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Non Defeated-Based Repair in Possibilistic *DL-Lite* Knowledge Bases

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Abstract

Possibilistic *DL-Lite* is an extension of the *DL-Lite* framework to deal with uncertain pieces of information. In this paper, we deal with inconsistency in *DL-Lite* in case where the assertions are uncertain and encoded in a possibility theory framework. We investigate three inconsistency-tolerant reasoning methods for possibilistic *DL-Lite* knowledge bases which are based on the selection of one consistent assertional base. We show that the three possibilistic assertional-based repairs are achieved in a polynomial time.

Introduction

In many applications, knowledge is often affected by uncertainty especially when it is provided by several and potentially conflicting sources. In (Dubois and Prade 1991), it was shown that handling uncertainty is in a complete agreement with possibility theory. This latter offers a very natural framework to deal with ordinal and qualitative uncertainty or preferences and priorities. This framework is particularly appropriate when the uncertainty/priority scale only reflects a priority relation between the different pieces of information. Recently, a particular attention was given to the extension of Description Logics (DLs) within the possibility theory setting (*e.g.* (Qi et al. 2011; Benferhat and Bouraoui 2013)). One of the interesting aspects of possibilistic knowledge bases and more generally weighted knowledge bases is the ability of reasoning with partially inconsistent knowledge.

In recent years, there is a growing interest in Ontology-Based Data Access (OBDA) applications. The OBDA problem takes as input a set of facts, an ontology and a conjunctive query and aims at finding if there is an answer to the query in the set of facts, eventually enriched by the ontology. *DL-Lite* (Calvanese et al. 2007), a lightweight family of description logics specifically fitted for applications that use huge volumes of data, has been recognized as a powerful formal framework for ODBA. *DL-Lite* guarantees a very low computational complexity for reasoning tasks and especially query answering.

A crucially important problem that arises in OBDA is how to handle inconsistency; otherwise the knowledge base is meaningless and useless. In such setting, inconsistency is always defined with respect to some assertions that contradict the ontology. Indeed, a *DL-Lite* terminology may be incoherent but never inconsistent. Many works (*e.g.* (Lembo et al. 2010; Bienvenu and Rosati 2013)), basically inspired by the approaches proposed in the database area, tried to deal with inconsistency in standard *DL-Lite* by proposing and adapting several inconsistency-tolerant inference methods that consist in accepting and leaving inconsistency while coping with it when performing inference (i.e. while answering queries). All the proposed approaches are based on the notion of assertion-based repair. An assertion-based repair (Lembo et al. 2010) for *DL-Lite* knowledge bases is simply a maximal set of facts consistent with the ontology (*i.e.* the TBox). The notion of priority in DLs was studied in (*e.g.* (Du, Qi, and Shen 2013)) in order to deal with inconsistent $\mathcal{SHIQ}$ DL knowledge bases by defining maximal repair w.r.t set inclusion in order to answer queries. In (Bienvenu, Bourgaux, and Goasdoué 2014), the authors redefine AR and IAR semantics proposed in (Lembo et al. 2010) when priorities are available among facts and investigate a method based on conflict resolution to find only if a query has an answer in a preferred repair or not. Unfortunately, there is to the best of our knowledge no approach for handling inconsistency in possibilistic *DL-Lite* knowledge bases in the context of OBDA. Namely, when uncertainty is bearing only on the assertions in the ABox whereas the TBox is assumed to be certain and stable.

In this paper, we address the problem of inference under inconsistency in possibilistic *DL-Lite*. We investigate three inconsistency-tolerant reasoning methods mainly based on the selection of one consistent assertion-based repair. We specifically focus on non-defeated inference. Interestingly enough, such relations allow efficient handling of inconsistency in *DL-Lite* bases without any extra computational complexity. An important feature when restoring consistency in *DL-Lite* is the use of the notion of deductive closure. Indeed, we study for each proposed inference method its sensitivity to the deductive closure. Note that the use of deductive closure is really proper to description logic languages contrarily to the propositional logic framework where it is hard to be defined.

Possibilistic *DL-Lite*: A Brief Presentation

Handling uncertainty can be conveniently and efficiently dealt with in possibility theory. In order to encode the available uncertain knowledge and exploit it to deal with inconsistency, a possibility theory based *DL-Lite* logic (Benferhat and Bouraoui 2013) is more appropriate. In this section, we recall the main notions of possibilistic *DL-Lite* logic, denoted by π-DL-$Lite$.

DL-Lite Logic

This section briefly recalls the main concepts of *DL-Lite* logic. For the sake of simplicity, we only consider DL-$Lite_R$ that underlies the *OWL2-QL* language. (Calvanese et al. 2007). The DL-$Lite_R$ language is defined as follows:

$$B \longrightarrow \quad A \mid \exists R \qquad C \longrightarrow \quad B \mid \neg B$$
$$R \longrightarrow \quad P \mid P^- \qquad E \longrightarrow \quad R \mid \neg R$$

where A is an atomic concept, P is an atomic role and P^- is the inverse of the atomic role. B (*resp.* C) is called basic (*resp.* complex) concept and role R (*resp.* E) is called basic (*resp.* complex) role. A *DL-Lite* KB is a pair $\mathcal{K}$=$\langle\mathcal{T},\mathcal{A}\rangle$ where $\mathcal{T}$ is called the TBox and $\mathcal{A}$ is called the ABox. A TBox includes a finite set of inclusion axioms on concepts and on roles respectively of the form: $B \sqsubseteq C$ and $R \sqsubseteq E$. The ABox contains a finite set of membership assertions on atomic concepts and on atomic roles respectively of the form $A(a)$ and $P(a,b)$ where a and b are two individuals. For the sake of simplicity, in the rest of this paper, when there is no ambiguity, we simply use *DL-Lite* instead of DL-$Lite_R$.

The semantics of a *DL-Lite* KB is given in term of interpretations. An interpretation I=$(\Delta^I,.^I)$ consists of an non-empty domain Δ^I and an interpretation function $.^I$ that maps each individual a to $a^I$$\in$$\Delta^I$, each A to $A^I$$\subseteq$$\Delta^I$ and each role P to $P^I$$\subseteq$$\Delta^I$$\times$$\Delta^I$. Furthermore, the interpretation function $.^I$ is extended in a straightforward way for complex concepts and roles for instance as follows: $(\neg B)^I$=$\Delta^I \setminus B^I$, $(P^-)^I$=$\{(y,x) \in \Delta^I \times \Delta^I | (x,y) \in P^I\}$ and $(\exists R)^I$=$\{x \in \Delta^I | \exists y \in \Delta^I \; such\, that \, (x,y) \in R^I\}$. An interpretation I is said to be a model of a concept (*resp.* role) inclusion axiom, denoted by $I \models B \sqsubseteq C$ (*resp.* $I \models R \sqsubseteq E$), iff $B^I \subseteq C^I$ (*resp.* $R^I \subseteq E^I$). Similarly, we say that I satisfies a concept (*resp.* role) membership assertion, denoted by $I \models A(a)$ (*resp.* $I \models P(a,b)$), iff $a^I \in A^I$ (*resp.* $(a^I,b^I) \in P^I$).

A *DL-Lite* TBox $\mathcal{T}$ is said incoherent if there exists at least a concept C such that for each interpretation I which is a model of $\mathcal{T}$, we have C^I=$\emptyset$. A KB $\mathcal{K}$ is said consistent if it admits at least one model, otherwise $\mathcal{K}$ is said inconsistent. Note that within the *DL-Lite* setting, the inconsistency problem is always defined with respect to some *ABox* since a *TBox* may be incoherent but never inconsistent.

Knowledge Representation in π-DL-$Lite$

Let $\mathcal{L}$ be a *DL-Lite* description language, a π-DL-$Lite$ KB is a set of possibilistic axioms of the form (φ,α) where φ is an axiom expressed in $\mathcal{L}$ and $\alpha$$\in$$]0,1]$ is the degree of certainty of φ. Namely, a π-DL-$Lite$ KB $\mathcal{K}$ is such that $\mathcal{K}$=$\{(\varphi_i,\alpha_i)$:i=$1,...,n\}$. Only somewhat certain information are explicitly represented in a π-DL-$Lite$ KB. Namely, axioms with a null degree (α=0) are not explicitly represented in the KB. The weighted axiom (φ,α) means that the certainty degree of φ is at least equal to α (namely $N(\varphi)$$\geq$$\alpha$). A π-DL-$Lite$ KB $\mathcal{K}$ will also be represented by a couple $\mathcal{K}$=$\langle\mathcal{T},\mathcal{A}\rangle$ where both elements in $\mathcal{T}$ and $\mathcal{A}$ may be uncertain. It is important to note that, if we consider all α_i=1 then we found a classical *DL-Lite* KB: $\mathcal{K}^*$=$\{\varphi_i : (\varphi_i,\alpha_i) \in \mathcal{K}\}$.

Given $\mathcal{K}$=$\langle\mathcal{T},\mathcal{A}\rangle$ a π-DL-$Lite$ KB, we define the α-cut of $\mathcal{K}$ (*resp.* $\mathcal{T}$ and $\mathcal{A}$), denoted by $\mathcal{K}_{\geq\alpha}$ (*resp.* $\mathcal{T}_{\geq\alpha}$ and $\mathcal{A}_{\geq\alpha}$), the subbase of $\mathcal{K}$ (*resp.* $\mathcal{T}$ and $\mathcal{A}$) composed of axioms having weights at least greater than α and the strict α-cut of $\mathcal{K}$ (*resp.* $\mathcal{T}$ and $\mathcal{A}$), denoted by $\mathcal{K}_{>\alpha}$ (*resp.* $\mathcal{T}_{>\alpha}$, $\mathcal{A}_{>\alpha}$), as a subbase of $\mathcal{K}$ (*resp.* $\mathcal{T}$ and $\mathcal{A}$) composed of axioms having weights strictly greater than α. We say that $\mathcal{K}$ is consistent if the standard KB obtained from $\mathcal{K}$ by ignoring the weights associated with axioms is consistent. In case of inconsistency, we attach to $\mathcal{K}$ an inconsistency degree. The inconsistency degree of a π-DL-$Lite$ KB $\mathcal{K}$, denoted $Inc(\mathcal{K})$, is syntactically defined as follow: $Inc(\mathcal{K})$=$max\{\alpha$:$\mathcal{K}_{\geq\alpha}$ is inconsistent$\}$.

The semantics of π-DL-$Lite$ KB is given by a possibility distribution, denoted π. This latter is a mapping from a set of *DL-Lite* interpretations Ω (namely, I=$(\Delta^I,.^I)$$\in$$\Omega$) to the unit interval $]0,1]$. For a complete presentation of the π-DL-$Lite$ semantics, see (Benferhat and Bouraoui 2013).

Negative Possibilistic Closure in π-DL-$Lite$

Let $\mathcal{K}$=$\langle\mathcal{T},\mathcal{A}\rangle$ be a π-DL-$Lite$ KB. In (Benferhat and Bouraoui 2013), it was shown that computing the inconsistency degree of $\mathcal{K}$ comes down to compute the one of $\langle\pi - neg(\mathcal{T}),\mathcal{A}\rangle$ where π-$neg(\mathcal{T})$ is the negated closure of $\mathcal{T}$. The negated closure will contain all the possibilistic negated axioms of the form $(B_1$$\sqsubseteq$$\neg B_2$,$\alpha)$ and $(R_1$$\sqsubseteq$$\neg R_2$,$\alpha)$ that can be derived from $\mathcal{T}$. The set π-$neg(\mathcal{T})$ is obtained by applying a set of rules that extend the ones defined in standard *DL-Lite* when axioms are weighted with certainty degrees. This notion is crucial for characterizing the concepts of consistency and how to deal with it in π-DL-$Lite$ KB. At the beginning π-$neg(\mathcal{T})$ is set to an empty set.

Rule1: Add all negated axioms of $\mathcal{T}$ to π-$neg(\mathcal{T})$.

Rule2: If $\langle B_1$$\sqsubseteq$$B_2$,$\alpha_1\rangle$$\in$$\mathcal{T}$ and $\langle B_2$$\sqsubseteq$$\neg B_3$,$\alpha_2\rangle$$\in$$\pi$-$neg(\mathcal{T})$ then add $\langle B_1$$\sqsubseteq$$\neg B_3$, $min(\alpha_1,\alpha_2)\rangle$ to π-$neg(\mathcal{T})$.

Rule3: If $\langle B_1$$\sqsubseteq$$B_2$,$\alpha_1\rangle$$\in$$\mathcal{T}$ and $\langle B_3$$\sqsubseteq$$\neg B_2$,$\alpha_2\rangle$$\in$$\pi$-$neg(\mathcal{T})$ then add $\langle B_1$$\sqsubseteq$$\neg B_3$, $min(\alpha_1,\alpha_2)\rangle$ to π-$neg(\mathcal{T})$.

Rule4: If $\langle R_1$$\sqsubseteq$$R_2$,$\alpha_1\rangle$$\in$$\mathcal{T}$ and $\langle\exists R_2$$\sqsubseteq$$\neg B$,$\alpha_2\rangle$$\in$$\pi$-$neg(\mathcal{T})$ or $\langle B$$\sqsubseteq$$\neg\exists R_2$,$\alpha_2\rangle$$\in$$\pi$-$neg(\mathcal{T})$ then add $\langle\exists R_1$$\sqsubseteq$$\neg B$,$min\ (\alpha_1$,$\alpha_2)\rangle$ to π-$neg(\mathcal{T})$.

Rule5: If $\langle R_1$$\sqsubseteq$$R_2$,$\alpha_1\rangle$$\in$$\mathcal{T}$ and $\langle\exists R_2^-$$\sqsubseteq$$\neg B$,$\alpha_2\rangle$$\in$$\pi$-$neg(\mathcal{T})$ or $\langle B$$\sqsubseteq$$\neg\exists R_2^-$,$\alpha_2\rangle$$\in$$\pi$-$neg(\mathcal{T})$ then add $\langle\exists R_1^-$$\sqsubseteq$$\neg B$,$min\ (\alpha_1$,$\alpha_2)\rangle$ to π-$neg(\mathcal{T})$.

Rule6: If $\langle R_1$$\sqsubseteq$$R_2$,$\alpha_1\rangle$$\in$$\mathcal{T}$ and $\langle R_2$$\sqsubseteq$$\neg R_3$,$\alpha_2\rangle$$\in$$\pi$-$neg(\mathcal{T})$ or $\langle R_3$$\sqsubseteq$$\neg R_2$,$\alpha_2\rangle$$\in$$\pi$-$neg(\mathcal{T})$ then add $\langle R_1$$\sqsubseteq$$\neg R_3$,$min\ (\alpha_1$,$\alpha_2)\rangle$ to π-$neg(\mathcal{T})$.

Rule7: If $\langle R$$\sqsubseteq$$\neg R$,$\alpha\rangle$$\in$$\pi$-$neg(\mathcal{T})$ or $\langle\exists R$$\sqsubseteq$$\neg\exists R$,$\alpha\rangle$$\in$$\pi$-$neg(\mathcal{T})$ or $\langle\exists R^-$$\sqsubseteq$$\neg\exists R^-$,$\alpha\rangle$$\in$$\pi$-$neg(\mathcal{T})$ then add $\langle R$$\sqsubseteq$$\neg R$,$\alpha\rangle$ and $\langle\exists R$$\sqsubseteq$$\neg\exists R$,$\alpha\rangle$ and $\langle\exists R^-$$\sqsubseteq$$\neg\exists R^-$,$\alpha\rangle$ to π-$neg(\mathcal{T})$.

Rule8: $\langle \varphi, \alpha_1 \rangle \in \pi\text{-}neg(\mathcal{T})$ and $\langle \varphi, \alpha_2 \rangle \in \pi\text{-}neg(\mathcal{T})$ then $\langle \varphi, max(\alpha_1, \alpha_2) \rangle \in \pi\text{-}neg(\mathcal{T})$ where φ is an axiom of TBox or ABox.

The first rule simply states that negative axioms that are explicitly stated in $\mathcal{K}$ are trivially entailed from $\mathcal{K}$, and hence can be added to $\pi\text{-}neg(\mathcal{T})$. The rules 2-7 simply express transitivity relation induced by the inclusion relations. Rules 8 deals with redundancy and simply states that an axiom does not need to appear several times in a KB. It is enough to keep the one having the highest degree. Note that the minimum operation used in the rules for propagating certainty degrees is justified by the fact that the joint distribution will not be affected if the derived inclusion relations are added to the KB. Lastly, when the degrees α_i are equal to 1, then $\pi\text{-}neg(\mathcal{T})$ simply collapses with the standard negated closure defined for standard *DL-Lite* KBs. In fact, $\pi\text{-}neg(\mathcal{T})$ extends standard *DL-Lite* when one only deals with fully certain pieces of information.

Computing Inconsistency Degree in π-DL-$Lite$

We now provide a characterization of the inconsistency degree of a π-DL-$Lite$ knowledge base by only focusing on $\langle \pi\text{-}neg(\mathcal{T}),\mathcal{A}\rangle$. First recall that the ABox only contains positive membership assertions (facts). Hence, the ABox alone is always consistent. Similarly, the TBox $\pi\text{-}neg(\mathcal{T})$ alone (namely, when ABox=$\emptyset$) is also consistent. Indeed, it is easy to define an interpretation I which is a model of $\pi\text{-}neg(\mathcal{T})$. For each $\langle B_1 \sqsubseteq \neg B_2, \alpha \rangle \in \pi\text{-}neg(\mathcal{T})$, we let $(B_i)^I = \emptyset$ if B_i is a concept and $(R)^I = \emptyset$ if B_i is of the form $\exists R$ or $\exists R^-$ and R is a role. I is then trivially a model of $\pi\text{-}neg(\mathcal{T})$. Hence, pieces responsible of inconsistency should involve both elements from $\pi\text{-}neg(\mathcal{T})$ and $\mathcal{A}$.

Besides, an inconsistency problem is always defined with respect to some ABox assertions and a TBox axiom, since a TBox may be incoherent but never inconsistent. A conflict, denoted by $\mathcal{C}$, is clearly an inconsistent subset of information that involve one element from $\pi\text{-}neg(\mathcal{T})$ and two elements from $\mathcal{A}$ (Benferhat and Bouraoui 2013). It is minimal (up to a particular case where B_1=B_2). Indeed, removing any element of a conflict restores consistency. A particular case is when $B_1 \sqsubseteq \neg B_1$ belongs to $\pi\text{-}neg(\mathcal{T})$. This corresponds to the situation of an unsatisfiable concept. A conflict hence involves one negative axiom from $\pi\text{-}neg(\mathcal{T})$ and one or two assertions. The following definition introduces the concepts of the degree of a conflict.

Definition 1. Let $\mathcal{C}$ be a conflict. The degree of conflict, denoted $Deg(\mathcal{C})$, is defined as: $Deg(\mathcal{C}) = min(\alpha_1, \alpha_2, \alpha_3)$ where $(D_1 \sqsubseteq \neg D_2, \alpha_1) \in \mathcal{C}$, $(X, \alpha_2) \in \mathcal{C}$ and $(Y, \alpha_3) \in \mathcal{C}$ with X (*resp.* Y) is a concept or role assertion according to the form of D_1 (*resp.* D_2).

The inconsistency degree of $\mathcal{K}$ ($Inc(\mathcal{K})$) using conflicts and their degrees is defined as follows:

Proposition 1. *Let $\mathcal{K}$=$\langle \mathcal{T}, \mathcal{A} \rangle$ be a π-DL-$Lite$ KB and π-$neg(\mathcal{T})$ be its negated closure. Then:*

$$Inc(\mathcal{K}) = Inc(\langle \pi - neg(\mathcal{T}), \mathcal{A} \rangle) = max\{Deg(\mathcal{C}) : \mathcal{C} \text{ is a conflict of } \langle \pi\text{-}neg(\mathcal{T}), \mathcal{A} \rangle\}$$

Proof. The proof can be found in (Benferhat and Bouraoui 2013). □

Inference Based on Possibilistic Entailment

Throughout the rest of this paper, we investigate inconsistency-tolerant inferences stemming from a possibilistic *DL-Lite* KB. We position ourselves in a context of OBDA. In such setting, the TBox acts as a schema used to reformulate the queries in order to offer a better access to the data stored in the ABox. A crucially important problem that arises in OBDA is how to handle inconsistency; otherwise the kb is meaningless and useless. Indeed, we assume that the TBox is stable and certain however assertions in the ABox are attached with certainty degrees.

Example 1. Let $\mathcal{K}$=$\langle \mathcal{T}, \mathcal{A} \rangle$ such that $\mathcal{T}$=$\{E \sqsubseteq F, \exists H \sqsubseteq F, F \sqsubseteq \neg G, \exists H^- \sqsubseteq I\}$ and assume that assertional facts of $\mathcal{A}$ come from distinct sources. Let $\{\alpha_1, \alpha_2, \alpha_3\}$ be a certainty scales such that $0 < \alpha_3 < \alpha_2 < \alpha_1$. $\mathcal{A}$ is as follows: $\mathcal{A}$=$\{(E(a), \alpha_1), (G(f), \alpha_1), (H(c,e), \alpha_2), (G(a), \alpha_2), (F(t), \alpha_2), (H(a,b), \alpha_3), (G(c), \alpha_3), (I(y), \alpha_3)\}$. As for the TBox $\mathcal{T}$, each axiom ψ of $\mathcal{T}$ will be encoded in the π-DL-$Lite$ setting as $(\psi, 1)$ (for instance $(E \sqsubseteq F$,1). For sake of simplicity, we omit weights attached to TBox axioms.

Within the possibility theory setting, possibilistic entailment is based on the selection of a consistent and not necessarily maximal sub-base of $\mathcal{A}$ which is consistent with $\mathcal{T}$. This sub-set is induced by the inconsistency degree of $\mathcal{K}$. Within an OBDA setting, the inconsistency problem is always defined w.r.t some ABox, since a TBox may be incoherent but never inconsistent. We define the notion of ABox conflict as a minimal inconsistent subset of assertions that contradict the TBox. More formally:

Definition 2. Let $\mathcal{K}$=$\langle \mathcal{T}, \mathcal{A} \rangle$ be an π-DL-$Lite$ KB. A sub-base $\mathcal{C} \subseteq \mathcal{A}$ is said to be an assertional conflict set of $\mathcal{K}$ iff

- $\langle \mathcal{T}, \mathcal{C} \rangle$ is inconsistent
- $\forall f \in \mathcal{C}$, $\langle \mathcal{T}, \mathcal{C} - \{f\} \rangle$ is consistent with f=(φ, α) is a fact.

Example 2. [Example continued] One can compute the following conflict sets: $\mathcal{C}_1$=$\{E(a)_{\alpha_1}, G(a)_{\alpha_2}\}$,$\mathcal{C}_2$=$\{H(c,e)_{\alpha_2}$,$G(c)_{\alpha_3}\}$,$\mathcal{C}_3$=$\{G(a)_{\alpha_2}$,$H(a,b)_{\alpha_3}\}$. We used φ_{α_i} as an abbreviation of (φ,α_i)

It is clear that in Definition 2, removing any assertional fact f from $\mathcal{C}$ restores the consistency of $\langle \mathcal{T}, \mathcal{C} \rangle$. Recall that when the TBox is coherent, a conflict involves exactly two assertions. Let us use $\pi(\mathcal{A})$ to denote the sub-set of $\mathcal{A}$ induced by the inconsistency degree (*i.e.*$\pi(\mathcal{A})$=$\mathcal{A}_{>Inc(\mathcal{K})}$). Note that if $\mathcal{K}$=$\langle \mathcal{T}, \mathcal{A} \rangle$ is consistent then $\pi(\mathcal{A})$=$\mathcal{A}$. The following definition introduces the π-DL-$Lite$ entailment (π-entailment).

Definition 3. Let $\mathcal{K}$=$\langle \mathcal{T}, \mathcal{A} \rangle$ be a π-DL-$Lite$ KB. A query Q is said to be a π-consequence of $\mathcal{K}$, denoted $\mathcal{K} \models_\pi Q$, iff: $\langle \mathcal{T}, \pi(\mathcal{A}) \rangle \models Q$.

Algorithm 1, COMPUTE-$\pi(\mathcal{A})$, details how to compute the subset $\pi(\mathcal{A})$. The first step of the algorithm consists in computing the π-$neg(\mathcal{T})$ of $\mathcal{T}$. We suppose that this is performed by a PINEGCLOSURE function.

Algorithm 1 COMPUTE-$\pi(\mathcal{A})$

Input: $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ where $\mathcal{A}$=$\mathcal{S}_1 \cup \ldots \cup \mathcal{S}_n$
Output: $\pi(\mathcal{A})$

```
π-neg(T) ←PINEGCLOSURE(T)
π(A) ← ∅,
inc ← 0, x ← 0
for all X⊑¬Y∈π-neg(T) do
    for all (φi, αi), (φj, αj) ∈ A do
        if ⟨X ⊑ ¬Y, {(φi, αi), (φj, αj)}⟩ is inconsistent then
            x ← min(αi, αj)
        inc ← max(inc, x),
        x ← 0
return π(A) ← A>inc
```

Then, for each negative inclusion axiom $(X \sqsubseteq \neg Y, 1)$ of π-$neg(\mathcal{T})$ the algorithm looks for the existence of a contradiction in the ABox. This is done by checking whether $\langle (X \sqsubseteq \neg Y,1),\{(\varphi_i, \alpha_i),(\varphi_j, \alpha_j)\}\rangle$ is consistent or not. Note that this step can be performed by a boolean query expressed in form $(X \sqsubseteq \neg Y, 1)$ to check whether $\{(\varphi_i, \alpha_i),(\varphi_j, \alpha_j)\}$ contradicts the query, or not.

Example 3. [Example continued] We have $E(a)$ and $G(a)$ contradict $(E \sqsubseteq \neg G, 1)$ (a negative inclusion axioms deduced from $\mathcal{T}$) and all assertion having weights equals to α_1 are consistent with $\mathcal{T}$. Therefore $Inc(\mathcal{K})$= α_2 and $\pi(\mathcal{A})$= $\{E(a)_{\alpha_1}, G(f)_{\alpha_1}\}$.

Proposition 2. *The computational complexity of π-entailment is $\mathcal{O}(cons)$ where $cons$ is the complexity of consistency checking of standard DL-Lite.*

Sketch of proof. The proof of the complexity of π-entailment can be found in (Benferhat and Bouraoui 2013). □

Possibilistic Deductive Closure

The inference relations given in Definition 3 can be either defined in $\langle \mathcal{T}, \mathcal{A} \rangle$ or on $\langle \mathcal{T}, Cl(\mathcal{A}) \rangle$ where *Cl*(.) denotes the deductive closure of a set of assertions. Note that the use of deductive closure is really proper to description logic languages. Let us first define the notion of deductive closure in standard *DL-Lite*.

Definition 4. Let $\mathcal{K}$=$\langle \mathcal{T}, \mathcal{A} \rangle$ be standard *DL-Lite* KB. Let $\mathcal{T}_p$ be the set of all positive inclusion axioms of $\mathcal{T}$. We define the deductive closure of a sub-base $\mathcal{S}$ of $\mathcal{A}$ w.r.t $\mathcal{T}$ as follows: $Cl(S)$=$\{B(a)$:$\langle \mathcal{T}_p, \mathcal{S}\rangle \models B(a)$ where B is a concept of $\mathcal{T}$ and a is an individual of $\mathcal{S}\} \cup \{R(a, b)$:$\langle \mathcal{T}_p, \mathcal{S}\rangle \models R(a, b)$ where R is a role of $\mathcal{T}$ and a,b are individuals of $S\}$.

The extension of deductive closure to the possibility-theory framework gives:

Definition 5. Let $\mathcal{K}$=$\langle \mathcal{T}, \mathcal{A} \rangle$ be π-DL-$Lite$ KB. Let β_1=1>...>β_n>0 be different weights in the KB. Let $\mathcal{S}_{\beta_i}$=$\{(\varphi, \alpha)$:$(\varphi, \alpha) \in \mathcal{A}$ and α=$\beta_i\}$. Then:

$$Cl(\mathcal{A})=\langle \mathcal{T}, Cl(\mathcal{S}_{\beta_1}) \cup Cl(\mathcal{S}_{\beta_1} \cup \mathcal{S}_{\beta_2}) \cup ... \cup Cl(\mathcal{S}_{\beta_1} \cup ... \cup \mathcal{S}_{\beta_n})$$

An important feature of π-inference is that it is insensitive to the deductive closure, more precisely:

Proposition 3. *Let $\mathcal{K}$=$\langle \mathcal{T}, \mathcal{A} \rangle$ be a π-DL-$Lite$ KB. Then $\forall Q$: $\langle \mathcal{T}, \mathcal{A} \rangle \models_\pi Q$ iff $\langle \mathcal{T}, Cl(\mathcal{A}) \rangle \models_\pi Q$.*

Proof. The intuition behind the proof is that first π-inference uses a consistency checking of the whole sets $\mathcal{S}_i$ to decide whether this latter should be kept or not for restoring the consistency of the KB. Besides, one can easily check that in standard *DL-Lite*, $\langle \mathcal{T}, \mathcal{A} \rangle$ is consistent iff $\langle \mathcal{T}, Cl(\mathcal{A}) \rangle$ is consistent. □

Unfortunately, π-entailment is not very satisfactory when handling inconsistency since following the definition of inconsistency degree, $\pi(\mathcal{A})$ is not guaranteed to be the maximal set of $\mathcal{A}$. In general, possibilistic inference suffers from an important drawback in the sense that some assertions from $\mathcal{A} \backslash \pi(\mathcal{A})$ that are not involved in any conflict are inhibited as we can see in the following example.

Example 4. [Example continued] One can see that $I(y)$ and $F(t)$ are not involved in any conflict. However $I(y) \notin \pi(\mathcal{A})$ and $F(t) \notin \pi(\mathcal{A})$.

Inference Based on Non-Defeated Entailment

One way to overcome this drawback consists in enlarging $\pi(\mathcal{A})$ by recovering all the inhibited non conflicting elements. Let us denote by $\mathcal{C}(\mathcal{A})$ the collection of conflict sets in $\mathcal{A}$ (Definition 2). The following definition introduces the notion of non conflicting assertions or (*free* assertions).

Definition 6. Let $\mathcal{K}$=$\langle \mathcal{T}, \mathcal{A} \rangle$ be π-DL-$Lite$ KB. An assertion $f \in \mathcal{A}$ is said to be *free* iff $\forall \mathcal{C} \in \mathcal{C}(\mathcal{A})$:$f \notin \mathcal{C}$ with f=(φ, α).

Intuitively, *free* assertions are those facts that are not involved in any conflict. The notions of *free* elements and *free*-entailment have been originally proposed in (Benferhat, Dubois, and Prade 1992) where KBs are encoded in a propositional logic setting. The definition of *free*-entailment is also equivalent to the definition of *IAR*-entailment (Lembo et al. 2010) for flat *DL-Lite* where its computation is achived in polynomial time. Let $\mathcal{S} \in \mathcal{A}$ be a set of facts, we denote by *free*($\mathcal{S}$) the set of *free* assertions in $\mathcal{S}$.

Definition 7. Let $\mathcal{K}$=$\langle \mathcal{T}, \mathcal{A} \rangle$ be π-DL-$Lite$ KB. Let α_1=1>...>α_n>0 be different weights in the KB and $\forall i$=1, ..., n :$\mathcal{S}_i$=$\{(\varphi, \alpha_i)$:$(\varphi, \alpha_i) \in \mathcal{A}\}$. The *non-defeated* assertional base of $\mathcal{A}$, denoted by $nd(\mathcal{A})$, is defined as follows:

$$nd(\mathcal{A})=free(\mathcal{S}_1) \cup free(\mathcal{S}_1 \cup \mathcal{S}_2) \cup \ldots \cup free(\mathcal{S}_1 \cup \ldots \cup \mathcal{S}_n)$$

where $\forall i$:$free(\mathcal{S}_1 \cup ... \cup \mathcal{S}_i)$ denotes the set of *free* facts in $(\mathcal{S}_1 \cup ... \cup \mathcal{S}_i)$.

Example 5 (Example continued). We have $free(\mathcal{S}_1) = \{E(a), G(f)\}$,$free(\mathcal{S}_1 \cup \mathcal{S}_2)$=$\{G(f)$,$H(c, e)$,$F(t)\}$,$free(\mathcal{S}_1 \cup \mathcal{S}_2 \cup \mathcal{S}_3)$=$\{I(y)$,$G(f)$,$F(t)\}$. Hence $nd(\mathcal{A})$=$\{E(a)$,$G(f)$, $H(c, e)$,$F(t)$, $I(y)\}$.

Note that given $i \geq 1$, there is no inclusion relation between $free(\mathcal{S}_1 \cup \ldots \cup \mathcal{S}_i)$ and $free(\mathcal{S}_1 \cup \ldots \cup \mathcal{S}_k)$ where k>i. Namely, if $f \in free(\mathcal{S}_1 \cup \ldots \cup \mathcal{S}_i)$ with f=(φ, α), this does not mean that $f \in free(\mathcal{S}_1 \cup \ldots \cup \mathcal{S}_k)$ since $\mathcal{S}_1 \cup \ldots \cup \mathcal{S}_i \subset \mathcal{S}_1 \cup \ldots \cup \mathcal{S}_k$ and $\mathcal{S}_1 \cup \ldots \cup \mathcal{S}_k$ may include new *free* assertional facts some of which can contradict ones which are *free* in $\mathcal{S}_1 \cup \ldots \cup \mathcal{S}_i$. However if $f \notin free(\mathcal{S}_1 \cup \ldots \cup \mathcal{S}_i)$ then $f \notin free(\mathcal{S}_1 \cup \ldots \cup \mathcal{S}_k)$.

From Definition 2, when $\mathcal{T}$ is coherent, a conflict involves exactly two assertional facts and dropping any element restores its consistency. When certainty degrees are available, restoring the consistency of a conflict sets comes down to throw out the elements which have the lowest level of certainty. More formally,

Definition 8. Let $\mathcal{C}$=$\{(\varphi_i, \alpha_i), (\varphi_j, \alpha_j)\}$ be a conflict in $\mathcal{A}$ then $\langle \mathcal{T}, \mathcal{C}^* \rangle$ is consistent where $\mathcal{C}^*$=$\mathcal{C}_{>_{Inc(\mathcal{C})}}$ and $Inc(\mathcal{C})$=$min(\alpha_i, \alpha_j)$.

It is clear that the *non-defeated* base $nd(\mathcal{A})$ of $\mathcal{A}$ is composed of all the facts that are not contained in the least certainty level of each conflict of $\mathcal{A}$. Indeed, if there is a conflict $\mathcal{C}$ in $\mathcal{A}$, then all the assertions that belong to $\mathcal{C}\cap\mathcal{C}^*$ have a certainty degree higher than $Inc(\mathcal{C})$.

Proposition 4. *Let* $\mathcal{K}$=$\langle \mathcal{T}, \mathcal{A} \rangle$ *be* π-*DL-Lite KB. Then* $nd(\mathcal{A})$=$\{(\varphi, \alpha)$*:* $\forall \mathcal{C}\in\mathcal{C}(\mathcal{A})$*,* $(\varphi, \alpha)\notin\mathcal{C}\setminus\mathcal{C}^*\}$

Proof. Let $\mathcal{C}$ be a conflict of $\mathcal{A}$ where k=$Inc(\mathcal{C})$ and $nd(\mathcal{A})$=$free(\mathcal{S}_1\cup\ldots\cup\mathcal{S}_n)$. Assume that $(\varphi, \alpha_i)\in\mathcal{C}^*$. This means that $k\geq i$ and $\varphi\in_i\mathcal{S}_1\cup\ldots\cup\mathcal{S}_k$. This means also that $\varphi\in_i free(\mathcal{S}_1\cup\ldots\cup\mathcal{S}_k)$. Moreover for $n>k$ $\varphi\in_i free(\mathcal{S}_1\cup\ldots\cup\mathcal{S}_n)$, since if (φ, α_i) is involved in another conflict, then following Definition 8 we drop the elements having the lowest certainty level. Therefore $\varphi\in nd(\mathcal{A})$. The converse follows similarly. $\square$

Example 6 (Examples continued). We have: $\mathcal{C}_1$=$\{E(a)_{\alpha_1}, G(a)_{\alpha_2}\}$,$\mathcal{C}_2$=$\{H(c,e)_{\alpha_2}$,$G(c)_{\alpha_3}\}$,$\mathcal{C}_3$=$\{G(a)_{\alpha_2}$, $H(a,b)_{\alpha_3}\}$. Indeed, $\mathcal{C}_1^*$=$\{E(a)_{\alpha_1}\}$, $\mathcal{C}_2^*$=$\{H(c,e)_{\alpha_2}\}$, $\mathcal{C}_3^*$=$\{G(a)_{\alpha_2}\}$, thus $nd(\mathcal{A})$=$\{E(a)$,$G(f)$, $H(c,e)$, $F(t)$, $I(y)\}$ which is the same result as in Example 5.

From Proposition 4, the set $nd(\mathcal{A})$ is the largest subset of $\mathcal{A}$ containing non defeated facts.

Definition 9. Let $\mathcal{K}$=$\langle \mathcal{T}, \mathcal{A} \rangle$ be π-DL-$Lite$ KB and $nd(\mathcal{A})$ be its *non-defeated* sub-base. A query Q is said to be a non-defeated consequence (*ND*-consequence) from $\mathcal{K}$, denoted by $\mathcal{K} \models_{\texttt{nd}} Q$, iff $\langle \mathcal{T}, nd(\mathcal{A}) \rangle \models Q$.

The following proposition shows that *ND*-inference is sensitive to the use of the deductive closure.

Proposition 5. *Let* $\mathcal{K}$=$\langle \mathcal{T}, \mathcal{A} \rangle$ *be* π-*DL-Lite KB and* $nd(\mathcal{A})$ *be its non-defeated sub-base. Then* $\forall Q$*: if* $\langle \mathcal{T}, \mathcal{A} \rangle\models_{nd}Q$ *then* $\langle \mathcal{T}, Cl(\mathcal{A}) \rangle\models_{nd}Q$*. The converse is false.*

Proof. The proof is immediate since *free*$(\mathcal{S}_i)\subseteq$*free*$(Cl(\mathcal{S}_i))$, $\forall i$=1,...,n. For the converse it is enough to consider $\mathcal{T}$=$\{E\sqsubseteq\neg B$,$B\sqsubseteq C$,$E\sqsubseteq C\}$ and $\mathcal{A}$=$\mathcal{A}_1$=$\{E(a), B(a)\}$. We have $nd(\mathcal{A})$=$\emptyset$ and $nd(Cl(\mathcal{A}))$=$\{C(a)\}$. Hence $C(a)$ is an *ND*-consequence of $\langle \mathcal{T}, Cl(\mathcal{A}) \rangle$ but it is not an *ND*-consequence of $\langle \mathcal{T}, \mathcal{A} \rangle$. $\square$

The computational complexity of the computation of the *non-defeated* sub-base of $\mathcal{A}$ is polynomial. This is obtained by a simple modification of Algorithm 1. Algorithm 2 shows how to compute $nd(\mathcal{A})$.

Proposition 6. *The complexity of ND-entailment is in P.*

Algorithm 2 COMPUTE-$nd(\mathcal{A})$

Input: $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ where $\mathcal{A}$=$\mathcal{S}_1 \cup \ldots \cup \mathcal{S}_n$
Output: $nd(\mathcal{A})$
$\pi - neg(\mathcal{T}) \leftarrow$ PINEGCLOSURE$(\mathcal{T})$,
$nd(\mathcal{A}) \leftarrow \mathcal{A}$,
$inc \leftarrow 0$,$\mathcal{C} \leftarrow \emptyset$.
for all $X \sqsubseteq \neg Y \in \pi - neg(\mathcal{T})$ **do**
  for all $(\varphi_i, \alpha_i), (\varphi_j, \alpha_j) \in \mathcal{A}$ **do**
    if $\langle X \sqsubseteq \neg Y, \{(\varphi_i, \alpha_i), (\varphi_j, \alpha_j)\} \rangle$ is inconsistent **then**
      $inc \leftarrow min\{\alpha_i, \alpha_j\}$
      $\mathcal{C} \leftarrow \mathcal{C} \cup \{\{\varphi_i, \varphi_j\} \setminus \{\varphi_i, \varphi_j\}_{>inc}\}$
return $nd(\mathcal{A}) \setminus \mathcal{C}$

Proof. The proof follows from the fact that computing $free$-subbase is done in polynomial time. *ND*-entailment proceeds to a linear number of computations of free subbases. $\square$

Inference Based on Linear-Based Entailment

Another way to recover the inhibited assertions is to define the *linear* assertional base from $\mathcal{A}$ that is consistent with $\mathcal{T}$.

Definition 10. Let $\mathcal{K}$=$\langle \mathcal{T}, \mathcal{A} \rangle$ be a π-DL-$Lite$ KB. Let α_1=1>...>α_n>0 be different weights in $\mathcal{A}$ and $\mathcal{S}_i$=$\{(\varphi, \alpha_i)$:$(\varphi, \alpha_i)\in\mathcal{A}\}$. The *linear* base of $\mathcal{A}$, denoted $\ell(\mathcal{A})$, is obtained as follows:

- For i=1:$\ell(\mathcal{S}_1)$=$\mathcal{S}_1$ if $\langle \mathcal{T}, \mathcal{S}_1 \rangle$ is consistent. Otherwise $\ell(\mathcal{S}_1)$=$\emptyset$.
- For i>1: $\ell(\mathcal{S}_1\cup\ldots\cup\mathcal{S}_i)$=$\ell(\mathcal{S}_1\cup\ldots\cup\mathcal{S}_{i-1})\cup\mathcal{S}_i$ if $\langle \mathcal{T}, \ell(\mathcal{S}_1 \cup \ldots \cup \mathcal{S}_{i-1}) \cup \mathcal{S}_i \rangle$ is consistent. Otherwise $\ell(\mathcal{S}_1\cup\ldots\cup\mathcal{S}_i)$=$\ell(\mathcal{S}_1\cup\ldots\cup\mathcal{S}_{i-1})$.

Clearly, $\ell(\mathcal{A})$ is obtained by discarding the layer $\mathcal{S}_i$ when its facts conflict with the ones involved in the previous layers. $\ell(\mathcal{A})$ is unique and consistent with $\mathcal{T}$.

Example 7 (Example continued). We have $\mathcal{T}\cup\{E(a), G(f)\}_{\alpha_1}\cup\{\{H(a,b)$, $G(c)$,$I(y)\}_{\alpha_3}\}$ is consistent. Then $\ell(\mathcal{A})$=$\{E(a)$,$G(f)$, $H(a,b)$,$G(c)$, $I(y)\}$.

The following proposition shows that the ℓ-inference is insensitive to the deductive closure, more precisely:

Proposition 7. *Let* $\mathcal{K}$=$\langle \mathcal{T}, \mathcal{A} \rangle$ *be a* π-DL-$Lite$ *KB. Then* $\forall Q$*:* $\langle \mathcal{T}, \mathcal{A} \rangle\models_{\ell}Q$ *iff* $\langle \mathcal{T}$*,* $Cl(\mathcal{A}) \rangle\models_{\ell}Q$

Proof. The proof is similar to the proof of Proposition 3 since ℓ-inference use also a consistency checking of the whole layer to decide whether this latter should be kept or not for restoring the consistency of the KB. Recall that a layer $\mathcal{S}_i$=$\{(\varphi, \alpha_i)$:$(\varphi, \alpha_i) \in\mathcal{A}\}$ where α_1=1>...>α_n>0 are the different weights in $\mathcal{A}$. $\square$

Algorithm 3 shows how to compute the set $\ell(\mathcal{A})$, the linear sub-base of $\mathcal{A}$.

Proposition 8. *The computational complexity of* ℓ*-entailment is in* $\mathcal{O}(n * cons)$ *where* n *is the number of strata in the DL-Lite KB and* $cons$ *is the complexity of consistency checking of standard DL-Lite.*

Algorithm 3 COMPUTE-$\ell(\mathcal{A})$

Input: $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ where $\mathcal{A}$=$\mathcal{S}_1 \cup \ldots \cup \mathcal{S}_n$
Output: $\ell(\mathcal{A})$
$\pi - neg(\mathcal{T}) \leftarrow$ PINEGCLOSURE($\mathcal{T}$)
$\ell(\mathcal{A}) \leftarrow \emptyset$,
$inc \leftarrow 0$
$\mathcal{V} \leftarrow \{\alpha_1, \alpha_2, ..., \alpha_n\}$ //vector of weights in $\mathcal{A}$
$S \leftarrow \emptyset$, $\mathcal{S}(\mathcal{A}) \leftarrow \emptyset$ // a collection of strata
for all $\alpha_i \in \mathcal{V}$ **do**
  for all $(\varphi_j, \alpha_j) \in \mathcal{A}$ **do**
    if α_i=α_j **then** $S \leftarrow S \cup \{(\varphi_j, \alpha_j)\}$
  $\mathcal{S}(\mathcal{A}) \leftarrow \mathcal{S}(\mathcal{A}) \cup S$, $S \leftarrow \emptyset$
for all $X \sqsubseteq \neg Y \in \pi - neg(\mathcal{T})$ **do**
  for all $\mathcal{S} \in \mathcal{S}(\mathcal{A})$ **do**
    for all $(\varphi_i, \alpha_i), (\varphi_j, \alpha_j) \in (\ell(\mathcal{A}) \cup \mathcal{S})$ **do**
      if $\langle X \sqsubseteq \neg Y, \{(\varphi_i, \alpha_i), (\varphi_j, \alpha_j)\}\rangle$ is inconsistent **then** $inc \leftarrow max\{\alpha_i, \alpha_j\}$
    if $inc = 0$ **then** $\ell(\mathcal{A}) \leftarrow \ell(\mathcal{A}) \cup \mathcal{S}$
    else $inc \leftarrow 0$
return $\ell(\mathcal{A})$

Sketch of proof. The proof of the complexity of ℓ-entailment is immediate since to see whether a stratum should be kept in the result of restoring consistency, one consistency check is needed. $\square$

Comparative Analysis

Obviously, the ℓ-entailment is more productive than π-entailment) since given i>1, if $\mathcal{S}_i$ is inhibited by $\models_\pi$ then $\mathcal{S}_j$ with j>i is not necessarily inhibited by $\models_\ell$. However the linear sub-base does not solve completely the drawback. Indeed, according to Definition 10, if an assertion in $\mathcal{S}_i$ conflicts with another one in a previous layer, then the whole $\mathcal{S}_i$ is inhibited including the assertions that are not involved in any conflict. For instance, from Example 4, $F(t)$ is not involved in any conflict but it is still inhibited in $\ell(\mathcal{A})$. The *ND*-entailment, ℓ-entailment and π-entailment don't have the same behavior in the sense that *ND*-entailment and ℓ-entailment are more productive than π-consequence. However, ℓ-entailment remains incomparable with the *ND*-consequence, since layers including non free assertions can be present in $\ell(\mathcal{A})$. Finally ℓ-entailment and π-entailment are insensitive to the negated closure. However *ND*-inference is sensitive to the use of the deductive closure. The following figure summarizes the relationships between *ND*-entailment, ℓ-entailment and π-entailment.

$$\begin{tikzcd}
 & \pi\text{-}inference \arrow[r, leftrightarrow] \arrow[dl] \arrow[dr] & cl(\pi)\text{-}inference \\
\ell\text{-}inference \arrow[d, leftrightarrow] & & ND\text{-}inference \arrow[d] \\
cl(\ell)\text{-}inference & & cl(ND)\text{-}inference
\end{tikzcd}$$

Figure 1: Relationship between inference relations

Conclusions

This paper dealt with an important issue regarding reasoning under inconsistency in π-DL-$Lite$. The core of our approach is exploiting the available certainty degrees for dealing with inconsistency. We defined inference modes based on possibilistic *DL-Lite* entailment and linear-based entailment. Both of them are not sensitive to the deductive closure of the ABox. The ND-entailment extends possibilistic entailment but remains incomparable with the linear inference as it is the case in the propositional setting. One way to go one step further than ND-inference is to use the possibilistic closure. This is proper to the DL-framework and it can hardly be defined in a propositional logic setting.

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