Toward a Computational Theory of Conceptual Metaphor

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Abstract

This paper provides a framework to construct a computational model of conceptual metaphor. We first analyze how conceptual metaphor is described by Algebraic Semiotic at linguistic level and by Institutional Theory (an abstract model theory) at a general logical level. By the Logic of Determination of Objects, which has been used in a system of semantic annotation and in a building ontologies system, we further provide a new computational model as a rival approach.

Introduction

George Lakoff and Mark Johnson in their works Conceptual *Metaphor in Everyday Language* (1980) and *Metaphor We Live By* (1980) extensively discussed how conceptual metaphor as a basic cognitive capacity of shaping our communication, action and the way we think. Conceptual metaphor shapes our thought and language in a way of viewing one idea as another (from one conceptual domain to another). Following this, the idea of conceptual blending has further been introduced by Gilles Fauconnier and Mark Turner: it is possible to yield a new conceptual space with emergent structure by blending of two thematically rather different conceptual spaces (Fauconnier and Turner 2003). A classical example for conceptual blending is a blend of the conceptual space of house and the conceptual space of boat, yielding the concept of houseboats and the concept of boathouses as new emergent structures. "Conceptual metaphor with conceptual blending" as a systematic whole, which was used to integrate two conceptual spaces, has been studied comprehensively by ontologists in computer science for developing various ontology designs (Kutz et al., 2010). In the literature, Joseph Goguen et. al. extensively developed algebraic semiotics methods to describe the structure of complex signs and the blend of such structure, so that it is possible to capture the essence of the transformations between two different concept domains at the logical level, called Institution Theory (Goguen and

Burstall 1999) (Goguen and Burstall, 1992, 1984). In this paper we introduce the *Logic of Determination of Objects* (LDO) (Desclés, Pascu 2011) as an alternative approach to study conceptual metaphor and conceptual blending.

The structure of the paper is as follows: In Section 2, we start our discussion on conceptual metaphor and conceptual blending that our paper has taken as describing human's basic cognitive capacity. Section 3 then discusses Algebraic Semiotics that moves toward Institution Theory that our paper is attempting to take to describe the underlying process of this cognitive capacity at the general level. In section 4, we start our discussion on LDO based on Combinatory Logic according to Curry and Feys (Curry amd Feys, 1958) and then analyze conceptual metaphor by LDO. Finally, section 5 is the conclusion.

Conceptual Metaphor and Conceptual Blending

Metaphor shapes our language and thought in the rhetoric sense of viewing one unfamiliar and abstract term *A* by means of borrowing some meaning of another term *B* that is more concrete and familiar that intuitively implies the understanding of one idea in terms of another. Recently, when we talk about metaphor in cognitive science we don't talk about metaphor in rhetoric sense. Rather, we talk about George Lakoff and Mark Johnson's works ``Metaphors we Live By". It refers to the understanding of one idea, or conceptual domain, in terms of another. Many abstract concepts can be defined metaphorically in terms of concrete experiences that we can comprehend. In the same spirit as metaphor in rhetoric sense, conceptual metaphor in cognitive linguistics intuitively implies the understanding of one idea, may be a coherent organization of human experiences, in terms of another. For example, "argument is war" is one conceptual metaphor which understands "argument" as "war", that is to say that we understand "argument" which belongs to a target domain by another source domain to which "war" belongs. We use this "concept of war" to shape the way that "concept of argument" was thought of, and moreover we shape the ways that we go in argument process. Generally speaking, there could be arbitrarily many mappings between the target domain and the source domain. However, only

limited numbers of them are commonly used by people to understand some concepts. This means some properties should be preserved from one to another, so that people can understand these concepts properly.

Gilles Fauconnier and Mark Turner develop a theory of cognition - conceptual blending - in their work *The Way We Think Conceptual Blending and the Mind's Hidden Complexities* (2003). According to this theory, some elements and vital relations from different conceptual spaces are able to be integrated subconsciously, and this kind integration is assumed to be ubiquitous to thought and language in our daily life. For example, John Searle in his works has given a general theory of social institutions (Searle, 1995, 2005) related to the construction of social institutions. In Searle's works, he stated that human have the ability of creating institutional facts from brute facts such as money, government, marriage, and so on. This creation of institutional facts could have a general logical form: ***x* counts as *y* in *c***, where *x* refers to brute facts, *y* refers to brute facts, and *c* refers to context. We can find that there is a subconsciously integrations of different conceptual spaces in the process of creating social institutions. For example, "a piece of paper" (a brute fact) counts as "100 USD" (an institutional fact), "a man" (a brute fact) was represented as "a president" (an institutional fact), etc. A piece of paper cannot present the so-called ``state function'' of being 100 USD only in virtue of the physical structure of the paper. Rather, there should be some collective assignment of a certain status. Similarly, our action of paying a bill by handing over some this 100 USD presupposes the existence of an institutionalized currency system. In the same spirit the man cannot present the state function of being a president only in virtue of the physical structure of the man.

Algebraic Semiotics and Institution Theory

A general logical system of conceptual metaphor and conceptual blending can be described by Goguen and Burstall's Institution Theory. Institution Theory comes from a series study on algebraic semiotics in 1980s. Algebraic semiotics originated from algebraic semantics in the mathematics of abstract data types. Some definitions shown in (Goguen and Harrell 2009, pp. 299–300) for algebraic semiotics and semiotic morphism will be given as follows[1]:

Algebraic Semiotics

The basic notion of *algebraic semiotics* is a (loose algebraic) theory, which consists of *type* and *operation declarations*, possibly with subtype declarations and

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[1] For the additional details omitted here to refer (Goguen and Harrell 2009).

axioms. A *semiotic system* is a theory, plus a *level ordering* on sorts and a priority ordering on constitutes at each level. *Sorts* classify the parts of signs and the values of attributes of signs. Signs of a certain sort are represented by terms of that sort, including but not limited to constants. *Constructors* build new signs from given sign parts as inputs. *Levels* express the whole-part hierarchy of complex signs, while priorities express the relative importance of constructors and their arguments; social issues play a key role in determining these orderings. Semiotic systems are formalized as algebraic theories with additional structure and semiotic morphisms are formalized as theory morphisms that also preserve these additional structures:

- *theory morphisms* consists of mappings between two theories that preserve the basic constituents, which are sort declarations, and operation declarations;
- *semiotic morphisms* are the mappings between semiotic systems (preserving levels and priorities), which are uniform representations for signs in a source space by signs in a target space.

Institution Theory

Further, along the development of algebraic semiotics, Goguen and Burstall discuss Institution Theory, which aims to capture the essence of the concept of "logical system". Next paragraph contains technical descriptions on semiotic morphisms. A semiotic morphism consists of the following:

- A category *Sign* of signatures (or grammars) with a set N of sorts partially ordered by a sub-sort relation.
- For each signature Σ, Sen is a function that builds the set of sentences Sen(Σ).
- A function $\rho : \Sigma_1 \rightarrow \Sigma_2$ between such sets as a signature morphism.
- For each signature morphism, the sentence translation map $\alpha(\rho) : \text{Sen}(\Sigma_1) \rightarrow \text{Sen}(\Sigma_2)$.

A semiotic morphism from $S_1 = (\Sigma_1, \text{Sen}(\Sigma_1))$ to $S_2 = (\Sigma_2, \text{Sen}(\Sigma_2))$ consists of a theory morphism that partially preserves the priority and level of orderings.

Following the work of algebraic semiotics, Institution Theory has introduced not only systematically mappings but also the underlying logical behaviors between semiotics. The following paragraph contains technical descriptions on the application to an abstract concept of logical system.

Given two logics $K_1 = \langle \Sigma_1, \models_1 \rangle$ and $K_2 = \langle \Sigma_2, \models_2 \rangle$; K_1, K_2 have the set Σ_1 and Σ_2 (of propositional symbols) as signatures, and a function $\rho: \Sigma_1 \rightarrow \Sigma_2$ between such sets as a signature morphism. A Σ-model M is a mapping from Σ to {true, false}. $\alpha(\rho): \text{Sen}(\Sigma_1) \rightarrow \text{Sen}(\Sigma_2)$ from the Σ_1-sentence to Σ_2- sentences. γ is a model translation function from K_2-models to K_1-models, such that $M_2 \models_2 \alpha(\phi_1)$ if

and only if $\gamma(M_2) \models_1 \phi_1$ holds for any $\phi_1 \in \text{Sen}(\Sigma)$ and any $M_2 \in K_2$-model.

The metaphor can be seen as a model translation function between K_2-models and K_1-models.

The Logic of Determination of Objects (LDO) and Conceptual Metaphor

In the literature, the Logic of Determination of Objects (LDO) was presented to account, in particular, for the distinction between typical and atypical instances of a concept. The primitives of this logic are the concepts and the objects. The concepts are operators in the sense of Frege (Frege,1971) and the objects are operands. The whole language of the LDO is an applicative system (Curry, 1958). The differences between LDO and the classical logic are: (1) objects in LDO are of two kinds: fully (totally, completely) determinate objects and more or less determinate objects; (2) objects in LDO are typical and atypical; (3) the duality between extension and intension of a concept is not kept. To account for the distinction between typical and atypical instances of a concept, (they all belong to the *expansion* or to the *extension* of this concept), it must be introduced the *intension* of this concept and articulate it to its *expansion* and its *extension* in such a way that one can describe atypical objects among the more or less determinate objects falling under this concept. The whole problem of typicality/atypicality led us no longer considered the duality between extension and intension (according to the law known as Port Royal law). For instance, the non-duality between intension and extension of a concept postulated in LDO was used to express in a categorization the difference between typical and atypical. Here, we try to use it to analyze the process of building a metaphor. We take into account also the structure of a concept composed by essence and intension in LDO.

Basic notion of LDO

LDO is a typed applicative system in the sense of Curry (Curry, Feys, 1958). It can be regarded as a formal theory of concepts and objects. That is, LDO is a typed applicative system LDO = $(\mathcal{F}, O, \mathcal{T})$ where: $\mathcal{F}$ is the set of concepts; O is the set of objects; $\mathcal{T}$ is a type theory. A concept is an operator, an object is always an operand. Types are associated with concepts and objects.

Types theory of LDO

We adopt a theory of types according to Curry (Curry and Feys, 1958):
- Primitive types are: J individual entity type, H truth value (sentence) type;
- **F**: functional type constructor;
- Rules:

      Primitive types are types;
      If α and β are types, then $\mathbf{F}\alpha\beta$ is a type;
      All types are obtained by one of the above rules.

In LDO:
- All objects are operands of type J; all propositions are of type H;
- All concepts are operators of type **F**JH.

An expression X of type α is specified by: X : α.

Application of a concept f **to an object** x**.** We denote:

$(f\,x) = \top$ if f is applied to x ("x falls under f")

$(f\,x) = \bot$ otherwise ("x does not fall under f")

The applicative scheme which expresses the application of a concept f to an object x is:

$$\frac{f : \mathbf{F}JH \qquad x : J}{(f\,x) : H}$$

In LDO, N_1 is the operator of negation defined as:

$((N_1 f)\,x) = \top$ if and only if $(f\,x) = \bot$

It has the classical logic property: $(N_1(N_1 g)) = g$

In LDO, N_0 is the negation of a sentence defined as:

$(N_0\,(f\,x)) = \top$ if and only if $(f\,x) = \bot$

LDO is an applicative language of operators applied to operands of different types (see Curry and Fey, 1958); it is composed of:

- Predicates defined on individual objects (concepts of type **F**JH) and the relators between individuals with respective types **F**JF JH, **F**JF JF JH, etc.);
- Connectives between propositions are of the type **F**H**F**HH;
- Fregean quantifiers: simple quantifiers with the type **FF**JHH; restricted quantifiers with the type **FF**JH**FF**JHH;
- Operators of negation with the type **F**HH (classical negation or intuitionist negation) defined only on propositions.
- Objects of type J.

LDO is also an illative (inferential) language with inferential rules.

Basic operators of LDO

The constructor of the "typical object": the operator τ

This operator denoted τ and called *the constructor of the typical object* builds an object totally indeterminate starting from a concept. Its type is **FF**JHJ; it canonically associates to each concept f, an indeterminate object τf, called "typical object". Its applicative scheme is:

$$\frac{\tau : \mathbf{FF}JHJ \qquad f : \mathbf{F}JH}{\tau f : J}$$

The object τf, is the "best representative" object of the concept f; it is totally indeterminate, typical and abstractly represents the concept f in the form of an "any typical object whatever"[2]. The typical object τf associated with f is unique. For example, if we take as concept f, the concept *to-be-a-man* then, the typical object associated is *a-man*. For the concept f, *to-be-a-computer*, τf is *a-computer*:

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[2] This expression was chosen to encode the notion captured by the word "quelconque" in French.

$$\frac{\tau: \textbf{FFJHJ} \qquad f: \textit{to-be-a-man}}{\tau f : \textit{a-man}}$$

The operator of determination: the operator δ.

The operator δ, called *the constructor of determination operators* builds a *determination operator*, starting from a given concept. The operator δ canonically associates a determination operator of the type **FJJ** to each concept f.

The type of operator δ is **FFJHFJJ**. Its applicative scheme is:

$$\frac{\delta: \textbf{FFJHFJJ} \qquad f: \textbf{FJH}}{\delta f : \textbf{FJJ}}$$

A determination operator δf is an operator which being applied to an object x constructs another object y: $y = ((\delta f)x)$[3]. The object y is more determinate than the object x, by means of the determination added by δf.

For example, if the concept f is *to-be-red*, then δf is *red*; if f is *to-be-on-the-table*, then δf is *which-is-on-the-table*.

The determination δf *red* applied to the object *a-book* gives the more determinate object *a-red-book*:

$$\frac{\textit{red}: \textbf{FJJ} \qquad \textit{a-book}: \text{J}}{\textit{a-red-book}: \text{J}}$$

The composition of determinations. Let x be an object and δf, δg two determinations.

In this case, we can write: $((\delta g \circ \delta f) x) = (\delta g (\delta f) x))$

The composition of determinations is associative and supposed to be commutative.

Chain of determination. A *chain of determination* Δ is a finite string of determinations which can be composed of each other: $\Delta = \delta g_1 \circ \delta g_2 \circ \ldots \circ \delta g_n$.

More or less determinate object.

A *more or less determinate object* is an object recursively obtained starting from the object τf by:

- τf is a more or less determinate object;
- If Δ is a chain of determinations, then $y = (\Delta x) = (\delta g_1 \circ \delta g_2 \circ \ldots \circ \delta g_n) x)$ is a more or less determinate object;
- Each more or less determinate object is obtained by the above rules.

Concepts and objects

The set of concepts $\mathcal{F}$ is provided with two relations:

- $\rightarrow$ called **comprehension**: The comprehension f$\rightarrow$g modelises the intuitive notion that "the concept f directly comprises the concept g" or "the concept g is directly comprised by the concept f. This relation is: reflexive and anti-symmetric. It is not transitive.

- $\Rightarrow$ called **direct necessary comprehension**. The direct necessary comprehension f $\Rightarrow$g modelises the fact that "the concept f directly contains in a necessary manner the concept g or "the concept g is necessarily directly contained in the concept f. The concept g is an "essential

component" of f. This relation is: reflexive, anti-symmetric and transitive. The definition of the essence of a concept is given based on this relation.

In this way, the set of concepts $\mathcal{F}$ is structured by relations $\rightarrow$ and $\Rightarrow$ and it become $(\mathcal{F}, \rightarrow, \Rightarrow)$. In LDO the set of concepts is more than a concept network considered by ontologies of a domain. The first one is structured by two relations, while the second one only by one relation.

The set of objects, O, contains a subset, O_{det}, which is the set of "fully (totally) determinate objects".

An object x is fully (totally) determinate if and only if for each determination δg with $g \in \mathcal{F}$: $(\delta g\, x) = x$

In LDO, objects are of two kinds:
- more or less determinate object: $x \in O$;
- fully (totally) determinate object: $x \in O_{det}$.

Nevertheless, all of them are of type J.

The set O is structured by the binary relation $<=$ defined as : $x <= y$ if and only if there is $g \in \mathcal{F}$ such that $y = (\delta g\, x)$.

This relation is reflexive, anti-symmetric and transitive.

Classes of concepts associated with a concept f.

(1) Characteristic intension of a concept. The *characteristic intension* of a concept f is the set of concepts which characterize f, in a sense of a pack of properties of f.

$$\text{Int-caract } f = \{g \ / \ f \rightarrow g\}$$

For example if f is *to-be-a-man*, then g can be *to-have-two-legs*; if f is *to-be-a-bird*, then g can be *to-fly*.

(2) The *intension* of a concept f.

$$\text{Int } f = \{g \ / \ f \rightarrow^* g\}$$

where $*$ stands for the transitive closure of the relation.

(3) The *essence* of a concept f. The essence of a concept f is the set of concepts necessarily comprised in f. If we remove a concept g from the essence of f, we destroy the concept f; it is not the same. If a concept g is in the essence of a concept f, then the negation of g cannot belong to this essence.

$$\text{Ess } f = \{g \ / \ f \Rightarrow g\}$$

For example if f is *to-be-a-man*, then g can be *be-derived-from-two-male-and-female-human-cells*. The concept g is essential for f. If f is *to-be-a-bird*, then g can be *to-lay-eggs*. But the concept *to-have-two-legs* is not essential for the concept *to-be-a-man*, since there are *one-legged-men*. Also the concept *to-fly* is not essential for the concept *to-be-a-bird*, since there are birds which cannot fly.

$$\text{Ess } f \subset \text{Int } f.$$

Classes of objects associated with a concept f and with the object τ f.

(1) Expansion (Etendue in French). The *expansion* of f, denoted by Exp(f), is the set of all objects of O (more or less determinate or totally determinate) to which f can be applied:

$$\text{Exp } f = \{x \in O\} \ / \ (f\, x) = \top\}$$

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[3] We use the prefixed notation of a function, that is (f x) for } f(x).

We can consider also the *expansion* of f, denoted by Exp τf, being the set of all objects of *O* (more or less determinate or totally determinate) which can be constructed starting from τf:

$$\text{Exp } \tau f = \{x \in O \,/\, x = (\Delta\ \tau f)\}$$

(2) Extension. The *extension* of f, denoted by Ext f, is the set of all totally determinate objects to which the concept f can be applied:

$$\text{Ext } f = \{x \in O_{det} \,/\, (f\,x) = \top\}$$

The *extension* of τf, denoted by Ext τf, is the set of all totally determinate objects which can be constructed starting from τf:

$$\text{Ext } \tau f = \{x \in O_{det} \,/\, x = (\Delta\ \tau f)\}$$

It is obviously that : Ext f ⊂ Exp f; Ext τf ⊂ Exp τf
In this paper we assume that:

Exp f = Exp τf   and   Ext f = Ext τf

An approach of the conceptual metaphor by the LDO

Based on LDO, we analyze the construction operated by "conceptual metaphor". It is a *complex transfer-operator* pairing from the *source concept—object* space to *the target concept- object* space. This operator is applied to concepts, more or less determinate objects or determinations. It is not only a simple transfer, it can change the category of the operand (i.e. a concept from essence from the source space can become the determination of a more or less determinate object in the target space).
Let us see some examples:

1° boat—people
2° *Cette faucille d'or dans le champ des étoiles (Victor Hugo, Booz endormi) (This gold sickle is in the stars field).*
3° *Il reste pétrifié au sol (He remains petrified ground).*
4° *The joy illuminates his face (La joie illumine son visage).*
5° *Sophie est un glaçon (Sophie is a real gold fish).*

We obtain the following four points by analyzing the cognitive nature of these conceptual metaphors in the framework of LDO:
(1) In the example 1°, it is straightforward that there is a determination (δg: on-a-boat) that occurred in the essence of source domain which transfered in the essenceof the target concept (a detailed analysis is given in figure 1.).
(2) In the examples 2° and 4°, we first observed that there is a mapping from the intension of the concept in the source domain to the target domain. Moreover, in the example 4°, an essence transfer even occurred in the source concept (a detailed analysis is given in figure 2).

(3) In examples 3° and 5°, the intension of the concept in source domain generates a determination of the typical object in the target domain (a detailed analysis is given in figure 3).
(4) In the example 2°, a determination of the object in source domain (*gold sickle* or *faucille d'or*) will be preserved to another determination of the object in the target domain (*the moon* or *la lune*) (a detailed analysis is given in figure 4).

Conceptual metaphor can be analyzed by a complex transfer operator from the source concept—object tuple to the source concept—object tuple in the framework of LDO

Conclusion

We conclude that not only has Institutional Theory, which is the general description of underlying logical behaviors between semiotics, been given to consider the conceptual metaphor as translations between two conceptual spaces, but LDO, which represents conceptual metaphor as a complex transfer operator using applicative expressions from Combinatory Logic, has been proposed to supply a suitable framework to construct a computational model of conceptual metaphor.

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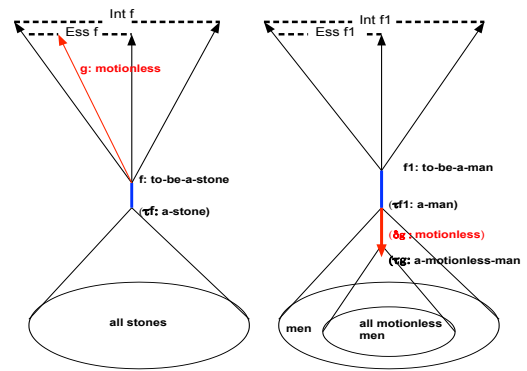Searle J., 1995. *The Construction of Social Reality*, Publisher,Free Press.

Fig. 3. The transfer of concept of the intension of the source concept (f) as a determination of the target typical object (τf1)
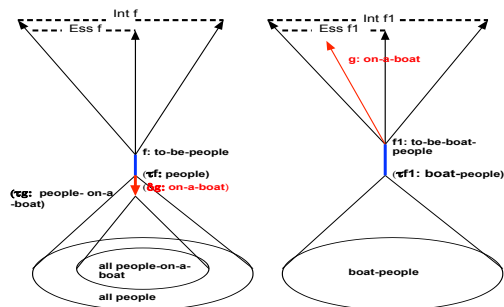

Fig. 1. The transfer of a determination of an source object (τg) in the essence of the target concept (f1)
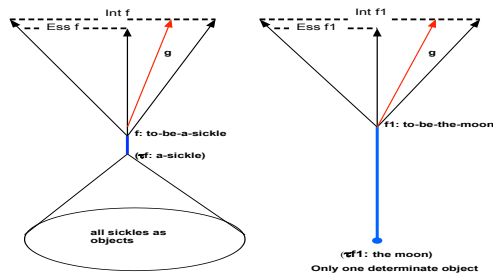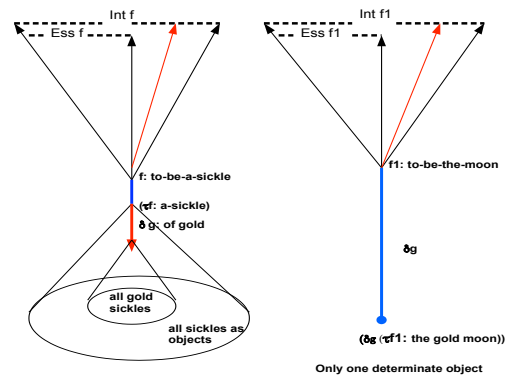

Fig. 4. The transfer of a determination of the source object (f) to a determination of the target object (f1)

Fig. 2. The transfer of a concept from the intension of the source concept (f) in the intension of the target concept (f1)