Quantum-like Models in Economics and Finances

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Abstract

We apply methods of quantum mechanics for mathematical modeling of price dynamics at financial market. We propose to describe behavioral financial factors (e.g., expectations of traders) by using the pilot wave (Bohmian) model of quantum mechanics. Our Bohmian model is a quantum-like model for the financial market, cf. with works of W. Segal, I. E. Segal, E. Haven. Our model expresses complexity of financial market: traditional description of price dynamics is completed by Schrödinger's dynamics for the pilot wave of expectations of traders. This is a kind of socio-economic model for financial market

Introduction

In economics and financial theory, analysts use random walk and more general martingale techniques to model behavior of asset prices, in particular share prices on stock markets, currency exchange rates and commodity prices. This practice has its basis in the presumption that investors act *rationally and without bias,* and that at any moment they estimate the value of an asset based on future expectations. Under these conditions, all existing information affects the price, which changes only when new information comes out. By definition, *new information appears randomly and influences the asset price randomly.* Corresponding continuous time models are based on stochastic processes (this approach was initiated in (Bachelier 1890)), see, e.g., the book (Mantegna & Stanley 2000) for historical and mathematical details.

This practice was formalized through the *efficient market hypothesis* which was formulated in the sixties, see (Samuelson 1965) and (Fama 1970) for details:

A market is said to be efficient in the determination of the most rational price if all the available information is instantly processed when it reaches the market and it is immediately reflected in a new value of prices of the assets traded.

Mathematically the efficient market hypothesis was supported by investigations of (Samuelson 1965). Using the hypothesis of rational behavior and market efficiency he was able to demonstrate how q_{t+1}, the expected value of price of a given asset at time $t+1$, is related to the previous values of prices $q_0, q_1, ..., q_t$ through the relation

$$E(q_{t+1}|q_0, q_1, ..., q_t) = q_t. \quad (1)$$

Typically there is introduced the σ-algebra $\mathcal{F}_t$ generated by random variables $q_0, q_1, ..., q_t$. The condition (1) is written in the form:

$$E(q_{t+1}|\mathcal{F}_t) = q_t. \quad (2)$$

Stochastic processes of such a type are called martingales. Alternatively, the martingale model for the financial market implies that the

$$(q_{t+1} - q_t)$$

is a "fair game" (a game which is neither in your favor nor your opponent's):

$$E(q_{t+1} - q_t|\mathcal{F}_t) = 0. \quad (3)$$

On the basis of information, $\mathcal{F}_t$, which is available at the moment t, one cannot expect neither

$$E(q_{t+1} - q_t|\mathcal{F}_t) > 0$$

nor

$$E(q_{t+1} - q_t|\mathcal{F}_t) < 0.$$

In this paper we develop a new approach that is not based on the *assumption that investors act rationally and without bias and that, consequently, new information appears randomly and influences the asset price randomly.*

Our approach can be considered as a special econophysical (Mantegna & Stanley 2000) model in the domain of behavioral finance. In our approach information about financial market (including expectations of agents of the financial market) is described by an *information field* $\psi(q)$ – *financial wave.* This field evolves deterministically[1] perturbing the dynamics of prices of stocks and options. Since psychology of agents of the financial market gives an important contribution into the financial wave $\psi(q)$, our model can be considered as a special *psycho-financial model.*

This paper can be also considered as a contribution into applications of quantum mechanics outside micro-world, see (Aerts & Aerts 1995), (Accardi 1997), (Khrennikov

[1]Dynamics is given by Schrödinger's equation on the space of prices of shares.

2004), (Khrennikov 2006). This paper is fundamentally based on investigations of (Bohm & Hiley 1993), (Hiley & Pylkkanen 1997) on the *active information* interpretation of Bohmian mechanics (Holland 1993) and its applications to cognitive sciences (Khrennikov 2004).

There were performed numerous investigations on applying quantum methods to financial market, see, e.g., (Haven 2002),(Haven 2003a), (Haven 2003b), that were not directly coupled to behavioral modeling, but based on the general concept that randomness of the financial market can be better described by the quantum mechanics, see, e.g., (Segal & Segal 1998): "A natural explanation for extreme irregularities in the evolution of prices in financial markets is provided by quantum effects."

Behavioral Finance and Behavioral Economics

We point out that there is no general consensus on the validity of the efficient market hypothesis. As it was pointed out in (Campbell & MacKinlay 1997): "... econometric advances and empirical evidence seem to suggest that financial asset returns are predictable to some degree. Thirty years ago this would have been tantamount to an outright rejection of market efficiency. However, modern financial economics teaches us that others, perfectly rational factors may account for such predictability. The fine structure of securities markets and frictions in trading process can generate predictability. Time-varying expected returns due to changing business conditions can generate predictability. A certain degree of predictability may be necessary to reward investors for bearing certain dynamic risks."

Therefore it would be natural to develop approaches which are not based on the assumption that investors act *rationally and without bias* and that, consequently, new information appears randomly and influences the asset price randomly. In particular, there are two well established (and closely related) fields of research *behavioral finance and behavioral economics* which apply scientific research on human and social cognitive and emotional biases to better understand economic decisions and how they affect market prices, returns and the allocation of resources. The fields are primarily concerned with the rationality, or lack thereof, of economic agents. Behavioral models typically integrate insights from psychology with neo-classical economic theory. Behavioral analysis are mostly concerned with the effects of market decisions, but also those of public choice, another source of economic decisions with some similar biases.

We recall that cognitive bias is any of a wide range of observer effects identified in cognitive science, including very basic statistical and memory errors that are common to all human beings and drastically skew the reliability of anecdotal and legal evidence. They also significantly affect the scientific method which is deliberately designed to minimize such bias from any one observer. They were first identified by Amos Tversky and Daniel Kahneman as a foundation of behavioral economics, see, (Tversky & Simonson 1993). Bias arises from various life, loyalty and local risk and attention concerns that are difficult to separate or codify. Tversky and Kahneman claim that they are at least partially the result of problem-solving using heuristics, including the availability heuristic and the representativeness.

Since the 1970s, the intensive exchange of information in the world of finances has become one of the main sources determining dynamics of prices. Electronic trading (that became the most important part of the environment of the major stock exchanges) induces huge information flows between traders (including foreign exchange market). Financial contracts are performed at a new time scale that differs essentially from the old "hard" time scale that was determined by the development of the economic basis (development of industry, natural and labour resources and so on) of the financial market. Prices at which traders are willing to buy (bid quotes) or sell (ask quotes) a financial asset are not only determined by the continuous development of industry, trade, services, situation at the market of natural resources and so on. Information (mental, market-psychological) factors play a very important (and in some situations crucial) role in price dynamics. Traders performing financial operations work as a huge collective cognitive system. Roughly speaking classical-like dynamics of prices (determined) by "hard" economic factors are permanently perturbed by additional financial forces, mental (or market-psychological) forces, see the book (Soros 1987).

In this paper we use methods of Bohmian mechanics to simulate dynamics of prices at the financial market. We start with the development of the classical Hamiltonian formalism on the price/price-change phase space to describe the classical-like evolution of prices. This classical dynamics of prices is determined by "hard" financial conditions (natural resources, industrial production, services and so on). These conditions as well as "hard" relations between traders at the financial market are mathematically described by the classical financial potential. As we have already remarked, at the real financial market "hard" conditions are not the only source of price changes. The information and market psychology play important (and sometimes determining) role in price dynamics.

We propose to describe those "soft" financial factors by using the pilot wave (Bohmian) model of quantum mechanics. The theory of financial mental (or psychological) waves is used to take into account market psychology. The real trajectories of prices are determined (by the financial analogue of the second Newton law) by two financial potentials: classical-like ("hard" market conditions) and quantum-like ("soft" market conditions).

Our quantum-like model of financial processes was strongly motivated by consideration by J. Soros (Soros 1987) of the financial market as a complex cognitive system. Such an approach he called the theory of *reflexivity.* In this theory there is a large difference between market that is "ruled" by only "hard" economical factors and a market where mental factors play the crucial role (even changing the evolution of the "hard" basis, see (Soros 1987)).

J. Soros rightly remarked that the "non mental" market evolves due to classical random fluctuations. However, such fluctuations do not provide an adequate description of mental market. He proposed to use an analogy with quantum theory. However, it was noticed that directly quantum formalism could not be applied to the financial market (Soros 1987). Traders differ essentially from elementary particles. Elementary particles behave stochastically due to perturbation effects provided by measurement devices.

According to J. Soros, traders at the financial market behave stochastically due to free will of individuals. Combinations of a huge number of free wills of traders produce additional stochasticity at the financial market that could not be reduced to classical random fluctuations (determined by non mental factors). Here J. Soros followed to the conventional (Heisenberg, Bohr, Dirac) viewpoint to the origin of quantum stochasticity. However, in the Bohmian approach (that is non-conventional one) quantum statistics is induced by the action of an additional potential, quantum potential, that changes classical trajectories of elementary particles. Such an approach gives the possibility to apply quantum formalism to the financial market.

A Brief Introduction to Bohmian Mechanics

In this section we present the basic notions of Bohmian mechanics. This is a special model of quantum mechanics in that, in the opposition to the conventional Copenhagen interpretation, quantum particles (e.g., electrons) have well defined trajectories in physical space.

By the conventional Copenhagen interpretation (that was created by N. Bohr and W. Heisenberg) quantum particles do not have trajectories in physical space. Bohr and Heisenberg motivated such a viewpoint to quantum physical reality by the Heisenberg uncertainty relation:

$$\Delta q \Delta p \geq h/2 \tag{4}$$

where h is the Planck constant, q and p are the position and momentum, respectively, and Δq and Δp are uncertainties in determination of q and p. Nevertheless, David Bohm demonstrated (Bohm & Hiley 1993), see also (Holland 1993), that, in spite of Heisenberg's uncertainty relation (4), it is possible to construct a quantum model in that trajectories of quantum particles are well defined. Since this paper is devoted to mathematical models in economy and not to physics, we would not go deeper into details. We just mention that the root of the problem lies in different interpretations of Heisenberg's uncertainty relation (4). If one interpret Δq and Δp as uncertainties for the position and momentum of an individual quantum particle (e.g., one concrete electron) then (4), of course implies that it is impossible to create a model in that the trajectory $q(t), p(t)$ is well defined. On the other hand, if one interpret Δq and Δp as *statistical deviations*

$$\Delta q = \sqrt{E(q - Eq)^2}, \;\; \Delta p = \sqrt{E(p - Ep)^2}, \tag{5}$$

then there is no direct contradiction between Heisenberg's uncertainty relation (4) and the possibility to consider trajectories. There is a place to such models as Bohmian mechanics. Finally, we remark (but without comments) that in real experiments with quantum systems, one always uses the statistical interpretation (5) of Δq and Δp.

We emphasize that the conventional quantum formalism cannot say anything about the individual quantum particle. This formalism provides only statistical predictions on huge ensembles of particles. Thus Bohmian mechanics provides a better description of quantum reality, since there is the possibility to describe trajectories of individual particles. However, this great advantage of Bohmian mechanics was not explored so much in physics. Up to now there have not been done experiments that would distinguish predictions of Bohmian mechanics and conventional quantum mechanics.

In this paper we shall show that the mentioned advantages of Bohmian mechanics can be explored in applications to the financial market. In the latter case it is really possible to observe the trajectory of the price or price-change dynamics. Such a trajectory is described by equations of the mathematical formalism of Bohmian mechanics.

We now present the detailed derivation of the equations of motion of a quantum particle in the Bohmian model of quantum mechanics. Typically in physical books it is presented very briefly. But, since this paper is oriented to economists and mathematicians, who are not so much aware about quantum physics, we shall present all calculations. The dynamics of the wave function $\psi(t, q)$ is described by Schrödinger's equation

$$i\, h \frac{\partial \psi}{\partial t}(t, q) = -\frac{h^2}{2m} \frac{\partial^2 \psi}{\partial q^2}(t, q) + V(q)\psi(t, q) \tag{6}$$

Here $\psi(t, q)$ is a complex valued function. At the moment we prefer not to discuss the conventional probabilistic interpretation of $\psi(t, q)$. We consider $\psi(t, q)$ as just a field.[2]

We consider the one-dimensional case, but the generalization to the multidimensional case, $q = (q_1, \ldots, q_n)$, is straightforward. Let us write the wave function $\psi(t, q)$ in the following form:

$$\psi(t, q) = R(t, q) e^{i \frac{S(t,q)}{h}} \tag{7}$$

where $R(t, q) = |\psi(t, q)|$ and $\theta(t, q) = S(t, q)/h$ is the argument of the complex number $\psi(t, q)$.

We put (7) into Schrödinger's equation (6). We have

$$ih \frac{\partial \psi}{\partial t} = ih\Big(\frac{\partial R}{\partial t} e^{\frac{iS}{h}} + \frac{iR}{h}\frac{\partial S}{\partial t} e^{\frac{iS}{h}}\Big) = ih\frac{\partial R}{\partial t} e^{\frac{iS}{h}} - R\frac{\partial S}{\partial t} e^{\frac{iS}{h}}$$

and

$$\frac{\partial \psi}{\partial q} = \frac{\partial R}{\partial q} e^{\frac{iS}{h}} + \frac{iR}{h}\frac{\partial S}{\partial q} e^{\frac{iS}{h}}$$

and hence:

$$\begin{aligned} \frac{\partial^2 \psi}{\partial q^2} \;\; &= \;\; \frac{\partial^2 R}{\partial q^2} e^{\frac{iS}{h}} + \frac{2i}{h}\frac{\partial R}{\partial q}\frac{\partial S}{\partial q} e^{\frac{iS}{h}} \\ &\quad + \frac{iR}{h}\frac{\partial^2 S}{\partial q^2} e^{\frac{iS}{h}} - \frac{R}{h^2}\Big(\frac{\partial S}{\partial q}\Big)^2 e^{\frac{iS}{h}} \end{aligned}$$

We obtain the differential equations:

$$\frac{\partial R}{\partial t} = \frac{-1}{2m}\Big(2\frac{\partial R}{\partial q}\frac{\partial S}{\partial q} + R\frac{\partial^2 S}{\partial q^2}\Big), \tag{8}$$

[2] We recall that by the probability interpretation of $\psi(t, q)$ (which was proposed by Max Born) the quantity $|\psi(t, q)|^2$ gives the probability to find a quantum particle at the point q at the moment t.

$$-R\frac{\partial S}{\partial t} = -\frac{h^2}{2m}\Big(\frac{\partial^2 R}{\partial q^2} - \frac{R}{h^2}\Big(\frac{\partial S}{\partial q}\Big)^2\Big) + VR. \qquad (9)$$

By multiplying the right and left-hand sides of the equation (8) by $2R$ and using the trivial equalities:

$$\frac{\partial R^2}{\partial t} = 2R\frac{\partial R}{\partial t}$$

and

$$\frac{\partial}{\partial q}(R^2\frac{\partial S}{\partial q}) = 2R\frac{\partial R}{\partial q}\frac{\partial S}{\partial q} + R^2\frac{\partial^2 S}{\partial q^2},$$

we derive the equation for R^2:

$$\frac{\partial R^2}{\partial t} + \frac{1}{m}\frac{\partial}{\partial q}(R^2\frac{\partial S}{\partial q}) = 0. \qquad (10)$$

We remark that if one uses the Born's probabilistic interpretation of the wave function, then

$$R^2(t,x) = |\psi(t,x)|^2$$

gives the probability. Thus the equation (10) is the equation describing the dynamics of the probability distribution (in physics it is called the continuity equation).

The second equation can be written in the form:

$$\frac{\partial S}{\partial t} + \frac{1}{2m}\Big(\frac{\partial S}{\partial q}\Big)^2 + \Big(V - \frac{h^2}{2mR}\frac{\partial^2 R}{\partial q^2}\Big) = 0. \qquad (11)$$

Suppose that

$$\frac{h^2}{2m} << 1$$

and that the contribution of the term

$$\frac{h^2}{2mR}\frac{\partial^2 R}{\partial q^2}$$

can be neglected. Then we obtain the equation:

$$\frac{\partial S}{\partial t} + \frac{1}{2m}\Big(\frac{\partial S}{\partial q}\Big)^2 + V = 0. \qquad (12)$$

From the classical mechanics, we know that this is the classical Hamilton-Jacobi equation which corresponds to the dynamics of particles:

$$p = \frac{\partial S}{\partial q} \text{ or } m\dot{q} = \frac{\partial S}{\partial q}, \qquad (13)$$

where particles moves normal to the surface $S = const$.

David Bohm proposed to interpret the equation (11) in the same way. But we see that in this equation the classical potential V is perturbed by an additional "quantum potential"

$$U = \frac{h^2}{2mR}\frac{\partial^2 R}{\partial q^2}.$$

Thus in the Bohmian mechanics the motion of a particle is described by the usual Newton equation, but with the force corresponding to the combination of the classical potential V and the quantum one U :

$$m\frac{dv}{dt} = -(\frac{\partial V}{\partial q} - \frac{\partial U}{\partial q}) \qquad (14)$$

The crucial point is that the potential U is by itself driven by a field equation - Schrödinger's equation (6). Thus the equation (14) can not be considered as just the Newton classical dynamics (because the potential U depends on ψ as a field parameter). We shall call (14) the *Bohm-Newton equation.*

We remark that typically in books on Bohmian mechanics (Bohm & Hiley 1993), (Holland 1993) it is emphasized that the equation (14) is nothing else than the ordinary Newton equation. This make impression that the Bohmian approach give the possibility to reduce quantum mechanics to ordinary classical mechanics. However, this is not the case. The equation (14) does not provide the complete description of dynamics of a systems. Since, as was pointed out, the quantum potential U is determined through the wave function ψ and the latter evolves according to the Schrödinger equation, the dynamics given by Bohm-Newton equation can not be considered independent of the Schrödinger's dynamics.

Classical Econophysical Model of the Financial Market

We repeat shortly the Bohmian model for the financial market proposed in (Choustova 2006). We start with a classical econophysical model of the financial market. This model will be developed to a quantum-like econophysical model.

Let us consider a mathematical model in that a huge number of agents of the financial market interact with one another and take into account external economic (as well as political, social and even meteorological) conditions in order to determine the price to buy or sell financial assets. We consider the trade with shares of some corporations (e.g., VOLVO, SAAB, IKEA,...).

We consider a *price system of coordinates.* We enumerate corporations which did emissions of shares at the financial market under consideration: $j = 1, 2, ..., n$ (e.g., VOLVO:$j = 1$, SAAB:$j = 2$, IKEA:$j = 3$,...). There can be introduced the n-dimensional configuration space $Q = R^n$ of prices, $q = (q_1, \ldots, q_n)$, where q_j is the price of a share of the jth corporation. Here R is the real line. Dynamics of prices is described by the trajectory $q(t) = (q_1(t), \ldots, q_n(t))$ in the configuration price space Q.

Another variable under the consideration is the *price change variable*: $v_j(t) = \dot{q}_j(t) = \lim_{\Delta t \to 0} \frac{q_j(t+\Delta t) - q_j(t)}{\Delta t}$, see, for example, the book (Mantegna & Stanley 2000) on the role of the price change description. In real models we consider the discrete time scale $\Delta t, 2\Delta t, \ldots$. Here we should use discrete price change variable $\delta q_j(t) = q_j(t + \Delta t) - q_j(t)$.

We now introduce an analogue m of mass as the number of items (in our case shares) that a trader emitted to the market. We call m the *financial mass.* Thus each trader j (e.g., VOLVO) has its own financial mass m_j (the size of the emission of its shares). The total price of the emission performed by the jth trader is equal to $T_j = m_j q_j$ – *market capitalization.*

We also introduce *financial energy* of the market as a function $H : Q \times V \to R$. If we use the analogue with classical mechanics, then we could consider (at least for mathematical modeling) the financial energy of the form: $H(q, v) = \frac{1}{2}\sum_{j=1}^{n} m_j v_j^2 + V(q_1, \ldots, q_n)$.

Here $K = \frac{1}{2}\sum_{j=1}^{n} m_j v_j^2$ is the *kinetic financial energy* and $V(q_1, \ldots, q_n)$ is the potential financial energy, m_j is the financial mass of jth trader.

The kinetic financial energy represents efforts of agents of financial market to change prices: higher price changes induce higher kinetic financial energies. If the corporation j_1 has higher financial mass than the corporation j_2, so $m_{j_1} > m_{j_2}$, then the same change of price, i.e., the same financial velocity $v_{j_1} = v_{j_2}$, is characterized by higher kinetic financial energy: $K_{j_1} > K_{j_2}$. We also remark that high kinetic financial energy characterizes rapid changes of the financial situation at market. However, the kinetic financial energy does not give the attitude of these changes. It could be rapid economic growth as well as recession.

The *potential financial energy* V describes the interactions between traders $j = 1, ..., n$ (e.g., competition between NOKIA and ERICSSON) as well as external economic conditions (e.g., the price of oil and gas) and even meteorological conditions (e.g., the weather conditions in Louisiana and Florida). For example, we can consider the simplest interaction potential: $V(q_1, \ldots, q_n) = \sum_{j=1}^{n}(q_i - q_j)^2$. The difference $|q_1 - q_j|$ between prices is the most important condition for *arbitrage.*

As in classical mechanics for material objects, we introduce a new variable $p = mv$, the *price momentum* variable. Instead of the price change vector $v = (v_1, \ldots, v_n)$, we consider the price momentum vector $p = (p_1, \ldots, p_n), p_j = m_j v_j$. The quantity $f_j(q) = -\frac{\partial V}{\partial q_j}$ is called the financial force. We can postulate the financial variant of the second Newton law:

$$m\dot{v} = f \tag{15}$$

"The product of the financial mass and the price acceleration is equal to the financial force."

Quantum-like Econophysical Model for the Financial Market

Our fundamental assumption is that agents at the modern financial market are not just "classical-like agents." Their actions are ruled not only by classical-like financial potentials $V(t, q_1, \ldots, q_n)$, but also (in the same way as in the pilot wave theory for quantum systems) by an additional information (or psychological) potential induced by a financial pilot wave.

Therefore we could not use the classical financial dynamics (Hamiltonian formalism) on the financial phase space to describe the real price trajectories. Information (psychological) perturbation of Hamiltonian equations for price and price change must be taken into account. To describe such a model mathematically, it is convenient to use such an object as a *financial pilot wave* that rules the financial market.

In some sense $\psi(q)$ describes the psychological influence of the price configuration q to behavior of agents of the financial market. In particular, the $\psi(q)$ contains expectations of agents.[3]

We emphasize two important features of the financial pilot wave model:

1. All shares are coupled on the information level. The general formalism (Bohm & Hiley 1993), (Holland 1993) of the pilot wave theory says that if the function $\psi(q_1, \ldots, q_n)$ is not factorized, i.e., $\psi(q_1, \ldots, q_n) \neq \psi_1(q_1) \ldots \psi_n(q_n)$, then any changing the price q_i will automatically change behavior of all agents of the financial market (even those who have no direct coupling with i-shares). This will imply changing of prices of j-shares for $i \neq j$. At the same time the "hard" economic potential $V(q_1, \ldots, q_n)$ need not contain any interaction term: for example, $V(q_1, \ldots, q_n) = q_1^2 + \ldots q_n^2$. The Hamiltonian equations in the absence of the financial pilot wave have the form: $\dot{q}_j = p_j, \dot{p}_j = -2q_j, j = 1, 2, \ldots, n$. Thus the classical price trajectory $q_j(t)$, does not depend on dynamics of prices of shares for other traders $i \neq j$ (for example, the price of shares of ERIKSSON does not depend on the price of shares of NOKIA and vice versa).[4]

However, if, e.g., $\psi(q_1, \ldots, q_n) = ce^{i(q_1 q_2 + \ldots + q_{n-1} q_n)} e^{-(q_1^2 + \ldots + q_n^2)}$, where $c \in C$ is some normalization constant, then financial behavior of agents at the financial market is nonlocal (see further considerations).

2. Reactions of the market do not depend on the amplitude of the financial pilot wave: waves $\psi, 2\psi, 100000\psi$ will produce the same reaction. Such a behavior at the market is quite natural (if the financial pilot wave is interpreted as an information wave, the wave of financial information). The amplitude of an information signal does not play so large role in the information exchange. The most important is the context of such a signal. The context is given by the shape of the signal, the form of the financial pilot wave function.

In fact, we need not develop a new mathematical formalism. We will just apply the standard pilot wave formalism to the financial market. The fundamental postulate of the pilot wave theory is that the pilot wave (field) $\psi(q_1, \ldots, q_n)$ induces a new (quantum) potential $U(q_1, \ldots, q_n)$ which perturbs the classical equations of motion. A modified Newton equation has the form:

$$\dot{p} = f + g, \tag{16}$$

where $f = -\frac{\partial V}{\partial q}$ and $g = -\frac{\partial U}{\partial q}$. We call the additional financial force g a *financial mental force.* This force

[3] The reader may be surprised that there appeared complex numbers C. However, the use of these numbers is just a mathematical trick that provides the simple mathematical description of dynamics of the financial pilot wave.

[4] Such a dynamics would be natural if these corporations operate on independent markets, e.g., ERIKSSON in Sweden and NOKIA in Finland. Prices of their shares would depend only on local market conditions, e.g., on capacities of markets or consuming activity.

$g(q_1, \ldots, q_n)$ determines a kind of collective consciousness of the financial market. Of course, the g depends on economic and other 'hard' conditions given by the financial potential $V(q_1, \ldots, q_n)$. However, this is not a direct dependence. In principle, a nonzero financial mental force can be induced by the financial pilot wave ψ in the case of zero financial potential, $V \equiv 0$. So $V \equiv 0$ does not imply that $U \equiv 0$. *Market psychology is not totally determined by economic factors.* Financial (psychological) waves of information need not be generated by some changes in a real economic situation. They are mixtures of mental and economic waves. Even in the absence of economic waves, mental financial waves can have a large influence to the market.

By using the standard pilot wave formalism we obtain the following rule for computing the financial mental force. We represent the financial pilot wave $\psi(q)$ in the form: $\psi(q) = R(q)e^{iS(q)}$, where $R(q) = |\psi(q)|$ is the amplitude of $\psi(q)$, (the absolute value of the complex number $c = \psi(q)$) and $S(q)$ is the phase of $\psi(q)$ (the argument of the complex number $c = \psi(q)$). Then the financial mental potential is computed as $U(q_1, ..., q_n) = -\frac{1}{R}\sum_{i=1}^{n} \frac{\partial^2 R}{\partial q_i^2}$ and the financial mental force as $g_j(q_1, \ldots, q_n) = \frac{-\partial U}{\partial q_j}(q_1, \ldots, q_n)$. These formulas imply that strong financial effects are produced by financial waves having essential variations of amplitudes.

Bohm-Vigier Model for the Financial Market

In principle, one can consistently combine the quantum-like approach with the efficient market hypothesis by considering the Bohm-Vigier stochastic model, instead of the completely deterministic Bohmian model. We follow here (Bohm & Hiley 1993). We recall that in the original Bohmian model the velocity of an individual particle is given by

$$v = \frac{\nabla S(q)}{m}. \tag{17}$$

If $\psi = Re^{iS/h}$, then Schrödinger's equation implies that $\frac{dv}{dt} = -\nabla(V + U)$, where V and U are classical and quantum potentials respectively. In principle one can work only with the basic equation (17).

The basic assumption of Bohm and Vigier was that the velocity of an individual particle is given by

$$v = \frac{\nabla S(q)}{m} + \eta(t), \tag{18}$$

where $\eta(t)$ represents a random contribution to the velocity of that particle which fluctuates in a way that may be represented as a random process but with zero average. In Bohm-Vigier stochastic mechanics quantum potential comes in through the average velocity and not the actual one.

We now shall apply the Bohm-Vigier model to financial market, see also (Haven 2006). The equation (18) is considered as the basic equation for the price velocity. Thus the real price becomes a random process (as well as in classical financial mathematics. We can write the stochastic differential equation, SDE, for the price:

$$dq(t) = \frac{\nabla S(q)}{m}dt + \eta(t)dt. \tag{19}$$

To give the rigorous mathematical meaning to the stochastic differential we assume that

$$\eta(t) = \frac{d\xi(t)}{dt}, \tag{20}$$

for some stochastic process $\xi(t)$. Thus formally: $\eta(t)dt = \frac{d\xi(t)}{dt}dt = d\xi(t)$, and the rigorous mathematical form of the equation (19) is $dq(t) = \frac{\nabla S(q)}{m}dt + d\xi(t)$. The expression (20) one can consider either formally or in the sense of distribution theory (we recall that for basic stochastic processes, e.g., the Wiener process, trajectories are not differentiable in the ordinary sense almost every where).

Suppose, for example, that the random contribution into the price dynamics is given by *white noise,* $\eta_{\text{white noise}}(t)$.It can be defined as the derivative (in sense of distribution theory) of the Wiener process: $\eta_{\text{white noise}}(t) = \frac{dw(t)}{dt}$, thus: $v = \frac{\nabla S(q)}{m} + \eta_{\text{white noise}}(t)$, In this case the price dynamics is given by the SDE:

$$dq(t) = \frac{\nabla S(q)}{m}dt + dw(t). \tag{21}$$

What is the main difference from the classical SDE-description of the financial market? This is the presence of the pilot wave $\psi(t, q)$, mental field of the financial market, which determines the coefficient of drift $\frac{\nabla S(q)}{m}$. Here $S \equiv S_\psi$. And the ψ-function is driven by a special field equation – Schrödinger's equation. The latter equation is not determined by the SDE (21). Thus, instead of one SDE, in the quantum-like model, we have the system of two equations:

$$dq(t) = \frac{\nabla S_\psi(q)}{m}dt + d\xi(t). \tag{22}$$

$$i\, h\frac{\partial \psi}{\partial t}(t, q) = -\frac{h^2}{2m}\frac{\partial^2 \psi}{\partial q^2}(t, q) + V(q)\psi(t, q). \tag{23}$$

We have only to make one remark, namely, on the role of the constant h in Schrödinger's equation, cf. (Haven 2002). In quantum mechanics (which deals with microscopic objects) h is the Planck constant. This constant is assumed to play the fundamental role in all quantum considerations. In our financial model we consider h as a price scaling parameter, namely, the unit in which we would like to measure price change. However, we do not exclude the possibility that there might be found a deeper interpretation of h.

On Views of G. Soros: Alchemy of Finances or Quantum Mechanics of Finances?

G. Soros is unquestionably the most powerful and profitable investor in the world today. He has made a billion dollars going against the British pound.

Soros is not merely a man of finance, but a thinker and philosopher as well. Surprisingly he was able to apply his general philosophic ideas to financial market. In particular, the project Quantum Fund inside *Soros Fund Management* gained hundreds millions dollars and has 6 billion dollars in net assets.

The book "Alchemy of Finance" (Soros 1987) is a kind of economic-philosophic investigation. I would like to analyze philosophic aspects of this investigation. The book consists of five chapters. In fact, only the first chapter - "Theory of Reflexivity" - is devoted to pure theoretical considerations.

J. Soros studied economics in college, but found that economic theory was highly unsatisfactory. He says that economics seeks to be a science, but science is supposed to be objective. And it is difficult to be scientific when the subject matter, the participant in the economic process, lacks objectivity. The author also was greatly influenced by Karl Popper's ideas on scientific method, but he did not agree with Popper's "unity method." By this Karl Popper meant that methods and criteria which can be applied to the study of natural phenomena also can be applied to the study of social events. George Soros underlined a fundamental difference between natural and social sciences:

The events studied by social sciences have thinking participants and natural phenomena do not. The participants' thinking creates problems that have no counterpart in natural science. There is a close analogy with QUANTUM PHYSICS, where the effects of scientific observations give rise to Heisenberg uncertainty relations and Bohr's complementarity principle.

But in social events the participants' thinking is responsible for the element of uncertainty, and not an external observer. In natural science investigation of events goes from fact to fact. In social events the chain of causation does not lead directly from fact to fact, but from fact to participants' perceptions and from perceptions to fact.

This would not create any serious difficulties if there were some kind of correspondence or equivalence between facts and perceptions.

Unfortunately, that is impossible, because the participants perceptions do not relate to facts, but to a situation that is contingent on their own perceptions and therefore cannot be treated as a fact.

In order to appreciate the problem posed by thinking participants, Soros takes a closer look at the way scientific method operates. He takes Popper's scheme of scientific method, described in technical terms as "deductive-nomological" or "D-N" model. The model is built on three kinds of statements: specific initial conditions, specific final conditions, and generalizations of universal validity. Combining a set of generalizations with known initial conditions yields predictions, combining them with known final conditions provides explanations; and matching known initial with known final conditions serves as testing for generalizations involved. Scientific theories can only be falsified, never verified.

The asymmetry between verification and falsification and the symmetry between prediction and explanation are two crucial features of Popper's scheme.

The model works only if certain conditions are fulfilled. It is the requirement of universality. That is, if a given set of conditions recurred, it would have to be followed or predicted by the same set of conditions as before. The initial and final conditions must consist of observable facts governed by universal laws. It is this requirement that is so difficult to meet when a situation has thinking participants. Clearly, a single observation by a single scientist is not admissible. Exactly because the correspondence between facts and statements is so difficult to establish, science is a collective enterprize where the work of each scientist has to be open to control and criticism by others. Individual scientists often find the conventions quite onerous and try various shortcuts in order to obtain a desired result. The most outstanding example of the observer trying to impose his will on his subject matter is the attempt to convert base metal into gold. Alchemists struggled long and hard until they were finally persuaded to abandon their enterprize by their lack of success. The failure was inevitable because the behavior of base metals is governed by laws of universal validity which cannot be modified by any statements, incantations, or rituals.

And now, we can at least understand why J. Soros called his book *Alchemy of Finance*, see (Soros 1987).

Soros considers the behavior of human beings. Do they obey universally valid laws that can be formulated in accordance with "D-N" model? Undoubtedly, there are many aspects of human behavior, from birth to death and in between, which are amenable to the same treatment as other natural phenomena. But there is one aspect of human behavior which seems to exhibit characteristics which are different from those of the phenomena from the subject matter of natural science: the decision making process. An imperfect understanding of the situation destroys the universal validity of scientific generalizations: given a set of conditions is not necessary preceded or succeeded by the same set every time, because the sequence of events is influenced by participants' thinking. The "D-N" model breaks down. But social scientists try to maintain the unity of method but with little success.

In a sense, the attempt to impose the methods of natural science on social phenomena is comparable to efforts of alchemists who sought to apply the methods of magic to the field of natural science. And here J. Soros presents (in my opinion) a very interesting idea about the "alchemy method in social science." He says, that while the failure of the alchemists was total, social scientists have managed to make a considerable impact on their subject matter. Situations which have thinking participants may be impervious to the methods of natural science, but they are susceptible to the methods of alchemy.

The thinking of participants, exactly because it is not governed by reality, is easily influenced by theories. In the field of natural phenomena, scientific method is effective only when its theories are valid, but in social, political, and economic matters, theories can be effective without being valid. Whereas alchemy has failed in natural sciences, social science can succeed as alchemy.

The relationship between the scientist and his subject matter is quite different in natural science as opposed to social science. In natural science the scientist's thinking is, in fact, distinct from its subject matter. The scientists can influence the subject matter only by actions, not by thoughts, and the scientists actions are guided by the same laws as all other natural phenomena. Specifically, a scientist can do nothing do when she wants to turn base metals into gold.

Social phenomena are different. The imperfect understanding of the participant interferes with the proper functioning of the "D-N" model. There is much to be gained by pretending to abide by conventions of scientific method without actually doing so. Natural science is held in great esteem: the theory that claims to be scientific can influence the gullible public much better than one which frankly admits its political or ideological bias.

Soros mentions here Marxism, psychoanalysis and laissez-faire capitalism with its reliance on the theory of perfect competition as typical examples.

Soros underlined that Marx and Freud were vocal in protesting their scientific status and based many of their conclusions on authority they derived from being "scientific." And Soros says, that once this point sinks in, the very expression "social science" became suspect.

He compares the expression "social science" with a magic word employed by social alchemists in their effort to impose their will on their subject matter by incantation. And it seems to Soros there is only one way for the "true" practitioners of scientific method to protect themselves against such malpractice - to deprive social science of the status it enjoys on account of natural science. Social science ought to be recognized as a false metaphor.

I cannot agree with this Soros statement. First of all there are a lot of boundary sciences. For example, psychoanalysis can be considered as a part of medicine. But medicine accompanied with biology and chemistry are of course the natural sciences. And S. Freud was famous and like many doctors succeeding him they helped many patients.

And J. Soros explains by himself his unusual statement. He says that it does not mean that we must give up the pursuit of truth in exploring social phenomena. It means only that the pursuit of truth requires to recognize that the "D-N" model can not be applied to situations with thinking participants. He asks us to abandon the doctrine of the unity of method and to cease the slavish imitation of natural sciences. He says that there are some new scientific methods in all kinds of science as quantum physics has shown.

Scientific method is not necessarily confined to the "D-N" model: statistical, probabilistic generalizations may be more fruitful. Nor should we ignore the possibility of developing novel approaches which have no counterpart in natural science. Given the differences in subject matter, there ought to be differences in the method of study.

Soros shows us the main distinction between the "D-N" model and his own approach. He says that the world of imperfect understanding does not land itself to generalizations which can be used to explain and to predict specific events. The symmetry between explanation and prediction prevails only in the absence of thinking participants. On the other hand, past events are just as final as in the "D-N" model; thus explanations turn out to be an easier task than prediction.

G. Soros proved his theory by becoming one of the most powerful and profitable investors in the world today. In my own work I use methods of classical and quantum mechanics for mathematical modeling of price dynamics at financial market and I use Soros' statement about cognitive phenomena at the financial market.

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