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Commonsense Inference in Dynamic Spatial Systems: Epistemological Requirements

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Abstract

We demonstrate the role of commonsense inference toward the modeling of qualitative notions of space and spatial change within a dynamic setup. The inference patterns are connected to those that are required to handle the frame problem whilst modeling inertia, and the causal minimisation of (Lin 1995) that is required to account for the ramifications of occurrences. Such patterns are both useful and necessary in order to operationalize a domain-independent qualitative spatial theory that is re-usable in arbitrary dynamic spatial systems, e.g., for spatial planning and causal explanation tasks. The illustration, grounded in the context of embedding arbitrary 'qualitative spatial calculi' within the situation calculus, utilizes topological and orientation calculi as examples.

1. Introduction

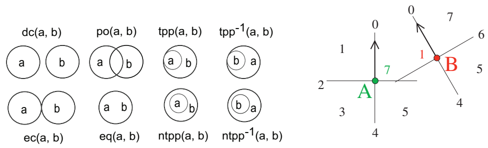
Research in the qualitative spatial reasoning domain has focused on the representational aspects of spatial information conceptualization and the construction of efficient computational apparatus for reasoning over those by the application of constraint-based techniques (Cohn and Renz 2007). For instance, given a qualitative description of a spatial scene, it is possible to check for its consistency along arbitrary spatial domains (e.g., topology, orientation and so forth) in an efficient manner by considering the general properties of a qualitative calculus (Ligozat and Renz 2004). So an important question that may be posed is: how do we integrate these specializations, which allow us to efficiently reason about a static spatial configuration, within a dynamic spatial system (Bhatt and Loke 2008) where spatial configurations undergo changes as a result of actions and events occurring within the system? More generally, how do we embed a specialized commonsense theory of space and spatial change within a general formalism to describe and reason about change? Indeed, this is closely connected to the agenda described by (Shanahan 1995), and is also related to the broader theme of the sub-division of endeavors and their integration in AI. Shanahan describes it aptly:

> 'If we are to develop a formal theory of commonsense, we need a precisely defined language for talking about shape, spatial location and change. The theory will include axioms, expressed in that language, that capture domain-independent truths about shape, location and change, and will also incorporate a formal account of any non-deductive forms of commonsense inference that arise in reasoning about the spatial properties of objects and how they vary over time'

This paper complements the results in (Bhatt 2009), where commonsense inference from the viewpoint of *phenomenal* and *reasoning* requirements is presented. Here, we demonstrate the utility of commonsense inference within the framework of the situation calculus for representing and reasoning about changing spatial domains. The reasoning tasks are directly connected to fundamental epistemological aspects concerning the frame and ramification problems, and are necessary for consistently preserving some of the high-level axiomatic aspects that characterize a generic qualitative spatial calculus (Section 2). Although we do not explicitly address all aspects pertaining to the task of 'spatial calculus embedding' (within situation calculus) herein, that is essentially the overall context. Here we solely focus on demonstrating the use of commonsense reasoning in the context of (AI–AII):

- AI maintaining compositional consistency of sets of spatial relations pertaining to an arbitrary number of integrated / non-integrated spatial calculi, i.e., calculi with / without integrated composition theorems. Here, compositional consistency for each spatial calculus is defined by the properties that are intrinsic to it and does not depend on the default reasoning approach. This aspect is connected to the ramification problem (Section 3.1).
- AII inertial aspects of a dynamic spatial system determining what remains unchanged, one instance of this being characterized by the intuition that the qualitative spatial relationship between two primitive spatial entities typically remains the same. Indeed, these aspects are connected to the frame problem (3.2).

Reasoning about changing spatial configurations in the presence of actions and events is useful in several scenarios of which the domain of cognitive robotics is a prime example. For instance, spatial re-configuration may be formulated as a planning task: given compositionally consistent models of an initial and desired spatial configuration, regress a situational-history (i.e., a sequence of actions) that would produce the goal configuration. Similarly, given an initial situation description and a temporally ordered set of partial observations denoting configurations of objects, abduce an



(a) $\mathcal{RCC} - 8$ Primitives (b) The $\mathcal{OPRA}_2$ relation $\vec{A}\ _2\angle_7^1\ \vec{B}$

Figure 1: Topological and Orientation Calculi

explanation that entails the observations. Indeed, the embedding and/or integration of commonsense notions of space and spatial change (e.g., qualitative spatial calculi) within the formal apparatus to reason about action and change is a necessary endeavor for operationalizing (spatial) calculi in practical application domains and for realizing the aforementioned spatial planning and causal explanation tasks.

2. Ontology of Space and Change

The situation calculus formalism used in this work, denoted $\mathcal{L}_{sitcalc}$, is a first-order many-sorted language with equality and the usual alphabet of logical symbols $\{\neg,\ \wedge,\ \vee,\ \forall,\ \exists,\ \supset,\ \equiv\}$. There are sorts for events and actions (Θ), situations (S), spatial objects (O) and regions of space (R), with corresponding (lowercase) variables for each sort. The use of the predicates including, $Holds$, $Poss$, $Occurs$, $Caused$ and the $Result$ function for a typical situation calculus theory will be self-evident. With $\mathcal{L}_{sitcalc}$ as a basis, a situation calculus meta-theory Σ_{sit} required from the viewpoint of the causal minimisation framework of (Lin 1995) is adopted :

Definition 1 (Foundational Theory Σ_{sit}). *The foundational theory Σ_{sit} of the situation calculus formalism consists of the following set of formulae: the property causation axiom determining the relationship between being 'caused' and being 'true', a generic frame axiom in order to incorporate the assumption of inertia, uniqueness of names axioms for the fluents, occurrences and fluent denotations, and domain closure axioms for propositional and functional fluents.* □

The spatial ontology that is required depends on the nature of the spatial calculus that is being modeled. In general, spatial calculi can be classified into two groups: topological and positional calculi. When a topological calculus such as the Region Connection Calculus (RCC) (Randell 1992) is being modeled, the primitive entities are spatially extended and could possibly even be 4D spatio-temporal histories (e.g., in a domain involving the analyses of motion-patterns). Alternately, within a dynamic domain involving translational motion in a plane, a point-based (e.g., Double Cross Calculus (Freksa 1992), $\mathcal{OPRA}_m$ (Moratz 2006)) or line-segment based (e.g., Dipole Calculus (Schlieder 1995)) abstraction with orientation calculi suffices. Figure 1(a) is a 2D illustration of relations of the RCC-8 fragment of the region connection calculus. This fragment consists of eight relations: disconnected (dc), externally connected (ec), partial overlap (po), equal (eq), tangential proper-part (tpp) and non-tangential proper-part ($ntpp$), and the inverse of the latter two tpp^{-1} and $ntpp^{-1}$. Similarly, Fig. 1(b) illustrates one primitive relationship for the Oriented Point Relation Algebra (OPRA) (Moratz 2006), which is a spatial calculus consisting of oriented points (i.e., points with a direction parameter) as primitive entities. The granularity parameter m determines the number of angular sectors, i.e., the number of base relations. Applying a granularity of $m = 2$ results in 4 planar and 4 linear regions (Fig. 1(b)), numbered from 0 to 7, where region 0 coincides with the orientation of the point. The family of $\mathcal{OPRA}_m$ calculi are designed for reasoning about the relative orientation relations between oriented points and are well-suited for dealing with objects that have an intrinsic front or move in a particular direction.

Definition 2 (Valid Regions within the Theory). *Let $\mathcal{U}$ denote the universe of the primitive spatial entities, whatever be their precise geometric interpretation in $\Re^n$. When extended, a region is valid if it has a well-defined spatiality, is measurable using some notion of n-dimensional measurability that is consistent across inter-dependent spatial domains (e.g., topology and size) and the region is convex and of uniform dimensionality.* □

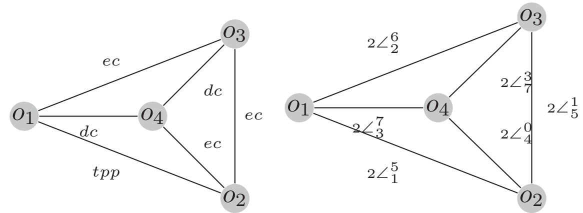
Definition 2 is one way to set the basic requirements for a particular application domain – these are necessary to accommodate the spatial calculi we use in the examples. The functional fluent $extension(o)$ denotes the extension of a physical object in space – to emphasize, this could be a region of space (for a topological calculus such as RCC), or a hypothetical entity such as a point or in general, an ordered tuple of points (for line-segment based orientation calculi) and also possibly a point with an additional direction parameter (for modeling a calculus such as $\mathcal{OPRA}_m$) on an absolute frame of reference. We suppose that the precise semantics vis-à-vis the concrete domain in $\Re^n$ is provided by a domain-specific qualifier. Finally, let $\mathcal{R} = \{\mathcal{R}_1,\ \mathcal{R}_2, \ldots,\ \mathcal{R}_n\}$ be a finite set of binary base relationships of a qualitative calculus over $\mathcal{U}$ with some spatial/spatio-temporal interpretation.[1] We reify the base relationships in $\mathcal{R}$ for representational purposes. i.e., relationships from each $\mathcal{R}$ are treated as concrete fluent denotations for spatial fluents denoting the spatial relationship between the primitive entities of $\mathcal{U}$ – let $\Gamma_{sp} = \{\gamma,\ \gamma_1, \ldots,\ \gamma_n\}$ denote such a set. For brevity, the object-region equivalence axiom (1) for spatial fluents (ϕ_{sp}) denoting spatial relationships (γ) between primitive spatial entities is used:

$$\begin{aligned} Holds(\phi_{sp}(o_1,\ o_2),\ \gamma,\ s) \equiv (\exists r_i, r_j).\, extension(o_1,\ s) = r_i \\ \wedge\ extension(o_2,\ s) = r_j \wedge Holds(\phi_{sp}(r_i,\ r_j),\ \gamma,\ s) \end{aligned} \tag{1}$$

From a high-level axiomatic viewpoint, a spatial calculus defined on $\mathcal{R}$ has the following properties:

- P1 $\mathcal{R}$ has the jointly exhaustive and pair-wise disjoint (JEPD) property, meaning that for any two entities in $\mathcal{U}$, one and only one spatial relationship from $\mathcal{R}$ holds in a given situation
- P2 the basic transitivity, symmetry or asymmetry or the relationship space is known
- P3 the primitive entities in $\mathcal{R}$ have a continuity structure, referred to its conceptual neighborhood (CND) (Freksa 1991), which determines the direct, continuous changes in the quality space (e.g., by deformation, and/or translational/rotational motion)

[1] Binary spatial relations are assumed here, but potential scenarios could also involve ternary orientation calculi.



(a) $\mathcal{RCC}-8$ (b) $\mathcal{OPRA}_2$

Figure 2: Complete N-Clique Descriptions

P4 for a calculus with n JEPD relationships, $[n \times n]$ composition theorems are pre-computed

P5 axioms of interaction that explicitly model interactions between interdependent spatial calculi, when more than one calculi are being applied in a non-integrated manner (i.e., with independent composition theorems)

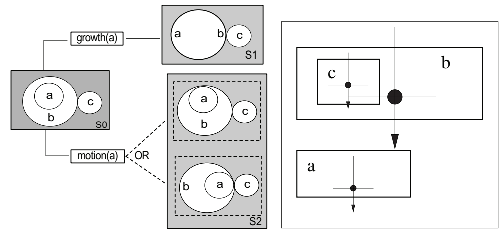
Whereas the JEPD property (P1) is necessary to model compositional reasoning and consistency maintenance, the CND structure (P) is useful in either projecting or abducing potential states for sets of qualitative spatial descriptions. By definition, for any spatial calculus, we assume that (P1–P5) are known apriori. Given the scope of this paper, we only discuss the modeling of requirements (P4) and (P5) herein. However, note that in order to realize a domain-independent spatial theory that is re-usable across arbitrary dynamic domains, it is necessary to preserve all the high-level axiomatic semantics in (P1–P5), and implicitly the underlying algebraic properties, that collectively constitutes a 'qualitative spatial calculus' (Ligozat and Renz 2004).

3. Commonsense and (Spatial) Calculi

3.1 Global Compositional Consistency

Corresponding to each situation (within a hypothetical branching-tree structured situation space), there exists a situation description that characterizes the spatial state of the system. Starting with the initial situation, it is necessary that the spatial component of such a state be a 'complete specification' without any missing information. Note that by complete specification, we do not imply absence of uncertainty or ambiguity. Completeness also includes those instances where the uncertainty is expressed as a set of completely specified alternatives in the form of disjunctive information. From the (spatial) viewpoint, for k spatial calculi being modeled, the initial situation description involving n domain objects requires a complete n-clique specification with $[n(n-1)/2]$ spatial relationships for each of the respective calculi (Fig. 2). Precisely, given that the foundational theory Σ_{sit} (Def. 1) consists of unique names axioms for fluents (i.e., $[\phi_{sp}(o_i,\ o_j) \neq \phi_{sp}(o_j,\ o_i)]$), static spatial configurations in actuality consist of $[(k \times [n(n-1)\,/\,2]) \times 2]$ unique functional fluents.

CI. Composition Theorems: From an axiomatic viewpoint, the notion of a spatial calculus, be it topological, orientational or other, defined on a relationship space $\mathcal{R}$ is (primarily) based on the derivation of a set of compositions between the primitive JEPD set $\mathcal{R}$. In general, for a calculus consisting of n JEPD relationships (i.e., $n = |\mathcal{R}|$), $[n \times n]$ compositions are precomputed. Each of these composition theorems is equivalent to an ordinary state constraint (2), which every n-clique spatial situation description (Fig.2) should satisfy.



(a) Compositional Theorems (b) Mutual Entailments

Figure 3: Compositional Consistency and Ramifications

$$(\forall s).\,[Holds(\phi_{sp}(o_1,\ o_2),\ \gamma_1,\ s) \wedge Holds(\phi_{sp}(o_2,\ o_3),\ \gamma_2,\ s) \supset Holds(\phi_{sp}(o_1,\ o_3),\ \gamma_3,\ s)] \tag{2}$$

CII. Axioms of Interaction: Axioms of interaction are only applicable when more than one spatial domain is being modeled in a non-integrated manner. Such axioms provide an explicit characterization of the relative entailments that exist between inter-dependent aspects of space. For instance, a spatial relationship of one type may directly entail or constrain a spatial relationship of another type (3a). Such axioms could also possibly be compositional in nature, making it possible to compose spatial relations pertaining to two different aspects of space in order to yield a spatial relation of either or both spatial types used in the composition (3b).

$$(\forall s).\,[Holds(\phi_{sp1}(o,\ o^{'}),\ \gamma,\ s) \supset Holds(\phi_{sp2}(o,\ o^{'}),\ \gamma',\ s)] \tag{3a}$$

$$(\forall s).\,[Holds(\phi_{sp1}(o_a,\ o_b),\ \gamma'_{sp1},\ s) \wedge Holds(\phi_{sp2}(o_b,\ o_c),\ \gamma'_{sp2},\ s) \supset Holds(\phi_{sp}(o_a,\ o_c),\ \gamma_{sp},\ s)] \tag{3b}$$

We further exemplify (CI–CII) for topological, size and orientation relationships in (4–5). Here, the following notion of global compositional consistency accounting for (CI–CII) suffices:

Definition 3 ($\mathcal{C}$-Consistency). *A situation is $\mathcal{C}$-Consistent, i.e., compositionally consistent, if the n-clique state or spatial situation description corresponding to the situation satisfies all the composition constraints of every spatial domain (e.g., topology, orientation, size) being modeled, as well as the relative entailments as per the axioms of interaction among inter-dependent spatial calculi when more than one spatial calculus is modeled.*

Although the details do not pertain here, it is instructive to point out that $\mathcal{C}$-Consistency is a key (contributing) notion in operationalizing the principle of 'physically realizable/plausible' situations for spatial planning and causal explanation tasks.

$\mathcal{C}$-Consistency and Ramifications Spatial situation descriptions denoting configurations of domain objects must be $\mathcal{C}$-Consistent (Def. 3). To re-emphasize, in addition to the compositional constraints over $\mathcal{R}$, $\mathcal{C}$-Consistency also includes those scenarios when more than one aspect of space is being modeled in a non-integrated way, i.e., relative dependencies between mutually dependent spatial dimensions that are modeled explicitly too should be satisfiable. Ensuring these two aspects of global consistency of spatial information is problematic because both compositional constraints as well as axioms of interaction contain indirect effects in them, thereby necessitating a solution to the ramification problem (Finger 1987). In the context of the situation calculus, (Lin 1995) illustrates the need to distinguish ordinary state constraints from indirect effect yielding ones, the latter being also referred to as *ramification constraints*. This is because when ramification constraints are present, it is possible to infer new effect axioms from explicitly formulated (direct) effect axioms together with the ramification constraints. Simply speaking, ramification constraints lead to what can be referred to as '*unexplained changes*', which is clearly undesirable within a qualitative theory of spatial change. These are further illustrated in examples (E1–E2):

E1. **Motion and/or Deformation**: Consider the basic case of compositional inference with three objects a, b and c: when a and b undergo a transition to a different qualitative state (either by translational motion and/or deformation), this also has an indirect effect, although not necessarily, on the spatial relationship between a and c since the relationship between the latter two is constrained by at least one of the $[n \times n]$ compositional constraints (2) of the relational space. As one example, consider the illustration in Fig. 3(a) – the scenario depicted herein consists of the topological relationships between three objects 'a', 'b' and 'c'. In the initial situation 'S_0', the spatial extension of 'a' is a *non-tangential part* of that of 'b'. Further, assume that there is a change in the relationship between 'a' and 'b', as depicted in Fig. 3(a), as a result of a direct effect of an event such as *growth* or an action involving the *motion* of 'a'. Indeed, as is clear from Fig. 3(a), for the spatial situation description in the resulting situation (either 'S_1' or 'S_2'), the compositional dependencies between 'a', 'b' and 'c' must be adhered to, i.e., the change of relationship between 'a' and 'c' must be derivable as an indirect effect from the underlying compositional constraints. The new relationship between a and c in situation S_2 can either result in: increased ambiguity, decreased ambiguity and in some cases no change at all.[2] In the case of the RCC-8 topological calculus, there exist a total of 64 composition theorems, 27 of which provide unambiguous information as to the potential relationship. All other compositions provide disjunctive information that may further be refined by the inclusion of complementary spatial calculi (Randell and Witkowski 2004). The support of modeling complementary axioms of interaction (3) is included for this purpose.

E2. **Interdependent Calculi**: The relative entailments between the topological and the size domains serve as the simplest example of interacting spatial calculi. Consider Table 1, which illustrates the mutual entailments between size relationships and the RCC-8 topological primitives (Gerevini and Renz 2002). For instance, size *equality* rules out all containment (*tpp*, *ntpp* and their inverses) relationships. Similarly, if it is known that object o is a tangential part of object

[2]The former two cases involve ramifications whereas the last case, further discussed in Section 3.2, pertains to inertia.

(a) Topology to Size

ϕ_{top}	ϕ_{size}	ϕ_{top}	ϕ_{size}
tpp	$\models$ <	dc	$\models$ no-info
ntpp	$\models$ <	ec	$\models$ no-info
tpp^{-1}	$\models$ >	po	$\models$ no-info
ntpp^{-1}	$\models$ >	eq	$\models$ =

(b) Size to Topology

ϕ_{size}	ϕ_{top}
=	$\models$ dc $\vee$ ec $\vee$ po $\vee$ eq
>	$\models$ dc $\vee$ ec $\vee$ po $\vee$ tpp^{-1} $\vee$ ntpp^{-1}
<	$\models$ dc $\vee$ ec $\vee$ po $\vee$ tpp $\vee$ ntpp

Table 1: Mutual Entailments for Topology and Size

o', then it can also be presumed that the size of object o is less than the size of o'. The other forms of interaction are compositional in nature and may be illustrated with topological and naive intrinsic orientational primitives. Consider the illustration in Fig. 3(b) where the composition of topological and orientation relations $front$ and $inside$ involving 3 objects a, b and c is depicted. Here, topological and orientation relationships between [b,c] and [a,b] respectively implies an orientation relation between [a,c]. This and other forms of interactions are formally exemplified in the section to follow.

Applying Lin's Causal minimisation A solution to the problem of ramifications for this particular case (of ensuring global compositional consistency of spatial scene descriptions) is obtainable from the general works of (Lin and Reiter 1994; Lin 1995). The solution basically involves appeal to causality (i.e., modeling all ramification yielding constraints in the form of causal rules) and applying nonmonotonic reasoning (using circumscription) to minimise the effects of occurrences whilst deriving the successor state axioms or the causal laws of the domain. Note that this manner of deriving the successor state axioms is an extension to the original approach proposed by (Reiter 1991), where only a solution to the frame problem is included under a general 'completeness assumption' stipulating that there are no indirect effects within the domain theory.

A reformulation of all ramification yielding state constraints as causal rules of the form proposed by (Lin 1995) is necessary: (4a) and (4b) exemplify one composition theorem each for the RCC-8 and the $\mathcal{OPRA}_2$ calculi respectively. Similarly, (5a–5c) respectively exemplify the noncompositional and compositional axioms of interaction with topological, size and naive orientation primitives.[3] Notice the difference between axioms (5a) and (5b) – whereas the latter is compositional in nature, the former is not. Furthermore, (5c) represents yet another form where spatial relationships from two calculi entail a relationship of both types.

$$(\forall s).\,[Holds(\phi_{top}(o_1,\ o_2),\ tpp,\ s)\ \wedge\ Holds(\phi_{top}(o_2,\ o_3),\ eq,\ s) \supset\ Caused(\phi_{top}(o_1,\ o_3),\ tpp,\ s)] \tag{4a}$$

$$(\forall s).\,[Holds(\phi_{ort}(\vec{o}_1,\ \vec{o}_2),\ \ 2\angle_2^6\ ,\ s)\ \wedge\ Holds(\phi_{ort}(\vec{o}_2,\ \vec{o}_3),\ \ 2\angle_1^6\ ,\ s) \supset\ Caused(\phi_{ort}(\vec{o}_1,\ \vec{o}_3),\ \ 2\angle_1^7\ ,\ s)] \tag{4b}$$

$$(\forall s).\,[Holds(\phi_{top}(o,\ o'),\ tpp,\ s) \supset\ Caused(\phi_{size}(o,\ o'),\ <,\ s)] \tag{5a}$$

[3]For readability, naive labels are used instead of $\mathcal{OPRA}_m$ primitives since the latter are non-linguistic and hence, counterintuitive.

$$(\forall s).\ Holds(\phi_{ort}(o_a,\ o_b),\ front,\ s)\ \wedge\ Holds(\phi_{top}(o_c,\ o_b),\ inside,\ s)$$
$$\supset\ Caused(\phi_{ort}(o_a,\ o_c),\ front,\ s) \tag{5b}$$

$$(\forall s).\ [Holds(\phi_{top}(o_1,\ o_2),\ ec,,\ s)\ \wedge\ Holds(\phi_{ort}(o_1,\ o_2),\ right,\ s)\ \wedge$$
$$Holds(\phi_{top}(o_2,\ o_3),\ ec,\ s)\ \wedge\ Holds(\phi_{ort}(o_2,\ o_3),\ right,\ s)\ \supset$$
$$Caused(\phi_{top}(o_1,\ o_3),\ dc,\ s)\ \wedge\ Caused(\phi_{ort}(o_1,\ o_3),\ right,\ s)] \tag{5c}$$

Indeed, the basic form of the ramification constraint stays the same, namely as a causal rule, and from an operational viewpoint, it is expected that all spatial domain constraints (both ramification and ordinary) shall be generated automatically from external / high-level (algebraic) specifications of qualitative spatial calculi. Let Σ_{rc} denote the set of all ramification constraints; $[\Sigma_{sit}\ \cup\ \Sigma_{rc}]$ refers to the conjunction of these constraints with the foundational situation calculus theory as per Def. 1. Strictly speaking, other aspects concerning a general spatial calculus (Section 2, P1–P3) that are not included in this paper would also be needed in this theory for the causal minimisation to work, but these are not conceptually connected to this paper and hence omitted. What is relevant is that applying causal minimisation results in causation axioms, explained shortly, determining all potential ways in which the spatial relationship ϕ_{sp} of any sort (e.g., topological, orientational) between two domain objects o_i and o_j (within the complete n-clique description) may acquire a particular situation-specific denotation γ. The manner in which these causation axioms get utilized is further elaborated on in Section 3.2. For now, the following is relevant:

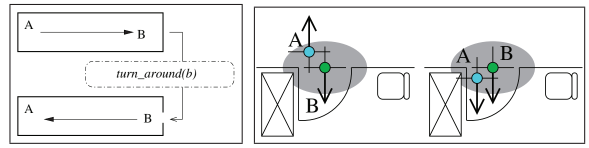
Proposition 1 ($\mathcal{C}$-Consistent Situation Space). *All spatial situation descriptions corresponding to 'legal' situations are $\mathcal{C}$-Consistent as per Def. 3.*

Proof. The proof sketch rests on the basic premise that the causal minimisation results in 'causation axioms' of the form in (6) (Lin 1995). Here, (A) and (B) correspond to the direct effects (not included in this paper) and indirect effects (Σ_{rc}) respectively that are either explicitly formulated or derivable from the theory.[4]

$$\begin{gathered} CIRC[\Sigma_{sit}\ \cup\ \Sigma_{rc}\ ;\ Caused] \\ \downarrow \\ Caused(\phi_{sp}(o_i,\ o_j),\ \gamma,\ s)\ \equiv\ \boxed{\{(A) \vee (B)\}} \end{gathered} \tag{6}$$

With (6) as a basic result, note that situation '*legality*' entails that permissible spatial changes are only those that adhere to the continuity constraints (Section 2, P3) of the relationship space $\mathcal{R}$ and other domain-specific pre-conditions. The direct effects of such continuous changes are covered by (A). Additionally, the formulation of all indirect-effect yielding constraints (Section 3.1, CI–CII) as causal rules, i.e., Σ_{rc}, ensures that the indirect effects that arise as a result of the permissible changes too are taken into consideration as a result of the causal minimisation. This implies that for all legal situations, the causation axiom entails all the compositional constraints and axioms of interactions of the relationship space $\mathcal{R}$. In other words, the legal situation space satisfies $\mathcal{C}$-Consistency. ■

[4] Note that the ternary 'Caused' relation always occurs on the right-side of the '$\supset$' connective (in all causal rules or explicitly formulated direct effects, and ramification constraints Σ_{rc}). Applying circumscription transforms the material implication to an equivalence – a syntactic transformation that follows from a standard result in circumscription (Lifschitz 1994, pg. 5). (Lin 1995) presents step-by-step operational details of the circumscriptive causal minimisation and (Lin 2003) realizes an implementation for the propositional case.



(a) Property Persistence (b) Positional Persistence

Figure 4: Incorporating Inertia

3.2 Incorporating Spatial Persistence

Global compositional consistency in section 3.1 dealt with the problem of ramifications, where spatial relationships undergo exceptional changes. With spatial persistence, there is essentially the need to incorporate the commonsense law of inertia, i.e., typically things stay the same. At least one other instance, addressing this line of investigation, can be found in the work of (Shanahan 1995). Within a real-valued coordinate system, Shanahan investigates the default reasoning pattern, also connected to the frame problem, required to model the commonsense law that '*space is typically empty*'. For instance, an agent would need to make such a default assumption before moving itself and/or other objects to a certain region of space or when other domain specific occurrences have happened. The patterns in the following complement this for the case where such a real-valued quantity space is reasoned upon qualitatively using formal spatial calculi.

Property/Relational Persistence Spatial property persistence, i.e., the intuition that the topological, orientational or other spatial relationship between two objects typically remains the same, is one default reasoning pattern rooted in the frame problem that is identifiable within the spatial context. For instance, assuming that dynamic topological and orientational information constitutes the state descriptions corresponding to the unique '*situations*', the problem is that of formalizing the intuition that the topological relationship between two objects or the orientation of an object relative to another '*typically*' remains the same, unless if there is '*cause*', whatever be the nature of such cause, to believe to the contrary. Consider Fig. 4(a), which qualitatively depicts the relationship of an *agent*, modeled as a directed line-segment ('b') to a containing object ('a') that is interpreted as a *room*. Given that the spatial relationship of the agent with that of the room is that of containment, the problem of spatial property persistence is that of formalizing the intuition that this containment relationship persists in the situation resulting from the occurrence of an action such as $turn\_around$.

Absolute Positional Persistence In addition to persistence at the qualitative or relational level, absolute positional persistence at the metric level is also required to formalize the intuition that the absolute spatial extension of an object, whatever that may be (Section 2), and its intrinsic orientation and/or implicit direction parameter typically stays the same. Depending on the nature of the spatial ontology that is adopted, the inertial aspects that need to be accounted for at the metric-level include:

I1 for spatially extended objects, their planar or volumetric extension typically stays the same. This implies that the '*qualified*' region of space occupied by an object typically stays the same as a result of occurrences.

I2 for point and line-segment approximated objects, its point-vector(s) and the additional direction parameter stays the same.

I3 for an empty region of space, the intuition that it typically remains empty.

Generic Frame Assumption Given the causal minimisation determining what changes directly or indirectly as a result of ramification constraints, the question of what does not change becomes almost trivial. In the context of the situation calculus formalism in use, a generic frame assumption of the form in (7) incorporating the principal of inertia whilst deriving the standard successor state axioms (Reiter 1991) is sufficient to handle all forms of persistence. Separate inertial assumptions are required to model each of (I1-I3), however their generic form remains the same as that required for property persistence as modeled by (7):

$$\begin{aligned}&Poss(\theta,\, s) \vee Occurs(\theta,\, s) \supset\\&[\neg(\exists\, \gamma')\ Caused(\phi_{sp}(o_i,\, o_j),\, \gamma',\, Result(\theta,\, s)) \supset\\&Holds(\phi_{sp}(o_i,\, o_j),\, \gamma,\, Result(\theta,\, s)) \equiv Holds(\phi_{sp}(o_i,\, o_j),\, \gamma,\, s)]\end{aligned} \tag{7}$$

What is essentially required to be done is to compile the causation axioms (6) within the generic frame axiom (7) to derive the final causal laws determining all changes as well as non-changes.

4. Discussion and Outlook

Qualitative spatial methods have primarily remained focused on reasoning with static spatial configurations. However, for applications such as cognitive robotics, these methods require different interpretation, where sets of spatial relations undergo change as a result of named occurrences in the environment. Consequently, the formal embedding of arbitrary spatial calculi – whilst preserving their high-level axiomatic semantics and low-level algebraic properties – has to be investigated from the viewpoint of formalisms such as the situation calculus, event calculus and fluent calculus. At a higher level of abstraction, this will result in the (native) incorporation of commonsense notions of space and spatial change within languages such as GOLOG and FLUX for their use in arbitrary dynamic domains. In general, the areas of commonsense reasoning, and action and change are mature and established tools, formalisms and languages from therein are general enough to be applied to the case of dynamic spatial systems, where relational spatial models undergo change as a result of interaction in the environment. In this paper, we highlighted (some) aspects of embedding arbitrary spatial calculi within the situation calculus formalism and the utility of commonsense inference patterns, connected to the frame and the ramification problems, whilst achieving the suggested embedding. This is primarily done with the aim of consistently preserving the high-level axiomatic properties determining the constitution of a qualitative spatial calculus. As research in qualitative spatial representation and reasoning moves form theory to practice, it will be necessary to integrated formal spatial calculi within general logic-based frameworks in AI, and to further broaden the interpretation of a (re-usable) qualitative spatial theory.

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