Computing the Aesthetics of Chess

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Abstract

Aesthetics in chess is a concept competent players are quite familiar with. For many, it is one of the main reasons they play. Much of the research with regard to chess however, has been towards making machines play better and to understand how humans think. This article explores the principles of chess aesthetics and proposes a computational model based on those principles. An experiment was carried out comparing composed chess problems to over-the-board games. The results suggest that the model can discern between the beauty in chess problems where aesthetics is more prominent and over-the-board games where it is not. Implementation of the model into a computer program could therefore lead to the automatic detection of aesthetic mating combinations in chess that will be of interest to humans, particularly chess players and problem composers.

Introduction

Some research into the principles of chess aesthetics has been done over the last few decades with promising results (Wilson 1969, Margulies 1977, Walls 1997). The principles derived are generally consistent with what chess players themselves have written about the beauty of the game (Lionnais 1951, Humble 1963, Osborne 1964, Bronstein 1983).

Given the nature of aesthetics, literature on the subject has mostly been the product of experience by professional players. Reviewing all this information it is possible to develop a computational model for aesthetics in chess. Since computers are able to analyze far more positions in the game than any human ever could such a model can be used to identify beautiful mating sequences in the game tree or from chess databases and present it to humans.

This is useful because humans appreciate such beauty and are often striving for it in their play or through composing chess problems. There are an extremely large number of positions possible on the chess board and many will appeal aesthetically to humans. However, only a very small percentage of them will ever be played or composed in a single human lifetime.

A computational model will allow for more beauty in chess to be discovered and appreciated than currently possible. It could also assist judges in chess problem competitions when evaluating the beauty of a composition. The model is based on principles that pertain to the most popular version of the game also known as Western or International chess. It does not cater for other variants of the game even though similar principles could theoretically be derived for them.

Chess Aesthetic Principles

Wilson proposed a method of evaluating chess problems using reference tables by attributing values to strategies like checks, blocks, castling and individual themes in the hope of providing a fair basis for comparing one composed chess problem to another (Wilson 1969). He did so because he felt human judges could not be consistent enough and composers were not treated justly for their hard work. Aesthetics being a significant element in chess problems (Ravilious 1994) was not explicitly accounted for in his method but assumed to be synergetic of the other factors.

Wilson's method was indeed able to produce a numeric score for chess problems for the purpose of comparison. It was even reasonably accurate. However, his proposal to use the method to replace human judges in chess problem composition contests was universally rejected and understandably so (Grand 1986). We cannot objectively claim one problem is necessarily 'better' than another when the existing methods (even to this day) rely on subjective evaluation by experienced judges and include concepts such as 'originality'.

Margulies on the other hand, derived 8 principles from the judgement of expert chess players who were shown pairs of chess positions and asked to select the more beautiful solution (Margulies 1977). His eight principles are:

1. successfully violate heuristics
2. use the weakest piece possible
3. use all of the piece's power
4. give more aesthetic weight to critical pieces
5. use one giant piece in place of several minor pieces
6. employ chess themes
7. avoid bland stereotypy
8. neither strangeness nor difficulty produces beauty (i.e. wildly improbable positions and difficult ones do not lead to judgements of beauty)

Walls applied some of these principles in his research on using chess beauty heuristics to play the game better than if using traditional heuristics (Walls 1997). He rejected the last three principles because he found them not applicable to mating problems where the quickest solution was desired. His results showed that beauty heuristics worked better in solving chess problems but he did not emphasize the concept of aesthetics in chess for its own sake.

Levitt and Friedgood explored the idea of beauty in chess further by identifying four elements namely *paradox, geometry, depth* and *flow* (Levitt and Friedgood 1995). For each, further classification and examples were provided. Sacrificing material and underpromotion come under what they termed *paradox of material* while *depth* refers to something more abstract such as the objective of a particular move only becoming clear several moves later.

Geometry implies physical formations on the board such as pieces being aligned and also includes themes such as the 'switchback' where a piece returns to the same square in a given move sequence. *Flow* refers to the absence of many confusing variations in a solution. A lot of what is discussed coincides with the principles as derived by Margulies but explained in more detail given their background as chess masters.

Lasker touches on aesthetics with the simple idea of 'achievement' and that being a master is not a prerequisite to appreciating beauty in chess (Lasker 1947). By achievement he meant that the move sequence has to be winning or successful in some way. Troyer writes about the aesthetics of chess problems by adding other factors notably *economy of force* (Troyer 1983). This is a characteristic quite common in compositions but it also occurs in actual games, especially in the endgame.

Other works exist which explore the concept of aesthetics in chess (Lionnais 1951, Humble 1963, Osborne 1964, Bronstein 1983) and with minor exceptions, they are generally in agreement with the principles mentioned above.

Principles Chosen for Computation

Ambiguous principles and those not amenable to computation could not be selected for use in the model. The chosen principles and themes were those capable of being represented by evaluation functions that adequately describe them. The selected principles are as follows.

1. *win with less initial material*
2. *successfully violate heuristics*
3. *use all of the piece's power*
4. *create geometry*
5. *use the weakest piece possible to mate*
6. *sacrifice material*
7. *checkmate economically*

The eighth chosen principle is 'employ chess themes' but since there are many only 11 were selected and given evaluation functions of their own. They are as listed below.

1. *fork*
2. *pin*
3. *skewer*
4. *x-ray*
5. *double attack*
6. *discovered attack*
7. *zugzwang*
8. *smothered/self-block*
9. *crosscheck*
10. *underpromotion*
11. *switchback*

Some principles were rejected for a variety of reasons such as overlapping with other principles, using imaginary pieces, relying on 'difficulty' (a concept relative to the player) and those involving multiple variations (this model analyzes only one variation of any move sequence). The chosen themes on the other hand are some of the more common and recognizable ones found in chess.

Lesser known themes such as the Grimshaw, Zwischenzug, Novotny and Bristol (Levitt and Friedgood 1995, Silman 1998) were not included because they are usually restricted to the domain of composed chess problems. Also, these themes lacked clearly identifiable parameters that are needed to develop suitable evaluation functions.

The Aesthetics Model

In general, the aesthetics model can be illustrated as follows.

a = cumulative aesthetic value of move sequence,
p = principles, t = themes

$$a = \sum \mathrm{p} + \sum \mathrm{t}$$

The model evaluates only White's moves (the default winning side) and tests for the presence of aesthetic principles and themes at every move. Each of the first seven principles is described using a specific evaluation function. The equation that follows shows these functions in sequence corresponding to the list of principles in the last section. Table 1 explains what the variables are.

$$\sum_{k=1}^{7} p_k = \frac{0.1(b_1 - w_1)}{6} + 0.1h + \frac{s_1}{10m_1} + \frac{0.1g_1}{2} + \frac{1}{g_2} + \frac{0.1\left[(w_1 - w_2) - (b_1 - b_2)\right]}{6} + \frac{1}{4(e+1)}$$

Variable	Meaning
b_1/b_2	material value for Black at initial/final position
e	superfluous material (i.e. not used in the actual mate)

g_1	number of consecutive pieces in a single file/rank/diagonal with no stray pieces present elsewhere on the board
g_2	piece power; Q=21, R=14, B=7, N=6, P=3
h	number of heuristic violations in the key (first) move
m_1	maximum power; Q=21, R=14, N=8, B=7, P=2
s_1	squares traversed
w_1/w_2	material value for White at initial/final position

Table 1: Meaning of variables used in evaluation functions for aesthetic principles 1 through 7

The last principle 'employ chess themes' is composed of 11 themes and described using evaluation functions as shown in the equation below. Each function corresponds in sequence to the themes as listed in the last section.

$$\sum_{n=1}^{11} t_n = \frac{\sum f + g_3}{50r} + \frac{(g_4 + d_1)}{100} + \frac{i}{100} + \frac{x}{100} + \frac{(g_5 + s_2 + d_2)}{100} + \frac{(s_3 + d_3)}{100} + \frac{\sum o + \sum m_2}{50} + \frac{\sum b_3}{d_4} + 0.1\left(\sum c\right) + \frac{m_3}{20} + 0.1(s_4 + 1)$$

Variable	Meaning
b_3	opponent pieces blocking flight squares
c	consecutive checks by both sides
d_1	distance in squares between the attacking piece and its target
d_2	distance between secondarily attacked piece and its attacker
d_3	distance between discovered attacked piece and its attacker
d_4	denominator; 10 if it is a true self-block and 50 otherwise
f	value of forked pieces
g_3	number of prongs used
g_4	value of target piece
g_5	value of primarily attacked piece
i	value of immediately attacked piece
m_2	legal opponent moves
m_3	material difference between a queen (9) and promoted piece
o	opponent pieces on the board
r	fork-related piece value; Q=5, R=4, B=3, N=3, P=1
s_2	value of secondarily attacked piece
s_3	value of discovered attacked piece
s_4	number of return moves
x	value of X-Rayed piece

Table 2: Meaning of variables used in evaluation functions for themes

When material value of the pieces needs to be calculated their traditional Shannon values are used (Q=9, R=5, B=3, N=3, P=1) unless stated otherwise (Shannon 1950). The king always defaults to a value of 10 regardless since its Shannon value of 200 is not suitable here. Mating squares also have a value of 10. For example, a knight may fork a queen and threaten a square which, if it is allowed to occupy in the next move would result in mate.

The model is designed to evaluate direct-mate in 3 move sequences and produces a scalar value which is basically the sum of the principles and themes that apply to that move sequence. Only moves made by White are evaluated because the model does not focus on Black as the winning side. Hence, all sequences must have White delivering the checkmate. It is a simple matter of symmetry to apply the same evaluation to Black if it was winning but that was avoided here for clarity.

Even though aesthetics in chess is not limited to mating sequences (e.g. many beautiful move sequences simply force a draw or win decisive amounts of material), it was set as a requirement because a checkmate is considered the highest achievement in the game and it does not require the inclusion of many ambiguous long term positional considerations.

The evaluation functions were formulated based on factors that relate the computational aspect of chess to its aesthetic principles. For example the principle, *use all of the piece's power* relates to how many squares a piece controls (Euwe 1982). This was interpreted to mean squares that are controlled by a piece when placed at the corner of an empty board. While this principle is intended for pieces with a long range attack it was found to be unfair to leave knights and pawns without any power so these pieces were defaulted to reasonable values of 8 and 2 respectively.

This principle would apply not only to the piece itself moving but also to the power involved in the actual checkmate. Denominators in certain evaluation functions were set to numbers that keep them relatively consistent with the typical value of the other principles and themes to avoid bias.

Not all principles and themes can apply to every move. For example, *winning with less material* is only relevant on the first move where evaluation of the move sequence begins. *Zugzwang* on the other hand has no relevance on the 3rd move since that is when the checkmate must happen while the *smothered* theme can only apply to a checkmate situation and is therefore limited to move 3. The table below lists the principles and themes that apply to each move in a mate-in-3 sequence. Refer to the last section for names.

Move	Applicable Principle/Theme
1	Principles 1-4; Themes 1-7, 9-11
2	Principles 3-4; Themes 1-7, 9-11
3	Principles 3-7; Themes 1-6, 8-11

Table 3: Applicable themes and principles

Human perception of a particular theme in chess might involve several moves on its own but this is different in the model since it evaluates themes on a per-move basis. For example, if a knight moves and attacks the enemy queen without the player's main intention of forking it and a nearby rook, the move still counts as having the theme 'fork' and a value for it is added based on the corresponding evaluation function. The recognition of a theme even when it was not intended by the player is valid because observers might still appreciate it.

Chess can be very creative and complex so it is sometimes difficult for people to identify if a theme or principle is even in effect. Therefore only 'successful' principles and themes that are clearly identifiable are considered. Taking the example of the knight fork but this time replacing the rook for a pawn, it is only evaluated as a fork if the pawn is unguarded or can be captured within the next 5 plies (half-moves) resulting in material gain. This confirmation is done as a side variation analysis.

If the knight itself could be captured by an enemy piece on the next move resulting in a loss of material for White, the theme does not count either. This is because not only is it an unsuccessful fork but also quite possibly an unfavourable exchange that undermines the theme. For the same reason, a more aesthetically pleasing fork involves stronger enemy pieces and more prongs. Conditions like these apply to most of the themes and principles to avoid ambiguity.

As another example the principle, *successfully violate heuristics* only takes into account heuristics of keeping the king safe, capturing enemy material and not leaving your own material *en prise* (in a position to be taken). The last heuristic usually does not apply if the key (first) move is a check because White's *en prise* piece(s) are not in any immediate danger. Also, only unique sets of pieces count as heuristic violations. This means that if White leaves both a pawn and a bishop *en prise* to the enemy queen, it only counts as one violation. On the other hand if the pawn was *en prise* to the enemy queen and the bishop to an enemy rook, it would count as two heuristic violations.

Positional heuristics are inconsequential given a mating sequence so they were not accounted for in the model. The principle *sacrifice material* on the other hand, is represented by an evaluation function that computes the difference between White's initial and final material value relative to Black's. It only applies if a positive value results indicating a sacrifice or unfavourable exchange has occurred. A rook sacrifice would therefore be equal in terms of aesthetics to sacrificing a bishop and two pawns in the move sequence. The evaluation function also caters for pawn promotions. Suppose White sacrifices a rook and knight but then is able to promote one of his pawns to a queen, the effect is somewhat neutralized.

In a sense, not everything the opponent does on the board is neglected by the evaluation. It is quite possible for Black to make aesthetically pleasing defensive moves against White but since the positions are necessarily won by White such moves lack the prerequisite of being 'successful' and need not be accounted for by the model. A more detailed explanation about these matters if desired can be obtained by contacting the author.

Experimental Results

An experiment was carried out to test the aesthetics model (Appendix). A reliable set of test data had to be acquired where the element of aesthetics in chess is more prominent to human perception than elsewhere. Composed chess problems met this requirement (Troyer 1983, Humble 1993, Ravilious 1994). The scope of the model also limited the problems to direct-mates in 3 only. Thirty problems were randomly selected from a variety of Internet resources and are those that have been composed and published within the last century.

Many of these problems were prize winners in competitions. They were tested for correctness using the Fritz 9 and Shredder 9 chess engines before analysis but limited to one variation after the key move where side variations existed. In some positions after White's key move, Black has more than one possible reply even though all lead to mate in 3. When this happens only one of those variations is selected for evaluation using the model. Thirty forced mate-in-3 sequences from over-the-board (OTB) games were also randomly selected and tested using the model since aesthetics is less prominent in that domain.

Only games played in tournaments in the last century by people with a master-level rating between ELO 2000-2500 were used. Games by players rated lower than that were not used to prevent poor play from unnecessarily influencing the results. Competent play is one of the prerequisites of aesthetic consideration and to undermine it with novice mistakes would be to treat the tournament games unfairly.

Each of the 60 positions was analyzed to determine its overall aesthetic value based on the model. The evaluation was done manually but repeated twice to ensure accuracy. A clear difference between chess problem and OTB scores was expected because the model is designed to detect and evaluate aesthetic principles. The probability that there would be no difference between the two groups was therefore slim. Figure 1 shows the results that were obtained. The difference between the means proved to be statistically significant. They have been sorted from highest to lowest for both groups.

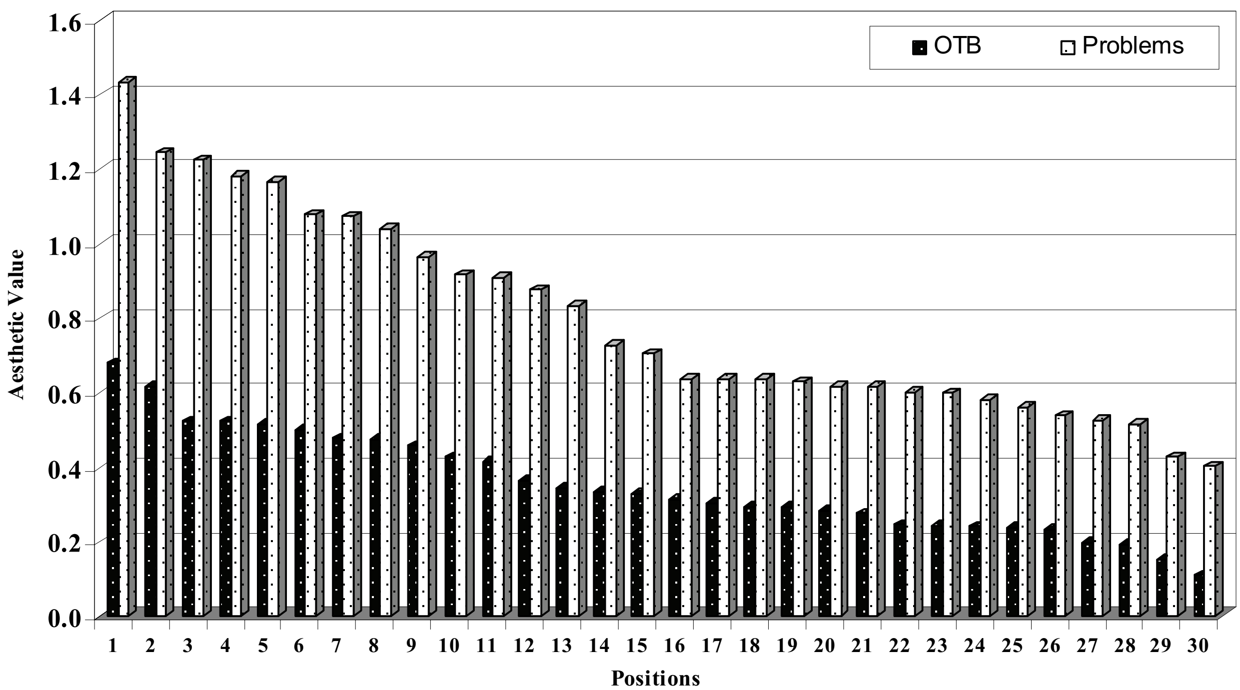


Figure 1: Comparative aesthetic value of chess problems and OTB mates

The chess problems scored a mean aesthetic value of 0.796 ± 0.28 compared to over-the-board mates which averaged only 0.353 ± 0.14 (two sample t-test using unequal variances, t (43) = 7.813, $p < 0.001$). This represents a 125% increase in favour of the problems and suggests that the model is able to recognize aesthetics in chess since it is more prominent in composed problems.

Over-the-board mates averaged less than half what the problems did since aesthetics is not a priority in tournaments under time constraints (Ravilious 1994). It is unlikely that something other than aesthetics is responsible for this discrepancy considering that the model is based on established aesthetic principles in chess that have even been verified experimentally (Margulies 1977).

The chess problems also had a range almost 45% wider (1.033) than OTB mates (0.57). This is the difference between the highest and lowest scoring positions from each group. It can be seen from Figure 1 that the some chess problems scored lower than the highest scoring OTB positions. This suggests that aesthetic content within a given problem may not be the same or even close to a different one and that a composed chess problem is not necessarily more aesthetic than an OTB position.

Nevertheless, such a finding is still consistent with the world of chess composition (Grand 1986). There are certainly unaesthetic chess problems that would pale in comparison to a spectacular combination performed in a tournament and particularly beautiful problems that might be almost impossible to duplicate in a real game. This is not to say that a chess problem which scores poorly in terms of aesthetics is necessarily 'weaker' than one that scores well but that merely in terms of beauty based on the principles of aesthetics, it might not rank as highly.

A difficult problem is not necessarily a beautiful one either. This is because chess compositions are usually judged on a number of additional factors as well such as originality, presence of duals and problem symmetry to name a few things. These considerations are very difficult to quantify so this model cannot be a substitute for human judges in that domain. It could however, assist judges in the evaluation of the aesthetic aspect of a composition. The actual purpose of the model is simply to facilitate recognition of beauty in chess and to expand current computational limits to include yet another facet of the game.

Implementation of the model into a computer program is possible but this has not been completed yet. It is still in the algorithmic stage. Essentially, the parameters of each principle and theme would need to be correctly identified from a move sequence and this is quite possible given the conditions mentioned earlier that minimize ambiguity. The process would be just as mechanical as was manually performed in the experiment, only quicker.

Figure 2 shows a pair of positions taken from the experiment and how their aesthetic values were calculated. The principles and themes for each move are also given. Knowledge of algebraic chess notation is assumed. The board coordinates are A through H from left to right and 1 through 8 from bottom to top.

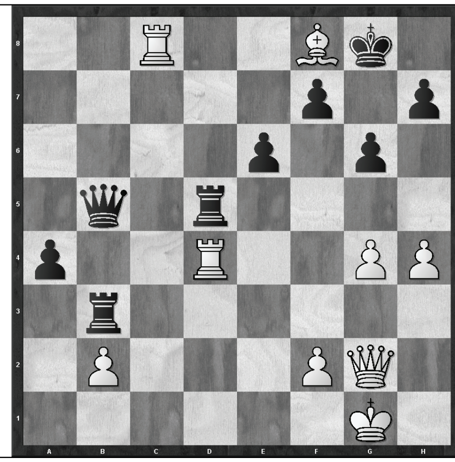
Problem			**OTB**		
					
Udo Degener, 6, Troll 3/91, 1991			*Gundula-Petra, EU-ch U20 Girls, 2000*		
1. Bf4 Bh4	*Principles 2 & 3*	0.1(2) + 2/70	1. Bh6+ Rd8	*Principle 3; Theme 6*	2/70 + 0.14
2. Ne3+ Kd4	*Principle 3; Theme 5*	3/80 + 0.17	2. Rdxd8+ Qe8	*Principle 3*	4/140
3. Nf5++	*Principle 3 (twice), 5 & 7; Themes 1 & 8*	3/80 + 3/80 + 1/6 + 1/28 + 15/150 + 1/50	3. Rxe8++	*Principle 3 (twice), 5 & 7; Theme 8*	1/140 + 2/140 + 1/14 + 1/76 + + 2/50
Total Score		**0.833**	**Total Score**		**0.343**

Figure 2: Examples of aesthetic value computation

In the chess problem, the first move is a violation of heuristics because the knight does not capture the bishop on e1 with a double check, winning material. Another violation of heuristics occurs because White does not capture the pawn on f7. There may be subjective and complex positional heuristics at play here but the model does not take that into account. White's next move scores by using the knight and creating a double attack on both the Black king and rook on f1.

The final move usually yields the most aesthetic value. Here the knight moves (a fixed three squares) and scores on that principle just as it did on move 2. It also scores the same for being the checkmating piece (its 3 square L-move ability is what mates the king). The knight scores yet again for being a weak piece as opposed to a queen or rook which, would also score but not as highly. It might seem as if the knight is given too much aesthetic preference here but this is not the case. A bishop or any other piece for that matter would be evaluated similarly on the final move.

Economically, White uses all its pieces for the checkmate except for the rook on h5 and pawn on b2. That amounts to (5+1) points of superfluous material. Finally, White scores for themes on move 3 with a clean fork of king and bishop and also because the Black king is smothered to a degree by the pawn on e4. The first move of the OTB position shows the bishop moving (scores by default on *use all of the piece's power*) and creating a discovered attack.

The second move shows White capturing the enemy rook with his d-file rook and scores for moving it 4 squares. Had the capture been done using the c-file rook, it would score 3/140 points less. The final move scores twice for the same principle (including once for checkmating) and also for the principle of *use the weakest piece possible to mate* just as the knight did in the chess problem. The rook, being a more powerful piece (Shannon value of '5') scores less than a knight, bishop or pawn. Economically the position is bad with many superfluous pieces except for the rook on e8 and bishop on h6. However, the smothered theme also comes into play here with two pawns blocking two possible flight squares for the king.

Conclusion

An aesthetics model for chess was introduced in this paper. The model incorporates the more prominent and computable aesthetic principles in the game. Each principle and theme was represented using a specific evaluation function with certain conditions. An experiment was then conducted which tested the model against 60 randomly selected chess problems and over-the-board forced mates-in-3.

A clear distinction between the problems and OTB positions was found which suggests that the model is able to recognize aesthetics in chess since it is more prominent in chess problems than in tournament games. The wider aesthetic range evident in the chess problems confirms that their beauty can vary significantly from one problem to the next. Examples of how the model works was given to illustrate the calculations that take place when analyzing entire move sequences as was done in the experiment.

Future work includes experiments that involve longer move sequences and refining the model with additional chess knowledge. Also, the author hopes to establish a positive correlation between human perceived aesthetic value and those attained computationally. Eventually, a computer program that incorporates the aesthetics model will be developed to automate evaluations and discover new beautiful mating sequences in chess.

Acknowledgements

I would like to thank John McCarthy (Stanford), John Troyer (University of Connecticut), GM Jonathan Levitt and IM David Friedgood for their feedback and comments.

Appendix

Internet Resources Used for Direct Mate in 3 Chess Problems

- Torsten Linß' Chess Problems
  http://www.math.tudresden.de/~torsten/problem/tlisproblems.html
- Vito Rallo's Chess Problems
  http://www.geocities.com/Colosseum/Field/4530/newpro2.htm
- Mat Plus: The Best of Chess Problems
  http://user.sezampro.yu/~mivel/TOURNEYS.htm
- Thinks.com: Classic Chess Problems
  http://thinks.com/chess/index.htm
- 132 Studies & Problems
  http://www.xs4all.nl/~dsu/index.html

Over-the-Board Forced Mate in 3 Games

- Obtained from *Big Database 2004* chess games database (2607013 games)

Chess Engines

- Fritz 9. 2005.
  http://www.chessbase.com/shop/product.asp?pid=247&user=&coin=
- Shredder 9. 2004.
  http://www.chesscentral.com/software/shredder-8.htm

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