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On the Minimal Labeling Problem of Temporal and Spatial Qualitative Constraints

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Abstract

Spatial and temporal reasoning is a crucial task for certain Artificial Intelligence applications. In this context, and since two decades, various formalisms representing the information through qualitative constraint networks (QCN) have been proposed. Given a QCN, the main two problems that are facing researchers are: deciding whether this QCN is consistent or not, and, the minimal labeling problem. In this paper, we propose an efficient algorithm aiming at solving the minimal labeling problem. This algorithm is based on subclasses of relations for which the property of $\diamond$-consistency implies the minimality of the QCN.

Introduction

In past, numerous qualitative calculi (Ligozat and Renz 2004) have been proposed to reason about temporal or spatial information. A qualitative calculus uses particular elements for representing the spatial or temporal entities and considers relations between these elements to constrain the relative positions of these elements. Particular kind of constraint satisfaction problems called qualitative constraint networks (QCN in short) can be used to represent all the temporal/spatial information of a system. A QCN allows to represent the possible configurations of temporal or spatial entities by specifying for each couple of entities (for the binary case) a set of possible relations among the base relations provided by the qualitative formalism in consideration. Each base relation corresponds to a particular relative position between two elements. Allen's calculus (Allen 1981) and RCC (Region Connection Calculus) (Randell, Cui, and Cohn 1992) are certainly the best known of the qualitative calculi respectively for temporal reasoning and spatial reasoning.

Given a QCN, two main problems may arise: the consistency problem and the minimal labeling problem. In the first problem, the main objective is to find if a solution of the QCN does exist or not. The second problem consists of determining all the feasible base relations (*i.e.* the base relations participating at least to one solution) for each of these constraints. For most of the qualitative formalisms, these two problems are in general NP-hard (Liu and Li 2012). For these formalisms, some studies have led to the characterization of tractable fragments, particularly, of subclasses for which these problems are polynomial problems.

In this paper, we are concerned by solving the minimal labeling problem (MLP in short) in a pratical way. This problem can be found for example in natural language processing applications, where certain spatial or temporal knowledge are acquired by manual annotation where other knowledge are inferred through an automatic reasoning mechanism.

A naive method to solve MLP consists of testing in an iterative way the feasibility of every base relation composing the constraints of the QCN through the use of a method that solves the consistency problem. For this, in a first step, the constraint containing the base relation to be tested is substituted by the singleton relation *singleton* composed of this base relation. In a second stage, the consistency test of the obtained QCN is realized. The detection of the consistency (resp. inconsistency) of the QCN allows to affirm that the base relation is feasible (resp. unfeasible). It is clear that inversely, the consistency problem can be solved through the resolution of the minimal labeling problem.

In the literature, we can note that the consistency problem has got much more attention than the minimal labeling problem. It is certainly in part explained by the fact that, as we have just seen it, these problems are equivalent under polynomial Turing reductions. Among the most efficient approaches aiming to solve the consistency problem of a QCN, one approach (Nebel 1996) consists of a backtrack search combining a splitting of the constraints into sub-relations of a tractable subclass and a filtering of the constraints realized by the weak composition method. The splitting of the selected constraint into sub-relations belonging to a tractable subclass in each step of the search allows to minimize the width of the search tree. The calculation of the closure by weak composition, also called path-consistency method, removes some unfeasible base relations through the operation of weak composition to obtain an equivalent $\diamond$-consistent sub-QCN and consequently, allows to reduce the search space similarly. Provided that every non trivially inconsistent and $\diamond$-consistent QCNs defined by relations of the used tractable subclass are consistent QCNs, this method is complete. This global approach can be enhanced under certain conditions, by using tree decompositions for example (Chmeiss and Condotta 2011; Sioutis and Koubarakis 2012), or also, by using concepts such as the eligible constraints (Condotta, Ligozat, and Saade 2007).

By checking the methods based on the above mentioned approach, we can observe that the detection of the consistency of a QCN is done by the characterization of one of its $\diamond$-consistent sub-QCN (*i.e.* one of its $\diamond$-consistent sub-QCN closed by weak composition) defined by some relations of the used tractable subclass. Furthermore, if for this tractable subclass every $\diamond$-consistent QCN is minimal, we can conclude that this sub-QCN is uniquely formed by feasible base relations. Starting from this, we define and study in this paper an efficient algorithm, called MinimizeSDCM, to solve the minimal labeling problem of a QCN.

The next section is devoted to reminders concerning QCNs. In the third section we present and study the algorithm MinimizeSDCM. After considering a running example of this algorithm, we report some experimental results. Finally, we conclude and give some perspectives of this work.

Preliminaries

A (binary) temporal or spatial qualitative calculus is based on a finite set B of base relations on a domain D. The elements of D represent temporal or spatial entities, and the elements of B represent all possible configurations between two entities. B forms a partition of $\mathsf{D} \times \mathsf{D}$, and it contains the identity relation Id on D, and is closed under the converse operation ($^{-1}$). A (complex) relation is the union of some base relations and is represented by the set containing them. Hence, the set 2^{B} will represent the set of relations. 2^{B} is equipped with the usual set-theoretic operations (union and intersection), the converse operation and the weak composition operation. The converse of a relation $r \in 2^{\mathsf{B}}$, denoted by r^{-1}, is the union of the converses of the base relations contained in r. The weak composition operation denoted by $\diamond$ is defined by: $\forall a, b \in \mathsf{B}, a \diamond b = \{c \in \mathsf{B} : \exists x, y, z \in \mathsf{D} \mid x\ a\ z \wedge z\ b\ y \wedge x\ c\ y\}$; $\forall r, s \in 2^{\mathsf{B}}, r \diamond s = \bigcup_{a \in r, b \in s}\{a \diamond b\}$. Note that for some calculi, $r \diamond s$ is identical to the usual relational composition $r \circ s = \{(x, y) \in \mathsf{D} \times \mathsf{D} : \exists z \in \mathsf{D} \mid x\ r\ z \wedge z\ s\ y\}$.

As illustration, the Interval Algebra (IA) (Allen 1981) is a temporal qualitative calculus whose domain is the set $\mathsf{D}_{\mathsf{IA}} = \{(x^-, x^+) \in \mathbb{Q} \times \mathbb{Q} : x^- < x^+\}$ since temporal entities are represented by intervals of the rational line. The set of base relations of this calculus is the set $\mathsf{B}_{\mathsf{IA}} = \{eq, p, pi, m, mi, o, oi, s, si, d, di, f, fi\}$. These thirteen binary relations represent all the orderings of the four endpoints of two intervals (see Figure 1(a)).

In what follows, we consider B as a set of base relations of a qualitative calculus. A Qualitative Constraint Network (QCN) is a pair composed of a set of variables and a set of constraints. Each variable represents an entity and each constraint represents a set of possible qualitative configurations between two variables. Formally, a QCN is defined as follows:

Definition 1 *A* QCN *is a pair $\mathcal{N} = (V, C)$ where: V is a non empty finite set of variables; C is a mapping that associates a relation $C(v, v') \in 2^{\mathsf{B}}$ with each pair (v, v') of $V \times V$. C is such that $C(v, v) \subseteq \{\mathsf{Id}\}$ and $C(v, v') = (C(v', v))^{-1}$ for every $v, v' \in V$.*

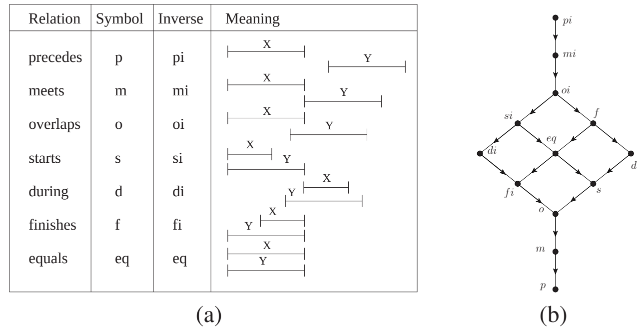


Figure 1: (a) The base relations of IA, (b) the interval lattice.

In the sequel, given a QCN $\mathcal{N} = (V, C)$ and $v, v' \in V$, $\mathcal{N}[v, v']$ will also denote the relation $C(v, v')$. Given a set of variables V, $\perp^V$ will denote the particular QCN where each constraint between each pair of variables $(v, v') \in V \times V$ is defined by the empty relation $\emptyset$. Given a QCN $\mathcal{N} = (V, C)$ we have the following definitions: A *partial solution* of $\mathcal{N}$ on $V' \subseteq V$ is a mapping σ defined from V' to D such that for every pair (v, v') of variables in V', $(\sigma(v), \sigma(v'))$ satisfies $C(v, v')$, i.e. there exists a base relation $b \in C(v, v')$ such that $(\sigma(v), \sigma(v')) \in b$. A *solution* of $\mathcal{N}$ is a partial solution of $\mathcal{N}$ on V. $\mathcal{N}$ is *consistent* iff it admits a solution. Two QCNs are *equivalent* iff they admit the same set of solutions. A *sub*-QCN $\mathcal{N}'$ of $\mathcal{N}$, denoted by $\mathcal{N}' \subseteq \mathcal{N}$, is a QCN (V, C') such that $C'(v, v') \subseteq C(v, v')$ for every pair $(v, v') \in V \times V$. Given a QCN $\mathcal{N}' = (V, C')$, $\mathcal{N}' \cup \mathcal{N}$ denotes the QCN (V, C'') defined by $C''(v, v') = C'(v, v') \cup C(v, v')$ for all $v, v' \in V$. An *atomic* QCN is a QCN such that each constraint is defined by a base relation. A *scenario* $\mathcal{S}$ of $\mathcal{N}$ is an atomic consistent sub-QCN of $\mathcal{N}$. A base relation $b \in C(v, v')$ with $v, v' \in V$ is *feasible* (resp. (*unfeasible*) iff there exists (resp. there does not exist) any scenario $\mathcal{S}$ of $\mathcal{N}$ such that $\mathcal{S}[v, v'] = \{b\}$. A QCN $\mathcal{N} = (V, C)$ is minimal iff for all $v, v' \in V$ and $b \in C(v, v')$, b is a feasible base relation. The unique equivalent minimal sub-QCN of a QCN $\mathcal{N}$ is denoted by $\mathcal{N}_{\mathsf{min}}$. It is called the minimal QCN of $\mathcal{N}$.

A QCN $\mathcal{N} = (V, C)$ is $\diamond$-*consistent* or *closed under weak composition* iff $\forall v, v', v'' \in V$, $C(v, v') \subseteq C(v, v'') \diamond C(v'', v')$. The *closure* under weak composition of $\mathcal{N}$, denoted by $\diamond(\mathcal{N})$, is the largest $\diamond$-consistent sub-QCN of $\mathcal{N}$ equivalent to $\mathcal{N}$. This sub-QCN can be obtained by iterating the triangulation operation $C(v, v') \leftarrow C(v, v') \cap (C(v, v'') \diamond C(v'', v'))$ for all $v, v', v'' \in V$ until a fix point is reached. This method can be implemented by an algorithm running in $O(n^3)$ time where $n = |V|$. Note that for some qualitative calculi as IA, path-consistency and $\diamond$-consistency are equivalent properties.

Given a set of base relations B and a set $\mathcal{A} \subseteq 2^{\mathsf{B}}$, we will denote by $\overline{\mathcal{A}}$ the closure of $\mathcal{A}$ under converse, intersection and weak composition. In the case where $\mathcal{A} = \overline{\mathcal{A}}$

we will say that $\mathcal{A}$ is a subclass of 2^{B}. Given a relation r of 2^{B} and a subclass $\mathcal{A} \subseteq 2^{\mathsf{B}}$ containing the total relation (the relation containing all base relations), $\mathcal{A}(r)$ denotes the smallest relation of $\mathcal{A}$ including r. Moreover, given a QCN $\mathcal{N} = (V, C)$, $\mathcal{A}(\mathcal{N})$ is the QCN $\mathcal{N}' = (V, C')$ defined by $C'(v, v') = \mathcal{A}(C(v, v'))$ for all $v, v' \in V$. In the sequel, all the considered subclasses will contain the singleton relations of 2^{B}. This is why from now on, we assume that given a subclass $\mathcal{A}$, $\mathcal{A}$ contains the singleton relations.

$\mathcal{C}$ will denote the subclass of convex relations of IA. $\mathcal{C}$ contains 83 relations. Ligozat (Ligozat 1996) introduces a lattice arranging the base relations of B_{IA}, see Figure 1(b). The convex relations of IA correspond to the intervals of this lattice. The ORD-Horn relations of IA (Nebel 1996), also called the preconvex relations, form a subclass denoted by $\mathcal{H}$ in the sequel. $\mathcal{H}$ is the maximal (for $\subseteq$) subclass of IA for which the consistency problem is polynomial. The subclass of the strict relations (Amaneddine and Condotta 2012), denoted by $\mathcal{S}$, is another subclass that we will consider in the sequel. $\mathcal{S}$ contains 82 relations and corresponds to those relations of IA wa can express by constraints of the Point Algebra (Vilain and Kautz 1986) whithout using inequations. Note that $\mathcal{C}$ and $\mathcal{S}$ are two distinct subsets of $\mathcal{H}$. For these subclasses, $\diamond$-consistency of a QCN (non trivially inconsistent) implies the consistency of this QCN. Moreover, for the subclasses $\mathcal{C}$ and $\mathcal{S}$, $\diamond$-consistency of a QCN implies the minimality of this QCN.

Function MinSubQCN($\mathcal{N}_{\mathsf{init}}$,$\mathcal{N}$,$e$)

input : two QCNs $\mathcal{N}_{\mathsf{init}} = (V, C_{\mathsf{init}})$ and $\mathcal{N} = (V, C)$ such that $\mathcal{N} \subseteq \mathcal{N}_{\mathsf{init}}$ and e a pair of variables.
output: a sub-QCN of $\mathcal{N}$.

1 **begin**
2 $\quad \mathcal{N} \leftarrow \mathsf{CWC}(\mathcal{N}, e)$;
3 $\quad$ **if** $\mathcal{N} = \perp^V$ **then**
4 $\quad\quad$ **return** $\perp^V$;
5 $\quad$ Select $(v, v') \in V \times V$ such that $\mathcal{A}(\mathcal{N}[v, v'])) \not\subseteq \mathcal{N}_{\mathsf{init}}[v, v']$;
6 $\quad$ **if** *a such pair does not exist* **then**
7 $\quad\quad$ **return** $\mathcal{N}$;
8 $\quad$ Split $\mathcal{N}[v, v']$ into sub-relations $r_1, \ldots, r_k \in \mathcal{A}$ such that $1 < k < |B|$;
9 $\quad \mathcal{N}' \leftarrow \mathcal{N}$;
10 $\quad$ **foreach** $i \in 1, \ldots, k$ **do**
11 $\quad\quad \mathcal{N}[v, v'] \leftarrow r_i; \mathcal{N}[v', v] \leftarrow r_i^{-1}$;
12 $\quad\quad \mathcal{N} \leftarrow \mathsf{MinSubQCN}(\mathcal{N}_{\mathsf{init}}, \mathcal{N}, (v, v'))$;
13 $\quad\quad$ **if** $\mathcal{N} \neq \perp^V$ **then**
14 $\quad\quad\quad$ **return** $\mathcal{N}$;
15 $\quad\quad \mathcal{N} \leftarrow \mathcal{N}'$;
16 $\quad$ **return** $\perp^V$

The Algorithm MinimizeSDCM

In this section, we propose and study an algorithm, called MinimizeSDCM (Minimize with a Subclass for which $\diamond$-consistency implies Minimality) to calculate the minimal QCN of a QCN $\mathcal{N}$ through a subclass $\mathcal{A}$ for which $\diamond$-consistency of a QCN implies its minimality. Before we describe this method, we describe a set of auxiliary functions. The function MinSubQCN takes as parameters two QCNs: $\mathcal{N}_{init}$ and $\mathcal{N}$ (with $\mathcal{N} \subseteq \mathcal{N}_{init}$), and possibly, a pair of variables e belonging to $V \times V$. This function is similar to the one proposed in (Condotta, Ligozat, and Saade 2007) for solving the consistency problem of QCNs. A non-trivially consistent QCN $\mathcal{N}'$ such that $\mathcal{N}' \subseteq \mathcal{N}$, $\mathcal{N}'$ is $\diamond$-consistent, and $\mathcal{A}(\mathcal{N}') \subseteq \mathcal{N}_{\mathsf{init}}$ is obtained in case where such a QCN exists. In the contrary case, it returns the QCN $\perp^V$. A backtrack search is realized by using the weak composition method for propagating constraints. The parameter e is null for the first call of the function, then, not null for the other recursive calls. It allows to realize an incremental constraint propagation through the function CWC (Closure by Weak Composition). The composed instructions CWC are not given but we can suppose that they are similar to those given in (Vilain and Kautz 1986). CWC takes as parameter a QCN and possibly a second parameter corresponds to a couple of variables of this QCN. It returns the closure by weak composition of the QCN used as parameter. When a pair of variables is given in parameter, it is supposed that the QCN used as parameter is $\diamond$-consistent by considering all triples of edges not containing e. In each step of the search of MinSubQCN, a constraint is selected and split into non-empty sub-relations of the subclass $\mathcal{A}$. Then, this constraint is iteratively defined by each of these sub-relations. The search continues by a recursive call of MinSubQCN. The notion of eligibility proposed in (Condotta, Ligozat, and Saade 2007) is implemented through the selection of the new constraint to be treated in line 5. Only an eligible constraint, *i.e.* a constraint defined by a relation whose closure with respect to $\mathcal{A}$ is not included in the constraint of $\mathcal{N}_{\mathsf{init}}$, can be selected. The notion of eligibility allows to minimize the depth of the search tree. We can notice that in the case where $\mathcal{N}$ admits a $\diamond$-consistent atomic sub-QCN, MinSubQCN finds and returns a non trivially inconsistent QCN. By using an approach similar to the one followed by (Nebel 1996), we can formally establish the following property:

Proposition 1 *Consider two* QCN $\mathcal{N}_{\mathsf{init}} = (V, C_{\mathsf{init}})$, $\mathcal{N} = (V, C)$ *such that* $\mathcal{N} \subseteq \mathcal{N}_{\mathsf{init}}$. *The call of the function* MinSubQCN($\mathcal{N}_{\mathsf{init}}$, $\mathcal{N}$) *returns a* QCN $\mathcal{N}'$ *non trivially inconsistent and* $\diamond$*-consistent such that* $\mathcal{N}' \subseteq \mathcal{N}$ *and* $\mathcal{A}(\mathcal{N}') \subseteq \mathcal{N}_{\mathsf{init}}$ *when such a* QCN $\mathcal{N}'$ *exists, it returns* $\perp^V$ *otherwise.*

The function MinimizeSDCM is the main method that allows the calculation of the minimal QCN of a QCN. It takes as parameter the QCN $\mathcal{N} = (V, C)$ for which we want to calculate its minimal QCN. As we will show in what follows, this method will be complete in the case where for the subclass $\mathcal{A}$, $\diamond$-consistency of a QCN implies minimality of this QCN. Roughly, MinimizeSDCM is divided into three successive steps. The first step during which different variables are initialized, a second step allows the calculation of the feasible base relations, and finally, a third step, where the result is returned. Let us describe these steps into details.

The different variables initialized during the first step are: $\mathcal{N}_{\mathsf{init}}$, $\mathcal{N}_{\mathsf{F}}$ and $\mathcal{N}_{\mathsf{NonF}}$. $\mathcal{N}_{\mathsf{init}}$ allows to save the initial state of

Function MinimizeSDCM($\mathcal{N}$,$\mathcal{C}$)

input : $\mathcal{N} = (V, C)$ a QCN on 2^{B}, $\mathcal{A}$ a subclass of 2^{B}.
output: A sub-QCN of $\mathcal{N}$

1 **begin**
// **Step 1:** Initialization
2 $\mathcal{N}_{\mathsf{init}} \leftarrow \mathcal{N}$;
3 $\mathcal{N}_{\mathsf{F}} \leftarrow \bot^V$; $\mathcal{N}_{\mathsf{nonF}} \leftarrow \bot^V$;
4 $\mathcal{N} \leftarrow \mathsf{PreTreatment}(\mathcal{N})$;
5 $\mathcal{N}_{\mathsf{nonF}} \leftarrow \mathcal{N}_{\mathsf{init}} \setminus \mathcal{N}$;
6 **if** $\mathcal{N}[v, v'] = \emptyset$ *for some* $v, v' \in V$ **then**
7 **return** $\bot^V$;
// **Step 2:** Minimization
8 **while** $\{(v, v') : (\mathcal{N}[v, v'] \setminus \mathcal{N}_{\mathsf{F}}[v, v']) \neq \emptyset\} \neq \emptyset$ **do**
9 Select $(v, v') \in V \times V$ such that $(\mathcal{N}[v, v'] \setminus \mathcal{N}_{\mathsf{F}}[v, v']) \neq \emptyset$;
10 $r \leftarrow \mathcal{N}[v, v'] \setminus \mathcal{N}_{\mathsf{F}}[v, v']$;
11 $\mathcal{N}[v, v'] \leftarrow r$; $\mathcal{N}[v', v] \leftarrow r^{-1}$;
12 $\mathcal{N}' \leftarrow \mathsf{MinSubQCN}(\mathcal{N}_{\mathsf{init}}, \mathcal{N})$;
13 **if** $\mathcal{N}' = \bot^V$ **then**
14 $\mathcal{N}_{\mathsf{nonF}}[v, v'] \leftarrow \mathcal{N}_{\mathsf{nonF}}[v, v'] \cup r$;
15 $\mathcal{N}_{\mathsf{nonF}}[v', v] \leftarrow (\mathcal{N}_{\mathsf{nonF}}[v, v'])^{-1}$;
16 **else**
17 $\mathcal{N}_{\mathsf{F}} \leftarrow \mathcal{N}_{\mathsf{F}} \cup \mathcal{A}(\mathcal{N}')$;
18 $\mathcal{N} \leftarrow \mathcal{N}_{\mathsf{init}} \setminus \mathcal{N}_{\mathsf{nonF}}$
// **Step 3: Return of the result**
19 **return** $\mathcal{N}_{\mathsf{F}}$

the QCN $\mathcal{N}$. The QCN $\mathcal{N}_{\mathsf{F}}$ will allow to store the base relations of $\mathcal{N}$ detected as feasible base relations during the treatment. At the end of the treatment, $\mathcal{N}_{\mathsf{F}}$ correspond to the minimal QCN of $\mathcal{N}$ given as parameter. The QCN $\mathcal{N}_{\mathsf{NonF}}$ allows the accumulation of the base relations of $\mathcal{N}$ which will be detected as unfeasible during the treatment. After the initialization of these variables, an optional preliminary treatment is performed over $\mathcal{N}$ (line 4), here the aim is to eliminate some unfeasible base relations with a fast method. This pre-treatment could be for example the calculation of the closure $\diamond$-consistency of $\mathcal{N}$. This pre-treatment must return an equivalent sub-QCN of $\mathcal{N}$. The base relations detected as unfeasible during this pre-treatment are added to the QCN $\mathcal{N}_{\mathsf{NonF}}$ (line 5). In the case where $\mathcal{N}$ is detected as trivially inconsistent we can assert that $\mathcal{N}_{\mathsf{init}}$ is not consistent. Its minimal QCN is then the QCN $\bot^V$ returned in line 7. In the contrary case, we continue the treatment. We note here that during this level, $\mathcal{N}$ is a sub-QCN equivalent to $\mathcal{N}_{\mathsf{init}}$. This property will stay satisfied until the end of the call of MinimizeSDCM.

In the second step of the treatment (line 8), the constraints of the QCN $\mathcal{N}$ are treated until all of its base relations are detected as feasible or as unfeasible. For this purpose, a pair of variables (v, v') is selected in line 9, for which, the corresponding constraint contains non-classified base relations. These base relations correspond to the relation r (line 10). The constraint of $\mathcal{N}$ between v and v' is then defined by this relation (line 11), a non trivially inconsistent and $\diamond$-consistent sub-QCN $\mathcal{N}'$ of $\mathcal{N}$ is then searched through a call of the method MinSubQCN (line 12). If such sub-QCN does not exist, we can state that $\mathcal{N}$ is inconsistent and therefore, the base relations of r are unfeasible. These base relations are then added to $\mathcal{N}_{\mathsf{NonF}}$ (lines 14-15) and removed from $\mathcal{N}$ (line 18). In the case where a sub-QCN $\mathcal{N}'$ of $\mathcal{N}$ non trivially inconsistent is found, the base relations of $\diamond(\mathcal{A}(\mathcal{N}'))$ are characterized as base relations belonging to the minimal QCN of $\mathcal{N}_{\mathsf{init}}$ and then added to $\mathcal{N}_{\mathsf{F}}$. $\mathcal{A}(\mathcal{N}')$ is defined by relations of the subclass $\mathcal{A}$. It is as well $\diamond$-consistent since $\mathcal{N}'$ is $\diamond$-consistent. Therefore, given that $\diamond$-consistency implies minimality for $\mathcal{A}$, we can affirm that $\mathcal{A}(\mathcal{N}')$ is minimal. Furthermore, and since $\mathcal{A}(\mathcal{N}') \subseteq \mathcal{N}_{\mathsf{init}}$ we can conclude that $\mathcal{A}(\mathcal{N}') \subseteq (\mathcal{N}_{\mathsf{init}})_{\mathsf{min}}$.

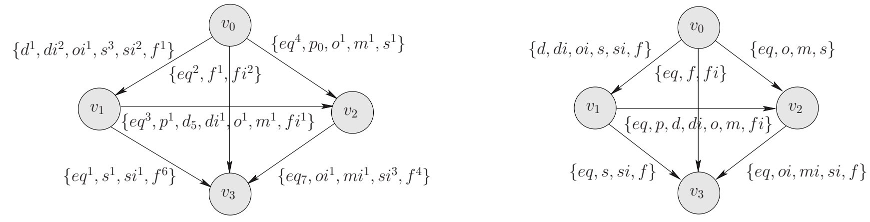
Let us prove now that after a certain number of finite loops the second step will end. We have previously seen that in case where the call of the function MinSubQCN (line 12) returns a trivially inconsistent QCN then the base relations of r were removed from $\mathcal{N}$. In the contrary case, this call returns a QCN $\mathcal{N}'$ non trivially inconsistent and $\diamond$-consistent. We can affirm that $\mathcal{A}(\mathcal{N}')$ is non trivially consistent as well. We have as well, $\mathcal{A}(\mathcal{N}'[v, v']) \cap \mathcal{N}'[v, v'] \neq \emptyset$. Therefore, $\mathcal{A}(\mathcal{N}'[v, v']) \cap r \neq \emptyset$. As a result, at least one of these base relations of r is added to $\mathcal{N}_{\mathsf{F}}$. From all what precedes, we can conclude that during each iteration of the second step, at least one of these base relations of $\mathcal{N}$ is added to $\mathcal{N}_{\mathsf{F}}$ or removed from it. Consequently, the second step is performed at most δ times with $\delta = \Sigma_{(v,v') \in V \times V}\{|\mathcal{N}_{\mathsf{init}}[v, v']|\}$.

Following the second step, all base relations of $\mathcal{N}_{\mathsf{init}}$ have been treated, we can then affirm that $\mathcal{N}_{\mathsf{F}}$ returned in line 19 corresponds to the minimal QCN of $\mathcal{N}_{\mathsf{init}}$. From all this, we can establish the following result:

Theorem 1 *Given a* QCN $\mathcal{N}$ *and a subclass* $\mathcal{A}$ *for which* $\diamond$*-consistency implies minimality, the algorithm* MinimizeSDCM *computes the minimal* QCN *of* $\mathcal{N}$.

Running Example

In the example we are proposing, we consider QCNs of IA and the set of the convex relations $\mathcal{C}$ as subclass $\mathcal{A}$ used by the method MinimizeSDCM. Consider the call of MinimizeSDCM with the QCN $\mathcal{N}$ described in Figure 2(a) as parameter. The indexes associated to each base relation Figure 2(a) identify the step during which the base relation has been characterized as feasible (index placed at the top) or non feasible (index placed at the bottom). The index 0 corresponds to base relations detected unfeasible during the pre-treatment (the closure of the weak composition in this example). Each index $i \in 1, \ldots, 7$ identifies the i^{th} iteration performed during the second step of the function MinimizeSDCM. The pair of variables (v, v') selected, the relation r corresponding to base relations to be treated and the QCN $\mathcal{N}'$ returned by MinSubQCN are indicated for each one of these iteration. Note that for this example, we have $\mathcal{C}(\mathcal{N}') = \mathcal{N}'$ for each iteration. These QCN are $\diamond$-consistent and defined by convex relations.



(v, v')	r	return of MinSubQCN
(v_1, v_2)	$\{eq, p, d, di, o, m, fi\}$	N'_1
(v_0, v_1)	$\{di, s, si\}$	N'_2
(v_2, v_3)	$\{eq, si, f\}$	N'_3
(v_2, v_3)	$\{eq, f\}$	N'_4
(v_1, v_2)	$\{d\}$	$N'_5 = \perp$
(v_1, v_3)	$\{f\}$	N'_6
(v_2, v_3)	$\{eq\}$	$N'_7 = \perp$

(c) Selected relations

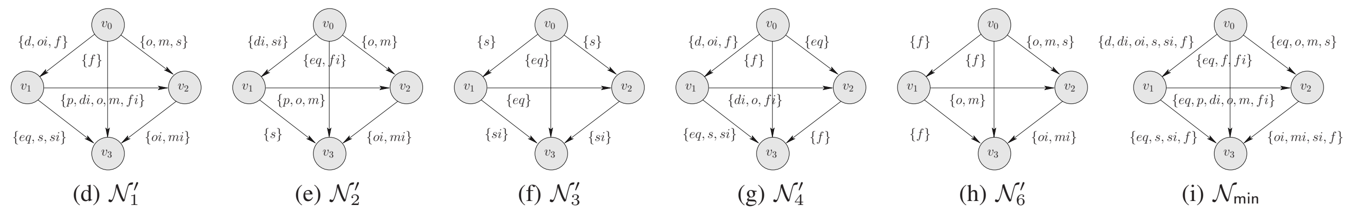


Figure 2: Tracing of the call of the function MinimizeSDCM

We have for example the base relation p belonging to the constraint between v_0 and v_2 has been detected as unfeasible during the pre-treatment phase, whereas the base relation d of the constraint between v_1 and v_2 has been detected feasible during the fifth iteration of the second step. As another example, the base relations di and si belonging to the constraint between v_0 and v_1 have been detected as feasible during the second iteration of the step 2. The minimal QCN of $\mathcal{N}$ returned at the end of the treatment corresponds to the QCN illustrated in Figure 2(i). This QCN is obtained after 7 iterations performed during the second step of MinimizeSDCM. This example shows that the iterative treatment performed during the second step can classify several base relations in a same iteration.

Experiments

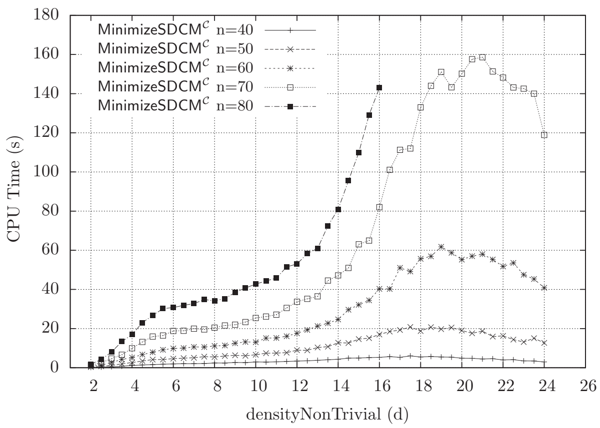
In order to study the behaviour of the algorithm MinimizeSDCM, we conducted some experiments concerning QCNs of the Interval Algebra. These QCNs have been generated from the model S (Nebel 1996). This model can randomly generate consistent QCNs according to three parameters n, d, and s, where n is the number of variables of the generated QCNs, d is the density of the non trivial constraints (constraints defined by a relation other than the total relation, *i.e.* a relation other than B) and s the average number of base relations of a non trivial constraint. For this model, the consistency of a generated QCN is guaranteed by adding a consistent scenario. A set of QCN generated through the model S using the parameters n, d and s will be denoted by $\mathsf{S}(n, d, s)$. The presented experiments concern instances issued from the series $\mathsf{S}(n, d, 6.5)$ with n varies between 40 à 80 with an incremental step of 10 and d varies between 2 à 24 with an incremental step of 2. For each series, we generated 100 QCNs.

Two subclasses of relations have been used to define the subclass $\mathcal{A}$ of the algorithm MinimizeSDCM, namely the subclass of convex relations $\mathcal{C}$ and the subclass of the strict relations $\mathcal{S}$. In what follows, MinimizeSDCM$^{\mathcal{C}}$ (resp. MinimizeSDCM$^{\mathcal{S}}$) refers to the algorithm MinimizeSDCM using $\mathcal{C}$ (resp. using $\mathcal{S}$) as tractable subclass. Minimize$^{\mathcal{H}}$ refers to a naive minimization algorithm (as described in introduction) based on the efficient algorithm proposed in (Condotta, Ligozat, and Saade 2007) to solve the consistency problem of a QCN. This method is similar to the function MinSubQCN previously presented and uses the preconvex subclass $\mathcal{H}$ as tractable subclass. The implementation of these functions has been done using the C programming language, for the corresponding experiments a timeout of 5 hours has been given for each series.

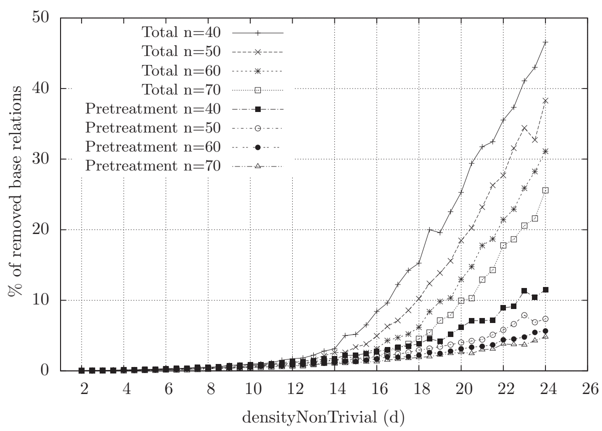
Figure 3(a) illustrates the cpu time required by MinimizeSDCM$^{\mathcal{C}}$ to solve the sequences $\mathsf{S}(n, d, 6.5)$ We note that for each fixed number n of variables, there exists a ceiling from which the calculation of minimal QCNs become very hard. For example, for $n = 60$ (resp. $n = 80$), this ceiling is placed around $d = 12$ (resp. $d = 10$). We note that similarly, after this phase of hight growing calculation time, a decreasing phase is found from a certain density of non trivial constraints. These phases can be partially explained by the structure of the generated QCNs.

From Figure 3(b), we can note that for each number of variables n, starting from a certain density of non trivial constraints, the percentage of unfeasible relations increases in a fast manner. This can explain the first increase of the calculation time. Note that for a given n, the number of base relation of the generated QCNs decreases when the density of the non trivial constraint increases. For example, for $n = 50$, the average number of base relations is 31298 for $d = 2$ and decreases progressively until it reached 24684 for $d = 24$. This decrease of the number of relation can explain the last phase in which the calculation time decreases.

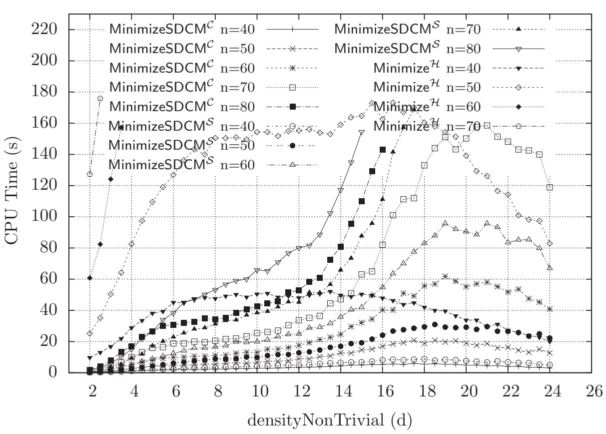
The figure 3(c) shows the cpu time put by the three methods MinimizeSDCM$^{\mathcal{C}}$, MinimizeSDCM$^{\mathcal{S}}$ and Minimize$^{\mathcal{H}}$. It is showing clearly that the methods MinimizeSDCM$^{\mathcal{C}}$ and MinimizeSDCM$^{\mathcal{S}}$ perform better than the method Minimize$^{\mathcal{H}}$. This is justified by the fact that the first two methods can treat many base relations during one single



(a)



(b)



(c)

Figure 3: Experimental results

iteration during the main treatment whereas Minimize$^{\mathcal{H}}$ can treat one single base relation. For illustrative purposes consider the series $\mathsf{S}(40, 14.0, 6.5)$. For this series, MinimizeSDCM$^{\mathcal{C}}$, MinimizeSDCM$^{\mathcal{S}}$ and Minimize$^{\mathcal{H}}$ realize respectively an average of 685, 854 et 8315 main iterations. During five hours, Minimize$^{\mathcal{H}}$ can not treat even one single series of $\mathsf{S}(80, d, 6.5)$. By examining Fgure 3(c), we notice that MinimizeSDCM$^{\mathcal{C}}$ is faster than MinimizeSDCM$^{\mathcal{S}}$. A possible explanation of this is that the subclass $\mathcal{C}$ allows a finest splitting of the relations of IA than the subclass $\mathcal{S}$: approximately 3.54 as average number of sub-relations for $\mathcal{C}$ and around 5.13 as average number sub-relations for MinimizeSDCM$^{\mathcal{S}}$. As a direct effect of this difference, the search performed by MinSubQCN can be faster by using the subclass $\mathcal{C}$ than the subclass $\mathcal{S}$. Furthermore, the sub-QCN returned by MinSubQCN through $\mathcal{C}$ can be more wide.

Conclusions

In this paper, we have introduced an algorithm called MinimizeSDCM allowing to solve the minimal labeling problem of a QCN given a tractable subclass for wich $\diamond$-consistency implies minimality. Our preliminary experimentation shows how this algorithm can be efficient. A future work is to conduct extensive experiments concerning other qualitative calculus than the IA. Also, a research perspective consists in defining and studying specific algorithms for the minimal labeling problems using tractable classes for which $\diamond$-consistency implies a property stronger than the minimality such that the global consistency.

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