is used to introduce the decision maker's attitude. If W is such that $w_j = 0$ for j = 1 to n - 1 and $w_n = 1$ we get $U(i) = \mathrm{Min}_j[C_{i1}]$, the pessimistic approach. If W is such that $w_1 = 1$ and $w_j = 0$ for j = 2 to n, we get $U(i) = \mathrm{Max}_j[C_i]$, optimistic approach. If we choose $w_j = 1/n$ for all j, then we get the average.

Mixed Strategies in DMUI

In the pessimistic, Maxi-Min approach nature is viewed as malevolent. Here the uncertain decision problem is viewed as if it were a zero sum game, nature is acting to try to minimize the payoff. Given this view it would appear natural to try to use some of the tools that are used in zero sum competitive games to help select the best solution alternative.

One strategy used in competitive games is to decide upon a *probability* of selecting each alternative rather then deciding upon a specific alternative. Here the decision maker decides upon a probability distribution P, wheret p_i is the probability that alternative A_i will be selected. The actual selection is obtained by the performance of a random experiment using P. We call the ***a mixed strategy***. The special case when one of the p_i's = 1 is called a pure strategy. Formally the advantage of using mixed strategies is extension of the space from the space of pure solutions to the space of mixed solutions.

We now investigate the use of a mixed strategy when the decision maker has a pessimistic point of view.. Our problem here can then be seen as trying to obtain the optimal probability distribution. Consider the decision problem shown below

	S_1	S_2
A_1	5	8
A_2	10	3

Let p be the probability of selecting A_1 and 1 - p be the probability of selecting A_2. If S_1 is the value of V, then the decision maker gets $U_1(p) = 5p + 10(1 - p)$ the expected payoff over the alternatives. If the value of V is S_2 than he gets $U_2(p) = 8p + 3(1 - p)$. Since he sees nature as purely adversarial, pessimistic, he assumes that the value S_q chosen by nature will be such that $U_q(p) = \text{Min}[U_1(p), U_2(p)]$. He must select the value of p to maximize this minimum. Since $U_1(p)$ increases as p decreases and $U_2(p)$ increases as p increases the p which give us the maximum of the minimum of the $U_i(p)$ occurs when $U_1(p) = U_2(p)$, hence p = 0.7.

We now provide a general formulation for selecting the best mixed strategy in this pessimistic environment. Let $U_j(P) = \sum_{i=1}^{m} C_{ij}\, p_i$ be his expected payoff if he uses P and S_j is the realized value for V. Based upon the decision makers pessimistic attitude the overalll value of selecting P is $U(P) = \text{Min}_j[U_j(P)]$. The problem is to select P such that U(P) is maximized. In [3] Yager looked at a number of properties of this approach.

Now we consider a mixed strategy in cases in which one sees nature as an ally, is optimistic. Let P be any mixed strategy here again $U_j(P)$ is the expected payoff if he uses P and S_j is the realized value. Because of the optimistic nature, he sees nature as trying to give him the most it can given his choice of P, his evaluation for any P is $U(P) = \text{Max}_j[U_j(P)]$. In this case he chooses P such that it maximizes U(P).

In [3] Yager shows that in this optimistic case the optimal choice is the pure stategy of selecting the alternative with with largest payoff. Thus for the optimistic decision maker, the optimal choice is to always select the alternative which has the maximal payoff. This result appears to be intuitively appealing in that if a decision maker perceives of nature as benevolent, an ally, then it is appropriate to be open with nature, don't use randomness to cause confusion, i.e. use a pure strategy. In addition, it would be wise toselect the alternative which will allow this ally to provide the decision maker with the best possible payoff, i.e. the row having the highest payoff in the matrix. Hence it appears appropriate that in the case of an optimistic decision make, there is no need to use a mixed strategy.

Now we consider the more general case where the decision maker's attitude is captured by an altitudinal vector W of dimension n, where n is the number of states of nature. Assume P is any mixed strategy, here again $U_j(P) = \sum_{i=1}^{m} C_{ij}\, p_i$, is the expected payoff if P is used and S_j is the realized value of the state of nature. Because of the decision maker's attitude, as conveyed by W, he believes that w_k is the probability that nature will select the state of nature having the k^{th} best expected payoff. Letting $b_k(P)$ be the k^{th} largest of the $U_j(P)$ we get that the overall evaluation of P, U(P), is the expected value of the $b_k(P)$, that is $U(P) = \sum_{k=1}^{n} b_k(P)\, w_k$, U(P) is effectively the OWA aggregation of the $U_j(P)$ with weighting vector W. Thus $U(P) = F_W(U_1(P), U_2(P), \ldots, U_n(P))$. As in the preceding, the decision comes down to selecting the P which maximizes U(P).

One important property shown in [3] is the following. Assume A_r and A_s are two alternatives such that A_r dominates A_s, $C_{rj} \geq C_{sj}$ for all j and for at least one j, $C_{rj} > C_{sj}$, then there always exists an optimal mixed strategy in which $p_s = 0$.

References

[1]. Luce, R. D. and Raiffa, H., Games and Decisions: Introduction and Critical Survey, John Wiley & Sons: New York, 1967.

[2]. Yager, R. R., "Decision making under Dempster-Shafer uncertainties," International Journal of General Systems 20, 233-245, 1992.

[3]. Yager, R. R., "A game theoretic approach to decision making under uncertainty," International Journal of Intelligent Systems in Accounting, Finance and Management 8, 131-143, 1999.