En-Route Sector Metering using a Game-theoretic Approach

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Abstract

Currently, Traffic Management Coordinators in the Air Route Traffic Control Centers establish flow constraints at various sector meter fixes, based on their best estimates of the predicted traffic demand into their sector. Sector flow rates are not coordinated between neighboring sectors or centers. Incorrect sector metering rates can lead often lead to a cascade effect resulting in delays in the schedules of aircraft several sectors away. In this paper we develop an approach for dynamic sector metering based on game theory. The approach is based on a Bayesian game with communication, wherein the sectors determine mutually beneficial metering rates (based on collaboration and exchange of metering rates and negotiated Scheduled Times of Arrival) chosen so as to optimize delay, controller workload and capacity. We formalize the model for a simplified scenario consisting of two sectors belonging to two different centers, attempting to set the flow rates at their boundaries. The simplified models for the two-player (sector) game capture the coupling of dynamics between the two sectors and possible interactions between incoming and outgoing traffic flows. The inbound aircraft from other sectors and outbound flow rate restriction to other sectors are generated from a stochastic time series model. To demonstrate feasibility we implement our approach on a simplified version of agent-based decision support to captures inter center/sector communication and decentralized decision-making

Introduction

As air traffic congestion and delay have increased in the post-deregulation era, the air transportation community has sought to identify more efficient methods of Traffic Flow Management (TFM) in order to best utilize existing capacity. Advanced TFM strategies have the potential to improve system throughput and reduce delay without requiring substantial rework of the National Airspace System (NAS) infrastructure (e.g., airspace redesign, Communication, Navigation and Surveillance (CNS) upgrades, etc.) and without imposing new aircraft equipage requirements.

Currently, Traffic Management Coordinators (TMCs) in the Air Route Traffic Control Centers (ARTCC) establish flow constraints at various sector meter fixes, based on their best estimates of the predicted traffic demand, or sectors affected by adverse weather conditions. Arrival rates into a sector are usually determined for sectors close to airports with high demand, and these rates typically flow down (radially outward) to en route sectors. Typically controllers add a safety buffer to this rate to accommodate for any unforeseen circumstances. Sector flow rates are not coordinated between neighboring sectors or centers based on the estimated demand on their sectors. Furthermore, metering times at sectors are assigned based on miles-in-trail restrictions (distance-based spacing). While this approach works reasonably well for low traffic densities, for higher densities (coupled with the dynamics and uncertainty of the traffic) this approach fails. Inappropriate sector metering rates near a busy airport can often lead to a cascade effect resulting in delays in the schedules of various aircraft several sectors away. In addition, it has been observed that fixed miles-in-trail restrictions based on quantization to maintain the metering rates can be inefficient. Controller workload can be improved by giving the controller the flexibility to impose variable in-trail restrictions between aircraft, as long as an average metering rate over a certain duration is maintained, and safety is not compromised.

Time-based spacing (or "metering") of arrival traffic flows has been shown to be more efficient than distance-based spacing, or "in-trail restrictions," as shown by Sokkappa in a theoretical study (Sokakapa. 1989). Sokkappa's findings were validated by Swenson, who documented significant improvements in delay and throughput versus in-trail spacing using NASA's Traffic Management Advisor (TMA) in operational field tests at Dallas–Fort Worth International Airport (DFW) (Swenson, Hoang, et. al. 1997)

Researchers at NASA Ames Research Center are pursuing a distributed scheduling concept to implement time-based metering in constrained, transition airspace (Farley, Foster, Hoang and Lee 2001). Instead of relying on a single, monolithic scheduler to compute a workable and efficient schedule for the entire region, as is the case for current time-based metering systems, a distributed scheduling scheme relies on a loosely integrated network of schedulers, each governing small airspace regions. A scheduler might govern an area as small as a single airspace sector or as large as an enroute Center. The envisioned time-based metering operation will be more sensitive to local ATC constraints and goals, but will still enable facilities to implement an efficient, time-based metering approach to air traffic management on a regional scale.

A key challenge in this work is to determine how—and to what extent—to couple the distributed schedulers in order to generate a set of schedules which are individually workable and collectively beneficial. That is, collectively they produce a significant throughput benefit.

This paper develops a game-theoretic approach for coupling distributed scheduling algorithms. The approach is based on a Bayesian game with communication, wherein distributed schedulers negotiate acceptance rates to optimize system-wide delay, controller workload, and throughput. One instance of the scheduler is assumed to be computing trajectory projections (estimated boundary-crossing times, or ETAs (Estimated Time of Arrival)) for each defined airspace region. The ETAs are exchanged between neighboring agents and are compared against negotiated acceptance rates. Overflow situations are resolved by negotiating changes in acceptance rates or by assigning delay to the offending aircraft. For the purposes of this initial investigation, a model was formalized based on a simplified airspace consisting of two neighboring sectors (Figure 1). The sectors set acceptance rates for arrivals across the shared boundary to their respective sector. This simplified model for a two-player (i.e., two-sector) game captured the coupling of dynamics between the two sectors, and it captured the possible interactions

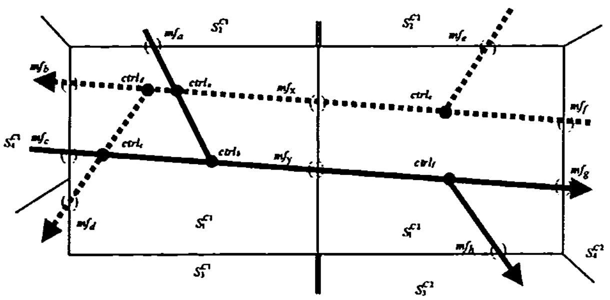


Figure 1: A layout of the airways in two sectors for the two player game-theoretic model for sector metering

between incoming and outgoing traffic flows. The inbound aircraft from other (non-playing) sectors and the outbound flow rate restrictions imposed by those other sectors were generated from a stochastic time-series model. The metering times assigned within each sector to comply with the negotiated acceptance rates were not based upon fixed, in-trail spacing restrictions. Rather, they were based upon assignment of minimum delay subject to the negotiated acceptance rates and minimum separation requirements only. Uncertainty in arrival time estimates was included. Low fidelity models for computation of estimated and scheduled boundary-crossing times (ETAs and STAs, (Scheduled Time of Arrival) respectively), trajectory conflicts, and controller advisories were also developed. A simplified implementation of the algorithms is adopted for this study, capturing inter-sector communication and decentralized decision-making.

Due to space restriction, we only briefly describe the domain. Readers may refer to (Nolan, 1994) for a more detailed description of current NAS operations.

Notation

$S_i^{C_u}$: Sector *i* in center Cu

mf : Sector meter fix (inbound or outbound)

ctrl: A control point (A control point is an intersection or merging point of two airways or a bifurcation point of an airway).

m_e: The indices of the *inbound* meter fixes with respect to the sector

m_o: The indices of the *outbound* meter fixes with respect to the sector

r_{mf}, r_{m_e}, r_{m_o} : The sector meter fix flow rate through meter fix *mf* or meter fix indexed by m_e or m_o

Δt: The time unit in the specified flow rate (i.e., 10 minutes if the flow rate is specified as 4/10 minutes).

$sar_{S_i^{C_u}}$: Sector arrival rate – the rate at which aircraft enter the sector $S_i^{C_u}$. This is equal to the sum of flow rates into the sector $S_i^{C_u}$ - $\sum_{m_e \in S_i^{C_u}} r_{m_e}$.

$sdr_{S_i^{C_u}}$: Sector departure rate – the rate at which aircraft depart the sector $S_i^{C_u}$. This is equal to the sum of flow rates from the sector $S_i^{C_u}$ into other sectors – $\sum_{m_o \in S_i^{C_u}} r_{m_o}$.

c : The capacity of a sector - the maximum number of aircraft that can be present in a sector at a given time. It differs from sector to sector depending the sector controller's ability to manage aircraft

t_0: The absolute value of time when a *stage* game starts. (The game theoretic approaches proposed for the sector metering problem consists of several stage games and is played an infinite number of times.)

ΔT: The duration of the stage game

- s: The minimum aircraft separation distance based on safety requirements
- v_{max}: The maximum cruise speed of an aircraft.
- v_{min}: The minimum cruise speed of an aircraft.
- δ: The minimum separation time that must be kept based on FAA's restriction on the minimum separation distance and the assumption of uniform cruise speed maintained by all aircraft (Note that δ is not same s/v_{max}).
- j, k: The aircraft are indexed in such a way that the PSTA of a low indexed aircraft is earlier than the PSTA of a higher indexed aircraft.
- l: A control point in sector's airways. The control points are indexed as l_1, l_2, ..., l_i, ...such that the STAs of the incoming aircraft must be determined at l_i before l_{i+1}.
- $d_{l_v l_w}$:The distance between two control points.
- $d_{m_e m_o}$:The distance between two meter fixes.
- $d_{m_e l_w}$:The distance between a meter fix and a control point.
- a_{j,m_e} :The j^{th} aircraft entering at m_e^{th} inbound meter fix
- $\text{ETA}_{a_{j,m_e},mf}$: Expected Time of Arrival of an aircraft j at a meter fix mf, a control point, inbound or outbound meter fix
- $\text{STA}_{a_{j,m_e},mf}$: Scheduled Time of Arrival of an aircraft j at a meter fix mf (STA is always equal or greater than ETA)
- $\text{PSTA}_{a_{j,m_e},m_e}$: Predicted STA of the aircraft a_{j,m_e} at m_e^{th} inbound meter fix such that $\text{PSTA}_{a_{j,m_e},m_e} < \text{PSTA}_{a_{j+1,m_e},m_e}$. (Predicted STAs are used only for optimization)
- $\text{PSTA}_{a_{j,m_e},l_w}$: Predicted STA of the aircraft a_{j,m_e} at l_w control point. (Predicted STAs are used only for optimization)
- $\text{PSTA}_{a_{j,m_e},m_o}$: Predicted STA of the aircraft a_{j,m_e} at m_o^{th} outbound meter fix. (Predicted STAs are used only for optimization)
- $\text{PETA}_{a_{j,m_e},m_o}$: Predicted ETA of the aircraft a_{j,m_e} at m_o^{th} outbound meter fix, given that $\text{PETA}_{a_{j,m_e},m_e} = \text{PSTA}_{a_{j,m_e},m_e}$. (Predicted ETAs are used only for optimization)
- $B(l, u, \alpha_1, \alpha_2)$: A beta probability distribution over $l \le x \le u$, where α_1 and α_2 are the two shape parameters

Formulation of the sector metering problem as a game-theoretic problem

Game-theoretic models are well suited to determining sector metering rates across sector boundaries as this involves negotiation between the sectors. It is only by negotiating the flow rates that sector controllers can reduce their workload and increase airspace safety. For example, one sector can negotiate with another sector to set a flow rate so that the former sector controller does not get overloaded with advisories, while the latter sector does not obtain any significant increase in its workload by holding or delaying aircraft. In this paper we formalize our approach using a simplified scenario consisting of two sectors belonging to two different centers, attempting to set the flow rates at their boundary. The simplified model for the two-player (sector) game captures the coupling of dynamics between the two sectors and possible interactions between incoming and outgoing traffic flows. Figure 1 shows the layout of the scenario chosen as a basis of our formulation.

In Figure 1 $S_i^{C_u}$ denotes Sector i in center u, mf denotes the sector meter fix, **ctrl** denotes a control point. The solid and the dashed lines show airways in the west-east and east-west directions respectively. The problem in this context is described as

> *The sectors $S_1^{C_1}$ and $S_1^{C_2}$ play a game to set an equilibrium flow (acceptance) rate across their sector boundary, that should be maintained for duration of ΔT seconds into the future. After the ΔT seconds, they play the game again to set the sector flow rates for the next ΔT seconds.*

The flow rate is said to be in equilibrium if, given r_{mf_x}, the sector $S_1^{C_1}$ cannot change r_{mf_y} in order to increase its *utility*, and given r_{mf_y} the sector $S_1^{C_2}$ cannot change r_{mf_x} in order to increase its *utility*. Here r_{mf}, denotes the sector meter fix flow rate through the meter fix. Note that we have assumed that the two sectors lie in two different centers. In this paper, we model the game as a Bayesian game with Communication.

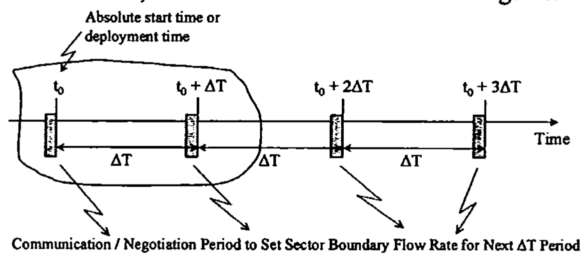
Bayesian Game with Communication

In this formulation, the game consists of two instances of TMA, one for each sector controller deciding a rate that is fixed for a duration of ΔT sec (the duration of the game). Each controller will "push" aircraft into the other sector and schedule aircraft to the sector boundary such that the actual time of arrival at the sector boundary conforms to the rate (action). Note that "conforming to a flow rate" is an important part of the action. For example, if the decision is to push 2 aircraft per ΔT, then the sector can push the two aircraft immediately, one after the other, and not send any aircraft for the rest of the interval. Alternatively, the sector could send one aircraft at the beginning of the interval and the other towards the end. The way the sector conforms to the flow rate will affect the utility of the other sector. Figure 2 summarizes the various components of the game.

This game can be modeled as a strategic game with incomplete information. Recall, in a strategic game, players take actions simultaneously and independently. In a game with incomplete information (i.e., payoff / utility due to their actions are not known to each other – Bayesian games) and absence of common knowledge (i.e., prior probabilities of the opponents' action that is known to or assumed by each player is not a common knowledge), the notion of equilibrium is hard to define. In fact, the games without common knowledge played by two players can be described as each player playing distinctly different games without each other's knowledge and hence equilibrium cannot be defined in such cases. Therefore, we transform the Bayesian game to be played by the sector controllers/players every ΔT seconds into a game with communication. The motivation is to exchange messages (or pre-play communication) in order to uncover the common knowledge and arrive at a Pareto-efficient equilibrium[1] of actions. In the Bayesian game with communication, a message can be considered as the decision regarding the flow rate made by the players. Note that in this context the action is determining the corresponding STAs of aircraft to the meter fix that conform to the prescribed flow rate. Therefore, the exchange of messages does not really reveal the actions to be played by the players[2]. In fact, a player may "cheat/defect" by not strictly conforming to the flow rate that was exchanged in the last message. Figure 3 shows the relationship between the duration of negotiation/ communication, the action and the duration of the game.

Game: The game consists of negotiating for a flow rate and executing an *action* (see action definition) accordingly over a ΔT period during which sector controllers use TMA to monitor and advise the aircraft pilots entering his/her sector and hand over the aircraft leaving his/her sector to the other sectors. At the beginning of every ΔT period, the player also communicates with the neighboring player by exchanging messages about the cross-boundary flow rates.
Player: Sector controller and his/her game-theoretic tool assisting him/her to set the flow rate.
Message: The rate at which a sector (e.g., S_1^{C1}) will push / hand off the aircraft into the neighboring sector (e.g., S_1^{C2}) during the game, that is the ΔT period.
Action: The action of a sector controller (e.g., S_1^{C1}) is to maintain the STAs of the aircraft leaving his/her sector into the neighboring sector (e.g., S_1^{C2}), conforming to the flow rate that he/she had exchanged as a message.

Figure 2 Components of the Bayesian Game with Communication



ΔT can be 1 hour, 4 hours, or morning, afternoon etc. or based on sector controller's shift period

Figure 3: Evolution of Time in the Game

Playing the Game

The solution of the sector-metering problem as formulated in the earlier section requires the identification and development of a methodology/approach to address the following aspects (inputs, models, optimization algorithms etc.) of the game:

- Identification of the utility function to be optimized in the game, and the inputs, variables and decision parameters of the game
- Development of traffic flow models needed for data generation (e.g. ETAs of the flights entering a sector) and optimization
- Development of an optimization algorithm, tailored to meet the specific needs of this problem. The players choose an action (i.e., decides STAs of the flights leaving its sector) that maximizes its utility.

[1] An equilibrium is Pareto efficient if a specified function of the utilities of the players at the equilibrium is maximum. For example, the function could be sum of the equilibrium utilities or product of equilibrium utilities (Zeuthen strategy)

[2] Unlike a typical strategic game discussed in literature, here the duration to complete the action is not instantaneous. As a result, a player / sector controller can see part of its opponent's action before he completes his own action. In that sense, the game being modeled is not strictly a strategic game as is discussed in the literature.

In the following subsections we discuss the technical approach adopted in this paper to address these aspects of the game

Sector Utility

We measure a sector's utility as a weighted sum of the following elements (e.g., for sector $S_i^{C_u}$, components of the utility function are):

1. **Aircraft delay due to traffic,** i.e. $\left(\sum_{m_o \in S_i^{C_u}} \sum_{a_{j,m_e}} \text{PSTA}_{a_{j,m_e},m_o} - \text{PETA}_{a_{j,m_e},m_o} \right)$ for each aircraft a_{j,m_e} leaving the sector at a meter fix indexed by m_o

2. **Ratio of estimated number of aircraft at a given time to capacity,** i.e. $\left([x_0 + (\sum_{m_e \in S_i^{C_u}} r_{m_e} - \sum_{m_o \in S_i^{C_u}} r_{m_o}) \times \Delta T]/c \right) . \sum_{m_e \in S_i^{C_u}} r_{m_e}$ (same as $sar_{S_i^{C_u}}$) and $\sum_{m_o \in S_i^{C_u}} r_{m_o}$ (same as $sdr_{S_i^{C_u}}$) are the arrival and departure rates of the sector and c is the capacity of the sector. x_0 is the number of aircraft at the beginning of ΔT period, i.e., just prior to t_0.

3. **Workload on the sector controller** (Number of advisories issued)

Variables, and Decision parameters

The variables and decision parameters of interest to us in this game include capacity, sector arrival rate, sector departure rate, ETAs and STAs to meter fixes. A key design issue is the ability to either measure or accurately estimate these variables. In some settings, some of these parameters can be directly obtained from high fidelity TMA tools such as Center-TRACON Automation System (CTAS) (Erzberger 1994). However, for the purpose of this investigation, we developed some simple approaches to determine these parameters.

1. Capacity (c): This is an upper bound on the number of aircraft that can be present in a sector at a given time, and is limited by the ability of the sector controller to manage more that a certain number of aircraft. With improved decision support tools, it is expected that this number will increase.

2. *sar*: The Sector arrival rate is the rate at which aircraft are entering into a sector. *sar* is the sum of the sector inflow rates. Thus, $sar_{S_1^{C_1}} = r_{mf_a} + r_{mf_c} + r_{mf_x}$, and $sar_{S_1^{C_2}} = r_{mf_e} + r_{mf_f} + r_{mf_y}$ .In this paper, $r_{mf_a}, r_{mf_c}, r_{mf_e}$, and r_{mf_f} are calculated from a time series model that predicts the inflow rate through the inbound meter fix for every ΔT period. These flow rates are not negotiable. r_{mf_x} and r_{mf_y} on the other hand are negotiable and are determined through a stage game.

3. *sdr*: The sector departure rate is the rate at which the aircraft are departing a sector. *sdr* is the sum of sector outflow rates. Thus, $sdr_{S_1^{C_1}} = r_{mf_b} + r_{mf_d} + r_{mf_y}$, and $sdr_{S_1^{C_2}} = r_{mf_g} + r_{mf_h} + r_{mf_x}$ In the proposed set up $r_{mf_b}, r_{mf_d}, r_{mf_e}$, and r_{mf_h} are calculated for every game (i.e., for every ΔT period) based on a time series model. These rates are also not negotiable. Based on the set-up of the game, we may consider some rates as completely inflexible (hard constraints) and some that can change subject to a penalty.

4. $\text{PETA}_{a_{j,m_e},m_o}$: The Predicted ETA is the ETA of an aircraft a_{j,m_e} (that has not *yet* entered the sector's airspace) at the outbound meter fix m_o,. Note that ETAs are calculated as soon as the aircraft enters the sector's airspace. ETAs of the aircraft at the sector boundary leaving the sectors S_1^{C1} and S_1^{C2} are typically calculated using extensions of the Route Analyzer (RA) and Trajectory Synthesis (TS) modules of the CTAS - Dynamic Planner (DP) (Wong 2000). These tools must be used to calculate PETAs too. However, in our experimental setup, we calculate the PETAs of the aircraft leaving the sectors S_1^{C1} and S_1^{C2} based on a simple calculation that does not consider speed restriction at merging points and the restrictions due to aircraft models and weather models in the calculation of PETAs, as is done currently with CTAS tools.

5. $\text{PSTA}_{a_{j,m_e},m_o}$: The Predicted STA of an aircraft at a meter fix or control point is the STA of the aircraft that has not *yet* entered the sector's airspace, but is predicted to enter the sector airspace through an inbound meter fix at a certain time in the future. Hence, PSTA of that aircraft is calculated assuming the entry-time of several such aircraft. $\text{PSTA}_{a_{j,m_e},m_e}$ is used to denote the PST of an aircraft a_{j,m_e} at the inbound meter fix m_e. $\text{PSTA}_{a_{j,m_e},m_e}$ is calculated given the flow rate r_{m_e} (which in turn is given by a time

series). Given $\text{PSTA}_{a_{j,m_e},m_e}$ of several such aircraft for all inbound meter fixes, $\text{PSTA}_{a_{j,m_e},m_o}$ at the outbound meter fix m_o can be computed using extensions of existing tools such as the DP. While the use of DP will provide a more accurate computation of PSTAs, due to time constraints in this effort we used relatively low fidelity heuristics to determine the PSTAs.

6. We measure the sector controller workload by the number of advisories issued by the controller and the complexity of those advisories (i.e., must the aircraft be prescribed a holding pattern or does it need to be diverted before it is returned to the original airway?). We assume that an advisory is issued when an aircraft's STA at any control point or exit point is not equal to its ETA. The complexity of the advisory is hard to measure, and at this stage of the development we do not consider complexity while computing the workload. Any advisory that results in a holding pattern is treated with an equal weight.

Repeated Strategic Bayesian Game

A Bayesian game with communication is similar to an extensive game (Osborne and Rubinstein 1994). In the case of an extensive game, one player exchanges a message, the other player responds with a message, and this exchange continues for a finite number of iterations until a terminal state is reached. The players then take the action simultaneously. Every ΔT seconds (duration of the game), the game with communication is played in the extensive form as follows (since the Bayesian game is played after every ΔT, it is called a repeated Bayesian game):

1. A sector, say $S_1^{C_1}$ predicts r_{mf_x}, calculates optimal r_{mf_y} and exchanges it with $S_1^{C_2}$. This involves
   a. Predicting $\text{PSTA}_{a_{j,mf_x},mf_x}$ of each aircraft a_{j,mf_x} using a set of probability distributions that conforms to r_{mf_x}.
   b. Calculating optimal $\text{PSTA}_{a_{j,m_e},mf_y}$ using $\text{PSTA}_{a_{j,mf_x},mf_x}$
   c. Calculating r_{mf_y} using $\text{PSTA}_{a_{j,m_e},mf_y}$ and sending r_{mf_y} to $S_1^{C_2}$.
2. Given r_{mf_y}, $S_1^{C_2}$ calculates an optimal r_{mf_x} and exchanges it with $S_1^{C_1}$.
3. Now $S_1^{C_1}$ calculates r_{mf_y} using the given r_{mf_x} and so on

The game reaches a terminal point when, given the arrival rate from the other sector, a sector's calculated rate of departure to the other sector remains the same as that computed earlier. Figure 4 shows a tree called an extensive

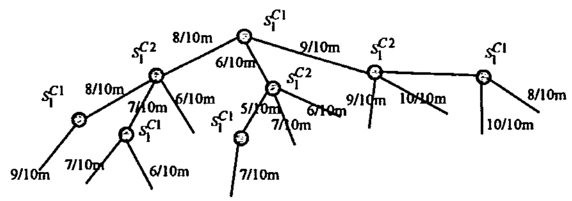


Figure 4: Extensive game tree of the Bayesian game with communication

tree representing the sequential actions of each sector-player. The design of the game involves designing the extensive tree in such a way that it leads to faster convergence to a terminal point.

In the above description of the Bayesian game with communication as an extensive game, the game starts with only one of the sector players initiating a message exchange (Satapathy 1999). However, in a strategic game form of the Bayesian game with communication, the prediction and computation is started by both sectors simultaneously and independently. Hence, in a strategic game form of the Bayesian game, two such extensive game trees can be constructed that progress simultaneously. The convergence occurs when both trees lead to a terminal point simultaneously. Note that it is possible that we may be unable to design the game such that it converges. In such a case, we impose a hard constraint on the number of times the messages can be exchanged and study the Pareto-efficient equilibrium of the strategic game played after the exchange of messages. The design task requires determining suitable constraints on the number of times the messages can be exchanged so that it results in equilibrium closer to the Pareto-efficient equilibrium. In this paper, we adopt a strategic form of the Bayesian game discussed above. The primary tasks associated with this game-theoretic formulation of determining equilibrium flow rate is to develop algorithms to:

1. Predict a probability distribution over the different possible sets of arrival times of the aircraft arriving from the neighboring sector that conforms to a flow rate.
2. Calculate the optimal outbound flow rate given all the constraints discussed in formulation.

Development of traffic flow models needed for data generation and optimization

Figure 5 presents the details of input data required for optimization to compute the flow rate. Details with respect to the sector $S_1^{C_1}$, are shown. The inputs are:

1. The inbound flow rates at the meter fixes mf_a and mf_c, and PSTAs conforming to these flow rates.
2. The inbound flow rate at the meter fix mf_x from the sector $S_1^{C_2}$ and the PSTAs conforming to the flow rate.
3. The outbound flow rate at the meter fixes mf_b and mf_d that the sector controller $S_1^{C_1}$ must maintain.

Based on this input data, the sector $S_1^{C_1}$ calculates the optimal STAs at the meter fix mf_y and thus calculates the optimal sector boundary flow rate from the computed optimal STAs. These optimal STAs are exchanged with the sector $S_1^{C_2}$ as Negotiable STAs (NSTAs). The optimal STAs sent by opponent sectors are called NSTA because the sector $S_1^{C_1}$ believes that these will be the STAs of the aircraft at the meter fix mf_y in the ΔT period. The sector $S_1^{C_2}$ is free to calculate PSTAs based on these NSTAs that conform to the exchanged optimal flow rate, or generate PSTAs that conform to the exchanged optimal flow rate. Similarly, the sector $S_1^{C_2}$ exchanges the optimal flow rate at meter fix mf_x and the NSTAs of the aircraft leaving through mf_x. The sector $S_1^{C_1}$ calculates PSTAs based on these NSTAs that conform to the exchanged optimal flow rate and uses the calculated PSTAs as inputs to the optimization routine as illustrated in Figure 5. We describe our approach for the input data collection and generation and the optimization routine in the following subsections.

The PSTAs of the aircraft entering from the opponent (sector) can be estimated based on NSTAs given by the opponent. However, in order for the opponent sector to send NSTAs, it needs to calculate NSTAs using the optimization routine, which in turn requires PSTAs from its opponent sector and other non-playing sectors. Because of the cyclical dependency of the data requirement, i.e., the data required for optimization depends on data output from the optimization, we resolve the issue by allowing the sectors to *only* exchange the set of aircraft (for the first optimization iteration) that they believe will enter from their own sector to a neighboring sector during the ΔT period. This set of aircraft can be calculated based on the PSTAs of the aircraft entering from the non-playing sectors. The PSTAs of the aircraft entering from non-playing sectors can be estimated based on the past observations. We describe each estimation procedure below:

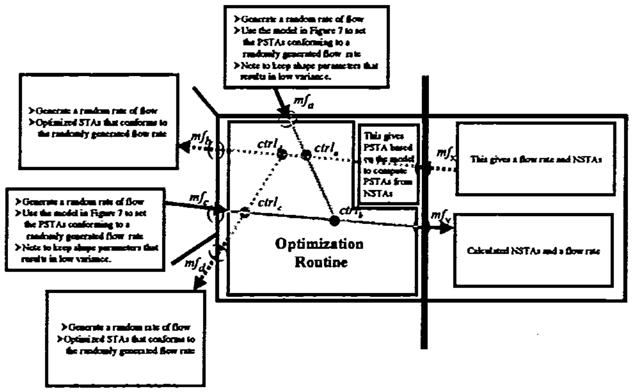


Figure 5: The input data to sector controller's optimization routine to calculate optimal outbound flow rate at the meter fix mf_y

<u>Estimation of the PSTAs from non-playing sectors based on past observations:</u>

The estimation of the PSTAs of aircraft from non-playing sectors involves three parts:

1. Prediction of flow-rate at a meter-fix that feeds aircraft from a non-playing sector
2. Generation of aircraft with its trajectory that will enter through the meter-fix
3. Estimation of PSTA of the aircraft that conforms to the predicted flow rate

The flow rate from non-playing sectors can only be predicted since no negotiation occurs between the sector-player and non-playing sector. We ***assume*** that the flow rates can be predicted from a time series model constructed using the historical data. The assumption that a time-series model is an accurate flow rate prediction model is a reasonable assumption. This assumption can be validated given the Enhanced Traffic Management System (ETMS) and Traffic Management Unit (TMU) log data. Note that the order of the time series is as important as the accuracy of the prediction model. Generally, higher order time series models are more accurate, but the parameters of higher order models are difficult to estimate given the ETMS and TMU log data. For simplicity, we assume a first-order time series, which can be expressed as follows:

$$r_{m_e}(n) = \mu + \phi\, r_{m_e}(n-1) + \varepsilon(n-1)$$

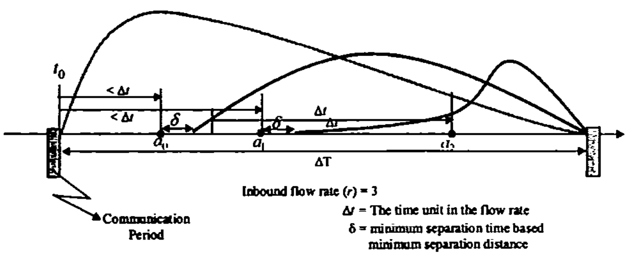


Figure 6: Estimation of PSTAs of the inbound aircraft from the probability distribution functions

where n is the index of the n^{th} stage game, μ and ϕ are the parameters of the first order time series. The parameter ε is randomly generated from a Gaussian distribution constructed from the difference between the last *(n-1)* predictions using the same time series model and the last *(n-1)* actual observations. Note that the order of the time series does not affect the convergence of iterative optimization, but it affects the fidelity of the solution.

rate. One requires a probability function(s) to represent the time when the aircraft arrive at the sector meter fix. As shown in Figure 7, the PSTA of the aircraft a_0 can lie anywhere in the ΔT period, but the probability of PSTA at a particular time is given by a probability distribution function (the red curve). Having PSTA of a_0 set (as shown by red dot), the PSTA of a_1 can be set to lie anywhere between $a_0 + \delta$ (where δ captures the minimum allowable separation distance) and $(t_0 + \Delta T)$ as long as PSTA of a_1 does not violate the inbound flow rate calculated based on the moving average method. The PSTAs of the other aircraft are estimated in a similar fashion.

The estimates of PSTAs of the aircraft are determined by using a set of probability distribution functions as illustrated in Figure 7. The model estimates PSTAs of all aircraft entering a given meter fix (e.g., m_e) such that the flow rate of less than or equal to r_{m_e} is maintained. For simplicity, r_{m_e} is denoted by r and $\mathrm{PSTA}_{a_{j,m_e},m_e}$ is denoted by PSTA_j

If r = 0,
No aircraft needs to be generated, return.
else

$$\mathrm{PSTA}_0 = t_0 + b_0 \quad \text{where}, b_0 \square B(0,\Delta T,\alpha_1^0,\alpha_2^0)$$

For all $j > 0$,

If $\mathrm{PSTA}_{j-1} + \delta < t_0 + \Delta T$

Generate j^{th} aircraft and assign PSTAs as follows:

$$\mathrm{PSTA}_j = \begin{cases} \mathrm{PSTA}_{j-1} + \delta + b_j \quad \text{where}, b_j \square B(0, t_0 + \Delta T - \mathrm{PSTA}_{j-1} - \delta, \alpha_1^j, \alpha_2^j) & \text{if } \sum_{i=1}^{j} g((\mathrm{PSTA}_{j-1} + \delta - \mathrm{PSTA}_{j-i}), \Delta t) < r \\ \mathrm{PSTA}_k + \Delta t + b_j \quad \text{where}, b_j \square B(0, t_0 + \Delta T - \mathrm{PSTA}_k - \Delta t - \delta, \alpha_1^j, \alpha_2^j) & \text{otherwise} \end{cases}$$

where,

$$g(a,b) = 1 \quad \text{if } a \le b; \qquad = 0 \quad \text{otherwise}$$

and

$$\mathrm{PSTA}_j + \delta - \mathrm{PSTA}_k \le \Delta t < \mathrm{PSTA}_j + \delta - \mathrm{PSTA}_{k-1}$$

Else
Do not generate any more aircraft for the ΔT period.

Figure 7 The model to estimate PSTAs of the aircraft at a particular meter fix, given its flow rate

The second and third part of the problem is how to estimate a set of PSTAs at the sector meter fix, given a predicted flow rate, and how to assign those PSTAs to randomly generated aircraft (objects). There can be a number of PSTAs of the aircraft arriving at the inbound meter fixes (e.g., mf_a and mf_c) that will conform to a specified flow

The model in Figure 7 estimates PSTAs only probabilistically. In order to estimate the PSTAs accurately (i.e., the PSTAs to be used in optimization do not vary significantly from the actual STA during the execution of the game), we need to ensure that the variance of the distributions is as low as possible. If the variance of the distribution (which is the error associated with the prediction) is low, then b_j in the model can be substituted with the mean of the distribution i.e., μ_{b_j} . Thus, the PSTA of the aircraft at the meter fixes can be estimated given the predicted inbound flow rates. The model ***assumes*** that the probability distributions associated with the estimation of each PSTA do not change from one stage game to the other, or the shape parameters of the distributions (α_1 and α_2) can be calculated from historical data and past observations (e.g., time series models).

Only a subset of the set of aircraft that is generated using this model can enter the neighboring sector. This subset is calculated as follows: Let $\text{PSTA}_{a_{j,m_e},m_e}$ be the estimated PSTA of the a_{j,m_e} aircraft. The aircraft a_{j,m_e} will enter the neighboring sector between t_0 and ($t_0+\Delta T$), if $\text{PSTA}_{a_{j,m_e},m_o}$ calculated for the sector boundary meter fix m_o using $\text{PSTA}_{a_{j,m_e},m_e}$, the distance between m_o and m_e and maximum cruise velocity of the aircraft a_{j,m_e} is less than ($t_0+\Delta T$).

<u>Estimation of the PSTAs given the set of aircraft:</u>

Estimation of PSTAs given the set of aircraft that enters between t_0 and ($t_0+\Delta T$) is similar to the estimation of PSTAs based on the past observation. The only difference is that the aircraft need not be generated. However, the flow rates still need to be predicted from a times-series like model and PSTAs assigned to the aircraft that conform to that flow rate. This approach is followed when the neighboring sector submits a set of aircraft that are likely to enter between t_0 and ($t_0+\Delta T$). In this model, as in the previous case, if $\text{PSTA}_{j-1}+\delta<(t_0+\Delta T)$ then $\text{PSTA}_j=(t_0+\Delta T)$ or $\text{PSTA}_j=\text{PSTA}_{j-1}+\delta$, whichever is greater.

<u>Estimation of the PSTAs based on exchanged NSTAs:</u>

The exchanged NSTAs and the flow rate are used to estimate PSTAs for next optimization cycle. Neither the NSTAs exchanged during communication nor the final NSTA that have been converged upon can be assumed to be the same as PSTAs. This is because the STAs during the execution of the game need not necessarily conform to the exchanged NSTAs. The difference between STAs observed during the execution and the exchanged NSTAs is captured in a correction model which is applied to the estimation of PSTAs. We ***assume*** that a time series model is appropriate to model the difference between STAs and NSTAs for the j^{th} aircraft entering at every n^{th} stage game. Therefore, the players are required to develop a time series model of the variation of the difference between NSTA and actual STA (STA-NSTA) with each stage-game played, for each j^{th} aircraft. Note that the NSTA in the time series model must be the NSTA that the players converge to before the game is played. Figure 8 illustrates how the (STA-NSTA) of two aircraft indexed 1 and 2 varies with the games that have been played.

In Figure 8, the indices of the games plotted on the x-axis represent consecutive games. In other words, the game indexed 2 is the game played for the period between (t_0 + 2ΔT) and (t_0 + 3ΔT) and the game index 1 (i.e., previous game) is the game played for the period between (t_0 + ΔT) and (t_0 + 2ΔT). In such cases, the time series may not make much sense since the aircraft indexed 1 in the game indexed 1 does not have any relationship with the aircraft indexed 1 in the game indexed 2. This is especially true if the ΔT is short (e.g., 1hr). Therefore, one may divide a day into several ΔT slots or stages, and represent the x-axis in Figure 8 as the games played at a particular ΔT stage over several consecutive days. By modeling the (STA-NSTA) in this way, we can extract the trends/patterns associated with a particular flight.

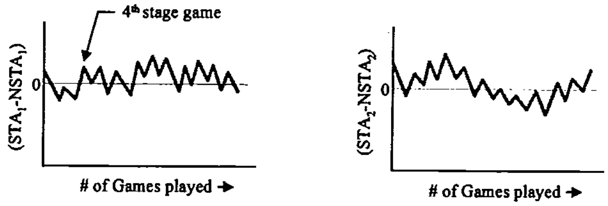


Figure 8: Variation of (STA-NSTA) of two aircraft indexed 1 and 2 with the each game

Let $\theta_{\text{STA}_j}(n)$ denote this correction time series model where n is the n^{th} stage game. This time series essentially predicts or estimates how much the actual STA will differ from the NSTA. The PSTA_j at the $(n+1)^{th}$ ΔT period game, given NSTA_j of the neighboring sector-player, are now calculated from a model similar to the model shown in Figure 7.

The estimated PSTA_j is the same as $\text{NSTA}_j+\theta_{\text{STA}_j}(n)$ for all aircraft except when the optimal flow rate (r_{opt}) sent by the neighboring sector-player is violated. In this case, PSTA_j is $\text{PSTA}_k+\Delta t+\delta$ (added to delay PSTA_j to a later time that does not violate the flow rate). This addition is justified, because according to a sector-player's past observation, the sector-player believes that aircraft will enter at the estimated PSTAs and will also maintain the flow rate. In other words, a sector believes that the neighboring sector-player will push the aircraft at its estimated PSTAs, since exchanged NSTAs only serve as a source of reference.

The error in the estimation is captured by the Gaussian error present in the time-series model. Similar to the estimation of PSTAs, where the error in the estimation is due to the large variance in the probability distribution, the error in this estimation is due to the inaccuracies inherent in the time series.

Optimization algorithm

The following data is available for the maximization problem:

1. $\mathrm{PSTA}_{a_{j,m_e},m_e}$ of the aircraft a_{j,m_e} at m_eth inbound meter fix such that $\mathrm{PSTA}_{a_{j,m_e},m_e} < \mathrm{PSTA}_{a_{j+1,m_e},m_e}$.
2. Trajectories of each aircraft a_{j,m_e} (e.g., we denote trajectory of an aircraft $a_{1,3}$ as [*3*, l_2, l_5, *2*]. This means the aircraft enters at the *3*rd inbound meter fix and leaves at *2*nd meter fix by passing through the control points l_2, l_5). Let the trajectory of an aircraft a_{j,m_e} be denoted by a set $\mathbf{P}_{a_{j,m_e}} = \left\{ p_0, p_1, \ldots p_i, \ldots p_{|\mathbf{P}_{a_{j,m_e}}|} \right\}$ such that p_i is a control point except p_0 and $p_{|\mathbf{P}_{a_{j,m_e}}|}$ which are meter fixes.
3. The rate of outbound flow at the m_oth outbound meter fix (r_{m_o}).
4. Minimum aircraft separation distance based on safety requirement (*s*).
5. Maximum allowable cruise speed (v_{max}).
6. Capacity (*C*) of the sector.

It is required to determine $\mathrm{STA}_{a_{j,\bullet},m_o}$ of the aircraft $a_{j,\bullet}$ [with trajectory containing m_o] at the m_oth outbound meter fix. The following steps illustrates the branch and bound algorithm:

1. Calculate ETAs of each aircraft at the control points and meter fixes using the following formula (Note that control points must be contained in the trajectory of the aircraft a_{j,m_e}):

$$\mathrm{ETA}_{a_{j,m_e},m_o} = \mathrm{PSTA}_{a_{j,m_e},m_e} + d_{m_e m_o} / v_{\max}$$

$$\mathrm{ETA}_{a_{j,m_e},l_i} = \mathrm{PSTA}_{i,m_e} + d_{m_e l_i} / v_{\max}$$

2. Create a bucket B_0 and add all the aircraft along with ETAs into the bucket as STAs of the corresponding aircraft at specified control points and meter fixes[1].
3. Use the algorithm shown in Figure 9 to create (branch) a list of buckets from B_0. Initially, the list of bucket contains only B_0.
4. A bucket is infeasible if it violate the following constraints:

$$v_{\max}(\mathrm{STA}_{a_{j,\bullet},l_i} - \mathrm{PSTA}_{a_{j,\bullet},m_e}) < d_{m_e l_i},$$

$$v_{\max}(\mathrm{STA}_{a_{j,\bullet},l_{i+1}} - \mathrm{STA}_{a_{j,\bullet},l_i}) < d_{l_i l_{i+1}}$$

$$\mathrm{STA}_{a_{j,\bullet},l_i} < \mathrm{PSTA}_{a_{j,\bullet},m_e}$$

[1] So bucket contains both ETAs and STAs of the aircraft, but both are equal in B_0

➔ For each bucket B_k from the list of buckets
- ➔ For each control point l_i (this set of points includes both control points and outbound meter fixes)
  - ➔ Separate the aircraft into two buckets $\hat{B}_k$ and $\breve{B}_k$ such that $\hat{B}_k$ contains ETAs of those aircraft whose trajectory contains l_i and $\breve{B}_k$ does not aircraft with point l_i
    - ➔ Sort the aircraft in $\hat{B}_k$ by the ascending order of $\mathrm{STA}_{a_{j,\bullet},l_i}$ of the aircraft $a_{j,\bullet}$ such that $\mathrm{STA}_{a_{j,\bullet},l_i} < \mathrm{STA}_{a_{j+1,\bullet},l_i}$
      - ➔ Starting from $j = 0$, if $v_{\max}(\mathrm{STA}_{a_{j+1,\bullet},l_i} - \mathrm{STA}_{a_{j,\bullet},l_i}) < s$, then make two copies of $\hat{B}_k$
      - ➔ In one copy ($\hat{B}'_k$), replace $\mathrm{STA}_{a_{j+1,\bullet},l_i}$ by $\mathrm{STA}_{a_{j,\bullet},l_i} + s / v_{\max}$ and in the other ($\hat{B}''_k$) replace $\mathrm{STA}_{a_{j,\bullet},l_i}$ by $\mathrm{STA}_{a_{j+1,\bullet},l_i} - s / v_{\max}$
      - ➔ Adjust $\mathrm{STA}_{a_{j+1,\bullet},l}$ of $a_{j+1,\bullet}$ at all control points l that comes after l_i to compensate the delay at l_i.
      - ➔ Adjust $\mathrm{STA}_{a_{j,\bullet},l}$ of $a_{j,\bullet}$ at all control points l that comes before l_i to ensure early arrival at l_i.
      - ➔ Add $\breve{B}_k$ to $\hat{B}'_k$ and $\hat{B}''_k$, and replace B_k in the list of buckets with these two buckets
      - ➔ Apply bucket elimination procedure to eliminate infeasible buckets.

➔ Repeat until the condition $v_{\max}(\mathrm{STA}_{a_{j+1,\bullet},l_i} - \mathrm{STA}_{a_{j,\bullet},l_i}) < s$ for all aircraft holds for all the buckets

Figure 9: Steps of the Branch and Bound Algorithm

$\sum_{a_{j,\bullet}} g((\text{STA}_{a_{j,\bullet},m_0} - \text{PSTA}_{a_{k,\bullet},\bullet}),0)/C > 1$, for all $a_{k,\bullet}$ and $a_{j,\bullet}$ as long as $\text{PSTA}_{a_{j,\bullet},\bullet} < \text{PSTA}_{a_{k,\bullet},\bullet}$ (This constraint evaluates the number of aircraft present in the sector's airspace when an aircraft enters the sector's airspace)

The calculated flow rate at m_0 ($r_{m_0}^{calc}$) does not violate r_{m_0}.

The flow rate $r_{m_0}^{calc}$ is calculated by the following equation:

$$\forall a_{j,\bullet},\ \text{if}\,(\text{STA}_{a_{j,\bullet},m_0} < (t_0 + \Delta T)),$$

$$r_{m_0}^{calc} = \max\Big(\sum_{a_{k,\bullet} \wedge (\text{STA}_{a_{k,\bullet},m_0} > \text{STA}_{a_{j,\bullet},m_0})} g(\text{STA}_{a_{k,\bullet},m_0} - \text{STA}_{a_{j,\bullet},m_0}), 0\Big)$$

5. For each bucket from the list of remaining buckets, calculate the utility value. Pick the bucket with maximum utility. Note that rerouting is required, if we have following conditions: $v_{\min}(\text{STA}_{a_{j,\bullet},l_i} - \text{STA}_{a_{j,\bullet},m_e}) > d_{m_e l_i}$ and $v_{\min}(\text{STA}_{a_{j,\bullet},l_{i+1}} - \text{STA}_{a_{j,\bullet},l_i}) > d_{l_i l_{i+1}}$

In our two sector-player game setup, the sector S_1^{C1} sends the optimal $\text{STA}_{a_{\bullet,\bullet},m_o}$ as $\text{NSTA}_{a_{\bullet,\bullet},m_o}$ to S_1^{C2} (where m_o is mf_x) and S_1^{C2} sends optimal $\text{STA}_{a_{\bullet,\bullet},m_o}$ as $\text{NSTA}_{a_{\bullet,\bullet},m_o}$ to S_1^{C1} (where m_o is mf_x) as long as optimal STAs is less than ($t_0 + \Delta T$). The sectors also exchange optimal flow rate $r_{m_0}^{calc}$ calculated using optimal $\text{STA}_{a_{\bullet,\bullet},m_o}$. The optimal flow rate calculated using a moving average method is as follows: the number of aircraft counted to cross the sector boundary meter fix m_o between $\text{STA}_{a_{\bullet,\bullet},m_o}$ and ($\text{STA}_{a_{\bullet,\bullet},m_o} - \Delta t$) is the flow rate when the aircraft $a_{\bullet,\bullet}$ crosses the meter fix m_o. The average of all such flow rate over the number of aircraft crossing between t_0 and (t_0 + ΔT) is the optimal flow rate r_{m_o} at the meter fix m_o for the period between t_0 and (t_0 + ΔT).

Figure 10 illustrates how optimization routine is invoked iteratively until both sector converges to and equilibrium flow rate and STAs. The sector S_1^{C1} converges to a flow rate $\sum_{m_o \in S_1^{C1}} r_{m_o}$ if at the i^{th} iteration, it finds $\sum_{m_o \in S_1^{C1}} r_{m_o}(i) \square \sum_{m_o \in S_1^{C1}} r_{m_o}(i-1)$. Similarly, The sector S_1^{C2} converges to a $\sum_{m_o \in S_1^{C2}} r_{m_o}$ if at the i^{th} iteration, it finds $\sum_{m_o \in S_1^{C2}} r_{m_o}(i) \square \sum_{m_o \in S_1^{C2}} r_{m_o}(i-1)$. Note that both sector need to converge at the same iteration in order to determine an agreed-upon $\sum_{m_o \in S_1^{C1}} r_{m_o}$ and $\sum_{m_o \in S_1^{C2}} r_{m_o}$. In that case, the flow rate $\sum_{m_o \in S_1^{C1}} r_{m_o}$ is $\sum_{m_o \in S_1^{C1}} r_{m_o}(i)$ and $\sum_{m_o \in S_1^{C2}} r_{m_o}$ is $\sum_{m_o \in S_1^{C2}} r_{m_o}(i)$. The sectors also need to converge at equilibrium PSTAs. The sector S_1^{C1} converges to equilibrium STA, if $\text{STA}_{a_{j,\bullet},m_o}(i) \square \text{STA}_{a_{j,\bullet},m_o}(i-1)$ for each aircraft $a_{j,\bullet}$ leaving any of the outbound meter fixes between t_0 and (t_0 + ΔT). The same is true for the sector S_1^{C2}.

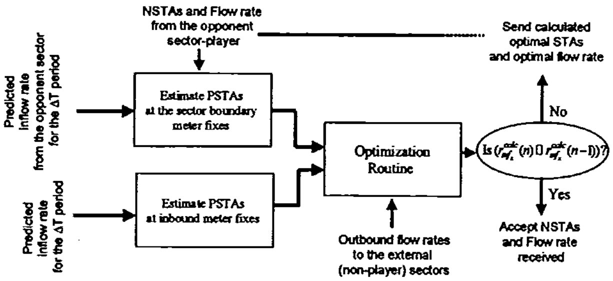


Figure 10: Iterative optimization process until calculated flow rate for two consecutive iterations are the same

The number of iterations required for convergence can be assessed from the airspace dynamics (i.e., how frequently the flow rates change from one stage game to another and the trajectories of the aircraft) and the predictability of the PSTAs. We study this through simulation described in the next section.

Implementation for Game Simulation

To study the robustness, sensitivity, stability (i.e. convergence) and uniqueness of the solution (i.e. equilibrium sector boundary flow rate), we developed and implemented software for optimization and simulation of the two-player game. As a part of the simulator we modeled some features of decision support tools and sector controller rules used for issuing holding directives. The simulation and the optimization were adequately coupled/interfaced since the optimization routine requires data (set of aircraft still flying the airspace when optimization gets started prior to ΔT) from simulation and vice versa. In the following sections we discuss the implementation aspect of the software in some detail.

User set parameters

The user is allowed to set up ΔT, start time (t_0), flow rate unit (Δt), minimum separation distance (s), minimum aircraft cruise velocity (v_{min}), maximum aircraft cruise velocity (v_{max}), capacity (C), and weights in the utility function. The default values of these parameters are: ΔT = 1hr (3600 seconds), t_0 = 00:02, Δt = 20 minutes, s = 5miles, v_{min} = 6miles/min, (v_{max} = 11miles/min, c = 10. The parameters such as airway segment information are read from files (sector1_config.txt & sector2_config.txt). Each meter fix has its own configuration file that sets up parameters required for the time series model for flow rate generation, the Gaussian error parameters associated with the time series model, the shape parameters of the Beta probability distributions used to determine PSTAs of the incoming aircraft, and the list of trajectories that can be randomly assigned to the aircraft entering through that meter-fix (this is true only for the inbound meter fix from the non-playing sectors). The choice of these parameters is discussed at length in the following section.

Data collection and generation

Tuning parameters for estimation of the PSTAs based on past observations

The first set of parameters required for the estimation of the PSTAs based on past observation are the values of μ, ϕ, and the mean and variance of the Gaussian error ε of the time series model. These parameters can be determined based on ETMS data. We used several combinations of μ (0 – 8), ϕ (0.0 – 0.25), Gaussian error mean (0 – 2) and Gaussian variance (0 – 0.44) for our experimental design. In a time series analysis, the mean and variance of the Gaussian error is calculated based on the past observations, but in our experimental design we used the constant Gaussian mean and variance. When the value of the ϕ, Gaussian mean and variance is 0.0 then the maximum inflow rate at the inbound meter fix for which the time series is relevant is μ at the specified flow rate unit (Δt). If there is some non-zero Gaussian variance associated with it, then the maximum inflow rate at the meter fix oscillates around μ. The value of the ϕ indicates how strongly the n^{th} stage flow rate depends on the $(n-1)^{th}$ stage.

The model described in Figure 7 estimates the PSTAs of each aircraft given the flow rate from the time series model for the n^{th} stage game. The shape parameters of the probability distributions are set in such a way that the distribution is more skewed towards the left for the first few aircraft entering the meter fix starting from t_0 and gradually the skew shifts towards the right as the PSTAs calculated from the model as described in Figure 7 approach ($t_0 + \Delta T$). For example, for the meter fix mf_a the shape parameters are arranged as: (2, 40), (3, 42), (5, 38), (6, 36), (8, 34), (10, 33), (12, 32) .... As far as the optimization is concerned, the $PSTA_j$ of the j^{th} aircraft is determined by replacing b_j (refer to the model described in Figure 7) by the mean of the distribution b_j, which is defined by the shape parameters as: $(\alpha_1/(\alpha_1+\alpha_2))$. When a $PSTA_j$ is determined to lie between t_0 and ($t_0 + \Delta T$), an aircraft object is generated which is assigned with that $PSTA_j$ to be the time at which it is expected to enter the meter fix. Each aircraft generated is assigned a trajectory.

The inbound meter fix configuration file lists all the trajectories that can be assigned to an aircraft if an aircraft originates from that meter fix. For example, the aircraft originating at the meter fix mf_a can have two trajectories - $mf_a \rightarrow ctrl_a \rightarrow ctrl_d \rightarrow mf_y \rightarrow ctrl_f \rightarrow mf_h$ and $mf_a \rightarrow ctrl_a \rightarrow ctrl_d \rightarrow mf_y \rightarrow ctrl_f \rightarrow mf_g$. The trajectories are assigned at random.

The generated aircraft objects are sent to the neighboring sector, if their $PSTA_j$ at the entering meter fix will result in their exiting into the neighboring sector between t_0 and ($t_0 + \Delta T$). Each sector obtains a list of such aircraft objects that just contains information about the sequence, but does not have information regarding the time at which the aircraft would enter the opponent's sector though the sector boundary meter fix. The sectors predict their flow rate and time of entry (i.e. $PSTA_j$ at the sector boundary meter fix).

Tuning parameters for estimation of the PSTAs given the set of aircraft

The set of aircraft objects sent by a sector is the sum of two subsets:

1. The subset of the generated aircraft objects for the period between t_0 and ($t_0 + \Delta T$) whose $PSTA_j$ at the sector boundary meter fix is between t_0 and ($t_0 + \Delta T$).
2. The subset of aircraft objects that are already in the airspace, but whose ETAs at the sector boundary meter fix is between t_0 and ($t_0 + \Delta T$)

Estimation and assignment of $PSTA_j$ at the sector boundary meter fix to each of the aircraft objects sent by the neighboring sector is similar to the previous case. The values of μ, ϕ, and the mean and variance of the Gaussian error ε of the time series model reflect the number of inbound meter fixes of the neighboring sector feeding into a particular sector boundary meter fix. For example, if μ_{mf_a} and μ_{mf_c} are the values of μ of the two time series models for the meter fix mf_a and mf_c, then the μ of the time series model for mf_y is approximately $(\mu_{mf_a} + \mu_{mf_c})$. Similarly other parameters of the time series model for mf_y reflect a similar relation with the parameters of the time series model for the meter fix mf_a and mf_c. The shape parameters of the probability distributions that determine the $PSTA_j$ of each j^{th} aircraft for the optimization are however set independently of the shape parameters of the probability distributions that determines the PSTAs at mf_a and mf_c.

Tuning parameters for estimation of the PSTAs based on exchanged NSTAs

The NSTAs are exchanged after the first optimization iteration. The PSTAs are calculated from NSTAs and the optimal flow rate sent by the opponent sector. Hence, there is no need to predict the flow rate or generated PSTAs using a probability distribution. Instead a time series model is used as a correction model to convert an NSTA to a PSTA. The parameters of this time series model (i.e., μ, ϕ_1,

ϕ_2, ϕ_3 ... etc. depending the order of the model) must be built or trained with data assuming several games have been played so that we have (STA_j –$NSTA_j$) data for each j^{th} aircraft. However, in order to play the games, we need to assume a time series model with its parameters that approximately reflects a realistic situation[1]. For this experimental design, we assumed a first order time series model with $\mu = 20$, $\phi_1 = 0.5$, Gaussian error mean = 0, and Gaussian variance = 10000. Note that the variance reflects the reliability of the correction model. A variance of 10000 indicates that the PSTAs calculated using the correction model might lie between ±300 seconds. For this experimental design we kept the same μ and ϕ_1 for each j^{th} aircraft.

Optimization

The optimization routine is invoked with the flight information of all the aircraft that will be flying through the sector's airspace between t_0 and ($t_0 + \Delta T$). For example, the optimization routine of the sector S_1^{CI} takes into consideration the following aircraft into its initial bucket:

1. The set of aircraft entering through mf_a between t_0 and ($t_0 + \Delta T$) as calculated by the model – "Estimation of PSTAs based on past observations".
2. The set of aircraft entering through mf_c between t_0 and ($t_0 + \Delta T$) as calculated by the model – "Estimation of PSTAs based on past observations"
3. The set of aircraft entering through mf_x between t_0 and ($t_0 + \Delta T$) as calculated by the model – "Estimation of PSTAs given the set of aircraft" (for the first optimization routine) or by the model – "Estimation of PSTAs given the NSTAs" (for subsequent optimization iteration).
4. The set of aircraft that has entered the airspace prior to t_0 and are still in the airspace after t_0.

A bucket is branched off into two buckets as described in the optimization routine. Figure 11 illustrates how a bucket is branched off into two buckets. Each new bucket of aircraft is considered further if the new STAs assigned to the aircraft indicating STAs at the control points can be maintained without exceeding the maximum cruise velocity.

The control point STAs assigned to the aircraft may need to be increased or adjusted depending on the holding

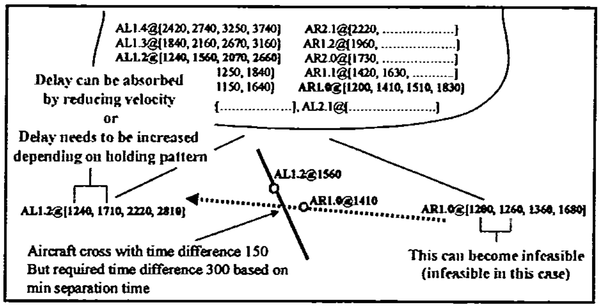


Figure 11: Illustration of how a bucket is branched into two buckets and what values are changed (In this case STAs at the control points are changed)

pattern to be enforced. Some other rules applied to check the validity of the STAs are illustrated in Figure 12. For example, in merging airways, the aircraft to be held depend on the location of the other aircraft causing it hold.

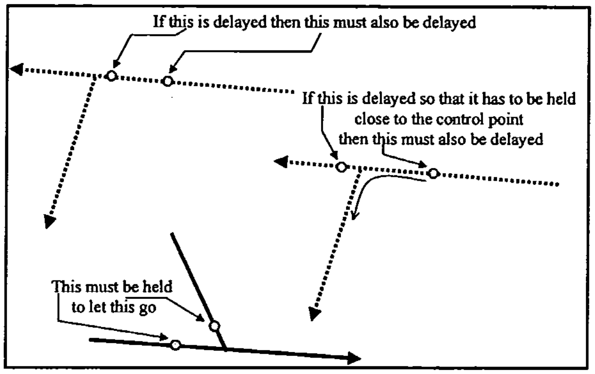


Figure 12: Safety criteria applied to adjust the STAs

Software and Simulation of the Game

We implemented the optimization and simulation modules using OpenCybele™ agent infrastructure in order to implement the interface easily (Refer www.opencybele.org for more details on OpenCybele). OpenCybele allows easy decomposition of problem into agents and activities and allows the programmers to write decision support tools that run concurrently with the simulation threads. In the implementation, the optimization for the two sectors takes place concurrently in different execution threads.

The simulator is used to simulate the behavior of a sector controller issuing advisories to aircraft and introducing randomness to the time of entry of aircraft from non-playing sectors. The main features of the simulator are as follows:

1. The rates at which aircraft are injected into the sectors during the simulation are different from the rates used in the optimization algorithm. This implies that the values of μ, ϕ, and Gaussian error parameters of the time series model used for simulation are slightly different from the

[1] One way to estimate the parameters of the time series is to collect the difference between STAs (not ETAs) at the sector boundary when an aircraft departs an airport and when it actually arrives at the sector boundary. This information can be collected from ETMS data and sector controller's log file.

ones used in optimization. This was done to show the robustness of our algorithm. We conjecture that a large difference in the aircraft-feed rate will demonstrate the need to develop accurate time series model or mandate exchange of NSTA similar to the exchange of NSTA by the two sector players in this experimental setup.

2. The probability distributions shape parameters used to calculate PSTAs is slightly different in simulation to demonstrate the robustness of the algorithm.

3. The simulator contains features to issue control advisories such as (1) reduce speed or (2) hold the aircraft. The simulator does not issue tactical advisories such as how to hold an aircraft due to limited time and resource. Since this part of the simulation can be eventually be replaced by other high fidelity TMA tools.

Analysis of Simulation Results

We ran several simulation scenarios, by changing the maximum inflow rate at meter fixes mf_a, mf_c, mf_e, and mf_f, to analyze the sector boundary flow rates determined by the game. These scenarios are described as follows:

1. The μ and ϕ of the time series model that restrains the maximum flow rate of aircraft through mf_a is set to 8 and 0.2 respectively. The mean and variance of the Gassuain error associated with model is set to 0 and 0.111. The flow rate through all other inbound meter fixes is set to zero for both sectors. Because of this setting, one observes at most (8±2) aircraft entering per 20 minutes into the sector $S_1^{C_1}$ and at most (8±2) aircraft are leaving per 20 minutes into the sector $S_1^{C_2}$ from $S_1^{C_1}$ which happens to be the negotiated sector boundary flow rate from $S_1^{C_1}$ to $S_1^{C_2}$. The sector boundary flow rate from $S_1^{C_2}$ to $S_1^{C_1}$ is zero. In the next stage game which occurs after (ΔT/Simulation speed) seconds (i.e., 3600/20 = 180 seconds), the maximum flow rate into the sector $S_1^{C_1}$ will be at most (8 + 0.2×(8±2) ±2) aircraft per 20 minutes.

2. The μ and ϕ of the time series model that restrains the maximum flow rate of aircraft through mf_a and mf_c are set to 5 and 0.2 respectively. The mean and variance of the Gassuain error associated with both models is set to 0 and 0.111. In this scenario, one may sometimes observe holding at the merging control point $ctrl_b$. As a result, the sector boundary flow rate from $S_1^{C_1}$ to $S_1^{C_2}$ will be less than the sum of the inflow rate through meter fixes mf_a and mf_c. The maximum inflow rate through one of these meter fixes will be (5±2). In the next stage game, the maximum inflow rate will be (5 + 0.2×(5±2) ±2) and so on. Note that during game execution, one may observe holding prior to the outbound meter fixes mf_g and mf_h in the sector $S_1^{C_2}$. This holding is due to the flow rate restriction set for the outbound flow rate through meter fixes mf_g and mf_h. The μ and ϕ of the time series model that sets this flow rate for every stage game are 3 and 0.2 respectively.

3. In the third scenario, μ and ϕ of the time series model that restrains the maximum flow rate of aircraft through mf_a and mf_c are both set to 4 and 0.2 respectively and the μ and ϕ of the time series model that restrains the maximum flow rate of aircraft through mf_e are set to 3 and 0.2. In this scenario, one may observe a non-zero negotiated sector boundary flow rate being set from both sectors. As the game continues, one may observe that the negotiated sector boundary flow rate from the sector from $S_1^{C_1}$ to $S_1^{C_2}$ will keep on rising higher than the negotiated sector boundary flow rate from the sector from $S_1^{C_2}$ to $S_1^{C_1}$. One may also observe that there will be aircraft holding in sector $S_1^{C_2}$ prior to sector boundary meter fix mf_x and on the segment connecting mf_x and $ctrl_e$ in order to maintain the low sector boundary flow rate at mf_x. That means, $S_1^{C_2}$ will witness some advisories due to the sector boundary flow restriction, which it would not have occured if there were no aircraft injected to the sector $S_1^{C_1}$ and/or east-west airway intersecting west-east airway at control points $ctrl_a$ and $ctrl_c$.

4. In the fourth scenario, aircraft enter through all the inbound meter fixes. μ and ϕ of the time series model that restrains the maximum flow rate of aircraft through mf_a and mf_c are set to 6 and 0.2 respectively and the μ and ϕ of the time series model that restrains the maximum flow rate of aircraft through mf_e and mf_f is set to 3 and 0.2. Because of the high inflow rate of aircraft into the sector $S_1^{C_1}$ in addition to some aircraft from $S_1^{C_2}$ to $S_1^{C_1}$, one may observe that the flow rate from $S_1^{C_2}$ to $S_1^{C_1}$ is *much* less than the sum of the inflow rate through mf_e and mf_f. Similar to the third scenario, the sector boundary flow rate through meter fix mf_x does not increase with stage game as the flow rate through meter fix mf_y increases. One may argue that the number of advisories incurred by the sector $S_1^{C_2}$ due to low flow rate set at meter fix mf_x will decrease as a result of increasing the flow rate. However, by increasing flow rate, the sector $S_1^{C_1}$ will try to increase its flow rate through mf_y. Consequently, the sector $S_1^{C_2}$ will incur additional advisories for holding aircraft prior to the outbound meter fix mf_g and mf_h because of the low flow rate restriction at the meter fix mf_g and mf_h compared to

the sector boundary flow rate at mf_y. In summary, the sector $S_1^{C_2}$ does not gain by increasing the flow rate at mf_x, but the sector $S_1^{C_1}$ gains by low flow rate set at the meter fix mf_x.

Observe that the metering times (i.e., time at which the aircraft enter or scheduled to enter) are not calculated based on miles-in-trail restriction in order to maintain the flow rate. The metering times are calculated based on a moving average method so that the maximum flow rate is the set flow rate. In other words, in any given flow rate unit (e.g., 20 minutes), the number of aircraft crossing a meter fix does not exceed the flow rate set for that meter fix. This is an important migration from current technique of setting metering times that creates an unnecessary number of advisories. The un-equal intervals of the scheduled arrival and departure times of aircraft demonstrate that we do not follow miles-in-trail restriction. However, we observed minimum separation distance between two aircraft. We also noticed that the average delay occurred in a particular ΔT period for both sectors are low while the density remains reasonably high.

An important and intuitive observation is that computation time for optimization does not depend on the number of aircraft, but their expected PSTA at the time of entry that may lead to holding and thus require the need to create more buckets. Therefore, if the initial bucket contains PSTAs of the aircraft that lead to conflict (minimum separation distance violation) at every control point between every two aircraft then the computation time may increase exponentially. However, the likelihood of such a scenario is not high in real world.

Also observe that the time (i.e., the number of optimization iterations) it takes for the sectors to converge to a solution (PSTAs at the sector boundary & the flow rate) depends on the following – (1) parameters of the correction model, (2) capacity constraints, (3) outbound flow rate constraint, and (4) the number of control points involving east-west airways intersecting west-east airways. In our experimental set up, since we did not consider capacity and outbound flow rate constraints in our optimization, and since the right sector does not have any control points involving east-west airway intersecting west-east airways, our optimization converges after every two iterations. This is because the right sector accepts any flow rate that the left sector sends. Lastly, we believe that the time for convergence depends on how many sectors are negotiating concurrently, and is important aspect to consider in multi-sector negotiation.

Conclusions

In this paper we developed a theoretical framework for sector metering based on a Bayesian game with communication, and implemented the optimization and simulation in OpenCybele agent infrastructure. For specific scenarios, we also demonstrated how the sector boundary flow rates are negotiated after every ΔT period depending on the rate at which aircraft are injected and the number of aircraft present in the airspace from the previous ΔT period. The sector boundary rates are not calculated based on miles-in-trail restriction. Based on our preliminary observations, discussions with Subject Matter Expertise (SME) and Aviation Consultants regarding these observations we conclude that our the approach provides an innovative and feasible solution to collaborative sector metering to reduce delays, improve efficiency and controller workload.

Future work includes the extension of the two-sector game and the development a DST for multi-sector metering using more realistic sector/center geometries, traffic flow interactions and ETMS data to tune the time series and optimization models. Issues related to convergence of the game, stability and sensitivity of the algorithms, and real-time issues will be addressed. High fidelity models for the sector utility will be developed.

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