Non-Rational Agents Explain GARCH Model: Agent Simulation for Behavioral Finance

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Abstract

GARCH (Generalized Autoregressive Conditional Hetero-scedasticity) is a macro level model to estimate the volatility of financial markets. Although the model is very fundamental in the financial and economic domain, however, there have been no clear explanation about the model from the micro-level financial behaviors. This paper develops agent-based simulation models, which consists of simple agents with rational and/or non-rational decision making functionalities for investment. Using the simulation model with both rational and non-rational agents, the paper has shown that the behaviors of the non-rational agents with the characteristics of prospect theory coincide with the estimation by GARCH model.

Key Words: Multiagent-based simulation for economic issues, Society dynamics, Self-organizing systems and emergent organization, Financial Engineering

INTRODUCTION

Understanding the mechanisms of financial markets is critical because of the rapid deployment of e-commerce and the rigidity of economic systems. So far, economic analysts have used macroscopic mathematical models to understand the mechanisms. The classical models in finance are analytical, thus, make assumptions regarding the market and the behavior of individuals operating in the market. The relations of such analytical models and the mental models of the individuals or agents have not become clear, yet (Arthur 1997), (Levy 2000). We do not have had rigid foundations of the validity of the macro-level analytical models. In this paper, we address agent-based approach and develop simulation models to evaluate financial market behaviors.

The rest of the paper is organized as follows: In Section 2, define the problems we would like to uncover. In Section 3, we describe our agent-based simulation models, then in Section 4, using the models, intensive experiments are carried out. Section 5 is devoted to the discussion of the experiments and related work. In Section 6, some concluding remarks will be given.

OBJECTIVES OF THE RESEARCH

The following three models are often used to estimate the volatility of stock market data: Ordinary Least Squire (OLS) Model, Generalized Autoregressive Conditional Heteroscedasticity (GARCH) Model (Bollerslev 1986), and Asymmetric GARCH Model (Glosten 1993). If the agents would follow the usual assumptions of the analytical models: they are rational about market information and maximize their expected utilities, they have homogeneous expectations regarding future distribution of returns, they all have same holding period, and so on. OLS model would coincide with the real market phenomena.

OLS Model:

$$\varepsilon_t \sim N(0, \sigma^2), \text{where } \sigma^2 \text{ is constant,}$$

GARCH Model:

$$\sigma_t^2 = a + \sum_{i=1}^{p} a_i \cdot \varepsilon_{t-i}^2 + \sum_{i=1}^{q} b_i \cdot \sigma_{t-i}^2,$$

Asymmetric GARCH Model:

$$\sigma_t^2 = a + \sum_{i=1}^{p} \left(a_i \cdot \varepsilon_{t-i}^2 + \gamma_i \cdot S_{t-i} \cdot \varepsilon_{t-i}^2\right) + \sum_{i=1}^{q} \cdot \sigma_{t-i}^2,$$

$$S_{t-i} = \begin{cases} 1 & \text{if } \varepsilon_{t-i} < 0 \\ 0 & \text{if } \varepsilon_{t-i} \geq 0, \end{cases}$$

where,t,y,ε, and σ^2 are time periods, estimation values, errors, and the variance of Normal distribution, respectively. The coefficients of a, b, and c are respectively parameters to be determined by real data. The analytic models so far are only able to explain how OLS model works.

To apply agent-based models to financial economic domain, in this research, we define the following two problems. These problems have been frequently reported in the finance literature using real world data, however, they have not been yet uncovered (Arthur 1997), (Levy 2000).

· Examining the Roles of non-rational Agents in the Market:

Friedman states in (Friedman 1953) that by the law of natural selection, non-rational investors should not survive and only rational investors would remain in the market. But, the statements seem not to hold. We also examine this via agent-based simulation.

- Agent Characteristics to Coincide with (A-)GARCH Model:
As shown above, (Asymmetric-)GARCH Model shows the best sit to the real data. Real market behaviors often show the price change similar to the ones GARCH Model generates. Using agent-based simulation, we would like to determine the agent characteristics to explain GARCH model. Our simulation models are at the similar complexity to the ones used in microscopic simulation (Levy 2000) and Sugarscape (Epstein 1996). The agents have a small set of decision rules to determine their attitudes about the market conditions. They are represented by simple equations. We will design the agents, which are neither game-theoretic ones nor simple ones with the KISS principle in (Axelrod 1997), because we will validate the agents' behaviors compared with real phenomena. We also will design the agents with complex functionalities as reported in (Takadama 1999), (Terano 1998). In OCS (Organizational Learning-oriented Classifier System) (Takadama 1999) the agents learn rules represented by classifiers using Genetic Algorithms and change good ones with each other. In TRURL (Terano 1998) the agents make decisions using multi attribute decision functions, communicate each other, and move around to gather together if they have similar knowledge. Compared with them, the agents in this paper are so simple that we are able to analyze the characteristics of them.

AGENT-BASED MODELING FOR THE FINACIAL MARKETS

In this section, we describe the agent-based models, which are extension of traditional finance models. First, we describe the components of the market, then determine the trading rules of the investor agents.

Components of the Market

The market we define on the computer consists of 1,000 investor agents. Each agent trade the assets as either individual stocks or riskless assets. At each simulation step, the benefits or losses will occur based on the Brownian motion, and each agent trades its asset based on its benefits/losses and past pricing information. One of the unique points of the agents we design is that the non-rational agents have the characteristics with Prospect Theory in socio-psychology. Prospect Theory is reported in e.g., Kahneman et al. (Kahneman 1979). The theory states the decision making model of human beings have the characteristics that (1) the decision is made based on the change from some reference points; (2) the decision attitude is different when the benefit or loss will occur; (3) the loss is considered to be larger than the benefit; and (4) the decision is made based on subjective probability of the events. In our agents model, the agents with Prospect Theory think the loss is twice as much as the benefit (Kagel 1995), (Shleifer 2000). Table 3 is a summary of the market components. Figure 1 also shows the outline of the agent architecture.

Table 1: The Components of the Agent-Based Market Simulator

Number of Asset	1,000	–
Assets (Two Kinds)	Stocks(1,000 units) Riskress Asset	Same Condition in Arthur
Changes of Benefits	Brownian Motion	Same Condition in Shleifer
Type of Investor Agents	Rational	Dividend Discount Model
	Prospect Theory	Estimate the loss twice larger
	Over Confidence	Estimate the Volatility Smaller
	Trend Predictor	Extrapolate the past ten days trading values

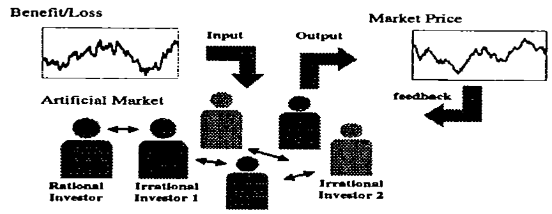

Figure 1: The Agent Architecture of the Simulator

Decision Making of the Investor Agents

All the investor agents determine their asset allocation based on the sum of (i) long term expected return ratio (SAA) and (ii) short term expected return ratio. The model is known as Black and Litterman Model in (Arthur 1997). The decision making strategies of the agents in our simulation model have very small difference. the SAA part is common to the all the agents, however, the decision on the short term expected return ratio is different. The rational agents determine the short term expected return ratio is calculated by *Forecast_short* statement in Figure 3. On the other hand, the ones of the irrational agents are determined by the same line of the following Figures 4, 5, and 6.

In the simulation cycle, the market price is determined by the value with which the demand and the supply are coinciding. The general decision-making algorithm is shown in Figure 2. In the following subsubsections, we explain how the investor agents determine their short term prediction values.

Prediction of Rational Investor Agents The prediction price of rational agents is determined by the dividend discount model. The benefits of the market are open to the agents in our model, the prediction price is computed based on the open market price. Figure 3 illustrates the decision making algorithm.

Prediction of Investor Agents with Prospect Theory The agents with Prospect Theory think the loss is twice as much as the benefit. More detail, they measure the loss twice more than the real one when the most recent price is lower

%Input Invester's Information
Equilibrium_Return: Average return ratio implied by the asset allocation (common to the all the investors)
Average return ratio : $2\lambda\sigma^2 stockweight + r_f$,
stockweight : Ratio of the amount of total stocks with the total assets in the market
σ_1*: Standard deviation of long term expected return ratio (common to all the agents, set to* 1.0×10^{-3}*)*
Forecast_short (): Short term expected return (different in each type of agents; described below)
σ_2 *(): Standard deviation of short term expected return ratio (different in each type of agents; described below)*
λ*: Ratio of risk avoidance (common to all the agents, set to 1.25)*
δ*: Required return ratio of the stock benefit (common to all the agent, set to 10%/200days)*
r_f *: Risk free rate (common to all the agent, set to 5%/200days)*
Fund: Total assets of the agents(initial value: 2000(common))
%Input Lower and Upper Bounds of Investment Ratio
Lower_Bound : Lower Bound of Investment Ratio
Upper_Bound: Upper Bound of Investment Ratio

%Calculate The Agents Return
$Return = (1/\sigma_1 * Equilibrium\_Return + 1/\sigma_2() * Forcast\_short())$
$/(1/\sigma_1 + 1/\sigma_2())$

% Solve the optimization problem to calculate
% the optimal stock weight:w
% $Max(1 + Return)w + (1 + r_f)(1 - w) - \lambda\sigma^2 w^2$
% $s.t.w \leq Lower\_Bound$
% $w \geq Upper\_Bound$
$w = (Return - rf)/(2\lambda\sigma^2)$
$w = Upper\_Bound, if w \leq Upper\_Bound$
$w = Lower\_Bound, if w \geq Lower\_Bound$

Figure 2: Decision Making Algorithm in Common

$Forecast\_short(Value_t)\{$
$Value_{t+1} = Profit_t/\delta + Profit_t$
$Forecast\_short = Value_{t+1}/Value_t - 1.0\}$
% $\sigma_2()\{\sigma_2 = 0.2/200^{0.5}\}$
% $Profit_t$*: Profit at period t (known)*
% δ *: Required return ratio of the stock (*$= 0.2/200^{0.5}$*)*

Figure 3: Decision Making Algorithm of Rational Agents

than the prediction price. The prediction price is determined by the same way of the rational agents. Figure 4 shows the decision making algorithm.

Forecast_short ($Value_t$*)*$\{$
$Value_{t+1} = Profit_t/\delta + Profit_t$
If($Value_{t-1} < ReferencePointValue$ *and* $Value_{t+1} < ReferencePointValue$*) Then*
$Value_{t+1} = ReferencePointValue - 2.25(RerencePointValue - Value_{t+1})$
Forecast_short=($Value_{t+1}/Value_t - 1)(1 + \varepsilon)$ *,where* $\varepsilon \sim N(0, 0.01)\}$
$\sigma_2()\{\sigma_2 = 0.2/200^{0.5}\}$
%ReferencePointValue : Past Trading Price(2 or 5 days ago)

Figure 4: Decision Making Algorithm of Agents with Prospect Theory

Prediction of Investor Agents with Over Confidence Characteristics The agents with over confidence characteristics think the risk of the stock market smaller. For example, the over confidence agents invest more to stocks than the rational agents. The prediction price is computed based on dividend discount model. Figure 5 shows the decision making algorithm.

$Forecast\_short(Value_t)\{$
$Value_{t+1} = Profit_t/\delta + Profit_t$
$Forecast\_short = Value_{t+1}/Value_t - 1.0\}$
% $\sigma_2()\{k = 0.2, \sigma_2 = k \times 0.2/200^{0.5}\}$
% $Profit_t$*: Profit at period t (known)*
% δ *: Required return ratio of the stock (*$= 0.2/200^{0.5}$*)*

Figure 5: Decision Making Algorithm of Agents with Over Confidence Characteristics

Prediction of Trend Predictor Agents The trend predictor agents determine the prediction price by extrapolating the averages of the most recent ten days stock price data. Figure 6 shows the decision making algorithm.

$Forecast\_short(Value_t)\{$
Trend The recent 10 days average of stock price return
$Value_{t+1} = Value_{t-1} \times Trend^2 + Profit_t \times Trend$
$Forecast\_short = Value_{t+1}/Value_t - 1.0\}$
% $\sigma_2()\{\sigma_2 = 0.2/200^{0.5}\}$

Figure 6: Decision Making Algorithm of Trend Predictor Agents

EXPERIMENTS

We implement the agent models in Matlab. The experiments using the agent models in the previous section are designed as follows: (1) How do the over confidence agents affect the market behaviors? (2) How do the agents with Prospect theory affect the market behaviors? These series of experiments (1) and (2) are to uncover the first research objective. (3) Experiments to measure the volatility change in the market. They address the second research objective.

Effects of Over Confidence Agents

We set the ratio of rational agents and trend predictor agents to 500:500, and the trend predictor agents have over confidence characteristics. From the experiments shown in Figures 7 and 8, the results are also contradictive with theoretical models. This is because the over confidence agents tend to make much more trade than the others.

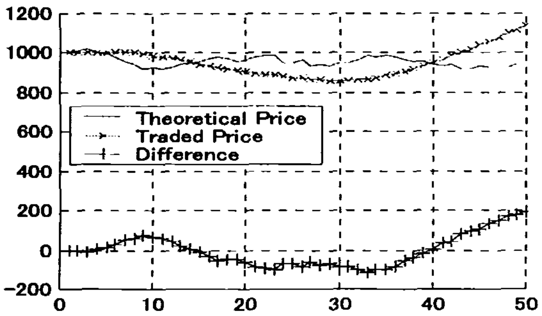

Figure 7: Changes of Trading Prices with Over Confidence Agents (500:500 no Constraints)

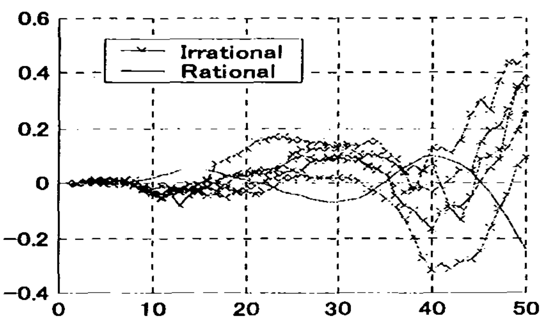

Figure 8: Changes of Cumulative Excess Returns with Over Confidence Agents (500:500)

Effects of Agents with Prospect Theory

We set the ratio of rational agents and agents with Prospect Theory to 500:500, and both kinds of agents makes decisions based on the expected utility maximization. From the experiments shown in Figures 9 and 10, the results are also contradictive with theoretical models. Interesting phenomena are that when the theoretical prices decrease, the trading prices also tend to randomly decrease. The cases also suggest that if the agents would not have rational decision making policies, the trading prices would become different from the theoretical ones and the rational agents would have less benefits.

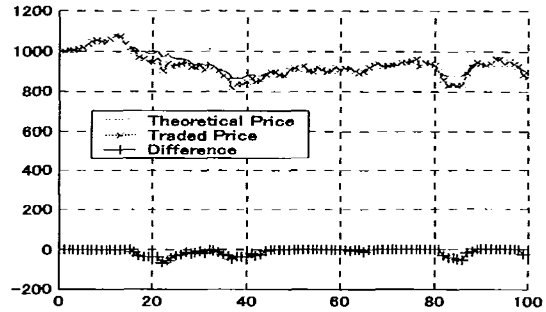

Figure 9: Changes of Trading Prices with Prospect Theory Agents (500:500 no Constraints)

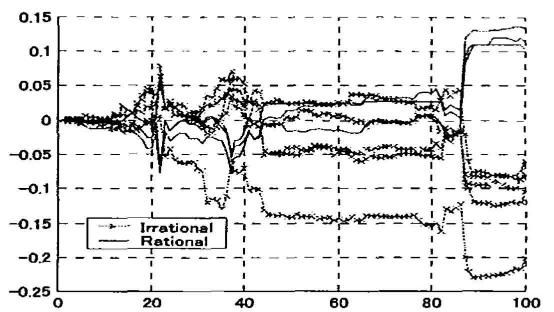

Figure 10: Changes of Cumulative Excess Returns with Prospect Theory Agents (500:500 no Constraints)

Analyzing Volatility of the Market

Before estimating the simulation models, We examine the three models with real data. In each model, we will estimate the value $y_t = c + \varepsilon_t$.

Table 1 and 2 are summaries to apply the above three models to real market data about MSCI (Morgan Stanley Capital International) stock index and Dow Jones individual stock market data, respectively. The date are taken during recent 5679 periods.

Table 2: Applying the Models to MSCI Index

	AUSTRALIA	AUSTRIA	BELGIUM	CANADA	DENMARK	FRANCE	GERMANY	HONG KONG	ITALY
OLS	18783	17765	17195	16558	17786	18617	19120	23476	20657
GARCH	17877	16823	16507	14681	17310	17891	18267	21436	19814
ASYMMETRY	17816	16815	16489	14645	17311	17848	18223	21279	19804

	JAPAN	NETHERLANDS	NORWAY	SINGAPORE	SPAIN	SWEDEN	SWITZERLAND	UK	USA
OLS	19785	17896	20400	19797	18927	20224	17277	17734	16324
GARCH	18718	16686	19387	17631	18024	18967	16521	17050	14911
ASYMMETRY	18665	16648	19358	17560	18019	18945	16444	17024	14823

On the parameters of GARCH and Asymmetric GARCH models, we use p=1, and p=1, that is, GARCH(1,1) and A-GARCH(1,1), which are most frequently used in the real data analysis. The values of each item represents how the real coincide with the model measured by AIC (Akaike Information Criteria). The shaded items are the best fit one in each row. From the AIC values, it is clear that Asymmetric GARCH model is the best for the estimation. These real data analysis suggests that we must develop novel theory to be able to well explain the real phenomena in the economic and financial domain.

Table 3: Applying the Models to Dow Jones Individual Stock Data

	ALCOA	AMEREX PRESS	AT & T	BOEING	CATERPIL LAR	COCA COLA	DISNEY (WALT)	DU PONT	EASTMAN KODAK	EXXON MOBIL
OLS	23768	24780	22619	23687	23638	22015	23870	22231	22907	20281
GARCH	23042	24006	21234	23121	23182	21100	23011	21648	22223	19364
ASYMMETRY	22922	23956	21234	23090	23141	21082	22865	21628	22191	19348

	GEN ELEC	GENERAL MOTORS	HEWLETT-PACKARD	INTEL	INTL BUS MACH	INTL PAPER	JOHNSON &JOHNSON	JPMORGAN	MCDONALDS	MERCK
OLS	21288	22990	26176	27728	22859	23243	21699	24524	21745	21423
GARCH	20301	22487	25650	27142	21904	22369	21178	23640	21169	20945
ASYMMETRY	20254	22498	25643	27104	21822	22385	21130	23582	21157	20917

	MINNESOTA MNG&MNFG	PHILIP MORRIS	PROCTER &	UNITEDTECHNOLOGIES	WAL MART
OLS	20804	22797	21879	22528	23949
GARCH	19930	22198	20514	21981	23297
ASYMMETRY	19915	22127	20454	21868	23285

To understand the characteristics of volatility of the market and the decision making procedures of the agents, we apply the three models OLS, GARCH, and Asymmetric GARCH to the agent simulator. To evaluate the accuracy of the models, we employ AIC already shown in Tables 1 and 2.

We set various kinds of agent sets as follows: (1) Rational agents only (2) The number of noise predictor agents and trend predictors is set to 500:500. The noise predictor agents predict random values during the simulation. (3) The number of trend predictor agents and rational agents is set to 500:500. (4) The number of agents with Prospect Theory and rational agents is set to 500:500. The number of the simulation experiments are 10. The results are summarized in Table 2. Each item in the table shows the value of AIC. The shaded items are best ones in each simulation run. The results have suggested that in Cases (1),and (2), as implied from the traditional analytic models, OLS model shows the best, however, in Case (3) and (4), GARCH and Asymmetric GARCH models are respectively show the best fit. Compared with the real data in Table 1 and experimental results in Table 2, we draw the conclusion that the proposed agent-base simulation model explains the gap between the macro level analytic models and micro level agent-based simulation.

RERALTED WORK AND DISCUSSION

Conventional finance theory have developed several prediction models for asset allocation price based on the assumptions that the investors behave rationally, that is, they make decisions in order to maximize their expected utilities. Ingersoll (Ingersoll 1987) have investigated the forms of the utility functions. Hawson et al. (Hawson 1993) have reported Bayesian theoretic decision making is rational if new information will sickeningly arrive when the time passes. Balck and Litterman model (Black 1992) also suggest that the effectiveness of Bayesian theoretic decision making. They consider that the maco level theoretical prices can be derived from the fundamentals of financial markets. However, as Shiller (Shiller 2000) has suggested, real markets are developing their own theories to explain the current prices. On the contrary to such conventional analytic models, in this paper, we employ agent-based simulation models, which are able to ground both macro level phenomena and micro level mental models. Recent agent-based (Axelrod 1997), (Epstein 1996) or microscopic simulation studies (Levy 2000) have frequently suggested the effectiveness of the approaches. However, the research in the literature tends to show too artificial results: The results are so good that the desire of a model builder is already built in the models, and they have very weak relationship to the real world phenomena (Terano 1998). In the poposed models in the paper, we have tried to address the issues. The experimental results using the proposed models have drawn the implications: (1) The "efficient market hypothesis" does not hold when there are some kinds non-rational agents in the market; (2) The roles of such non-rational agents are essential to explain the real world phenomena. in the Market; and (3) Using macro level (A-)GARCH model, micro level Prospect Theory, and agent-based simulation we are able to bridge the theoretical models and real phenomena. These statements are answers to the issues addressed in Section 2.

Table 4: Summary of the Comparison of Model Accuracy

Simulation	Number	1	2	3	4	5	6	7	8	9	10
rational	OLS	8761	8909	8816	8808	8720	8971	8623	8686	8873	8725
	GARCH	8765	8909	8819	8809	8722	8971	8621	8683	8877	8728
	ASYMMETRY	8766	8908	8820	8810	8724	8973	8623	8684	8878	8730
noise50%	OLS	9704	9686	9796	9782	9701	9782	9677	9795	9640	9737
	GARCH	9708	9689	9798	9780	9703	9780	9678	9798	9641	9739
	ASYMMETRY	9709	9685	9800	9782	9704	9782	9680	9799	9642	9740
trend50%	OLS	6589	6396	6699	6323	6688	6444	6433	6495	6642	6601
	GARCH	6347	6168	6447	6151	6474	6253	6192	6292	6461	6427
	ASYMMETRY	6348	6170	6449	6153	6476	6254	6193	6293	6463	6428
Prospect	OLS	10853	10594	10781	10639	10829	10767	10873	10768	10724	10743
2days	GARCH	10530	10504	10694	10552	10727	10693	10777	10629	10597	10692
	ASYMMETRY	10485	10464	10634	10506	10664	10626	10726	10580	10557	10657
Prospect	OLS	10365	10606	10535	10664	10564	10722	10555	10409	10462	10488
10days	GARCH	10332	10544	10474	10591	10512	10642	10526	10317	10399	10434
	ASYMMETRY	10179	10396	10316	10428	10342	10453	10368	10162	10264	10290

CONCLUDING REMARKS

This paper have proposed agent-based simulation models for economic and the finance domain in order to bridge the gap between macro- and micro-level concepts. The models consist of simple agents with rational and/or non-rational decision making functionalities for investment to virtual markets. Although the models are so simple, but the experimental results using both rational and non-rational agents have shown that (1) the efficient market hypothesis does not hold even if there are rational decision making agents in the model, (2) the non-rational decision making agents remains, even when there are some rational ones, and (3) the behaviors of the agents with the characteristics of prospect theory coincide with the estimation by GARCH model. Our future work includes (1) to develop automated tuning methods for the simulators, (2) to examine both macro and micro level theories to bridge real world phenomena and theoretical models, and (3) to develop much mode complex systems to simulate various levels of social and economic behaviors

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