On the Correspondence between Abstract Dialectical Frameworks and Nonmonotonic Conditional Logics

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Abstract

The exact relationship between formal argumentation and nonmonotonic logics is a research topic that keeps on eluding researchers despite recent intensified efforts. We contribute to a deeper understanding of this relation by investigating characterizations of abstract dialectical frameworks in conditional logics for nonmonotonic reasoning. We first show that in general, there is a gap between argumentation and conditional semantics when applying several intuitive translations, but then prove that this gap can be closed when focusing on specific classes of translations.

1 Introduction

It is well-known that argumentation and nonmonotonic resp. default logics are closely connected: In (Dung 1995) it is shown that Reiter's default logic can be implemented by abstract argumentation frameworks, a most basic form of computational model of argumentation to which many existing approaches to formal argumentation refer. On the other hand, it is clear that argumentation allows for nonmonotonic, defeasible reasoning, and in (Rienstra, Sakama, and van der Torre 2015) computational models of argumentation are assessed by formal properties that have been adapted from nonmonotonic logics. Nevertheless, argumentation and nonmonotonic reasoning are perceived as two different fields which do not subsume each other, and indeed, often attempts to transform reasoning systems from one side into systems of the other side have been revealing gaps that could not be closed (cf., e.g., (Thimm and Kern-Isberner 2008; Kern-Isberner and Simari 2011; Heyninck 2019)). While one might argue that this is due to the seemingly richer, dialectical structure of argumentation, in the end the evaluation of arguments often boils down to comparing arguments with their attackers, and comparing degrees of belief is a basic operation in qualitative nonmonotonic reasoning. Therefore, in spite of the abundance of existing work studying connections between the two fields, the true nature of the relationship between argumentation and nonmonotonic reasoning has not been fully understood.

We aim at deepening the understanding of the relationships between argumentation and nonmonotonic logics and establishing a theoretical basis for integrative approaches by focusing on most fundamental approaches on either side: *Abstract Dialectical Frameworks* (ADFs) (Brewka et al. 2013) for argumentation, and *Conditional Logics* (CL) (Nute 1984; Spohn 1988) for nonmonotonic logics. ADFs are an approach to formal argumentation, which subsumes many other argumentative formalisms in a generic, logic-based way. On the side of nonmonotonic logics, conditionals have been shown (and often used) to implement nonmonotonic inferences and provide expressive formalisms to represent knowledge bases; some of the most popular nonmonotonic inference systems (e. g., system Z (Goldszmidt and Pearl 1996)) make use of conditionals. Both ADFs and CL can be considered as high-level formalisms implementing properly the basic nature of the respective field without being restricted too much by subtleties of specific approaches, and both are based on 3-valued logics.

In this paper we investigate the correspondence between abstract dialectical frameworks and system Z. Syntactically, both frameworks focus on pairs of objects such as (ϕ, ψ). In conditional logic, these pairs are interpreted as conditionals with the informal meaning "if ϕ is true then, usually, ψ is true as well" and written as $(\psi|\phi)$. In abstract dialectical frameworks, these pairs are interpreted as acceptance conditions, and interpreted as "if ϕ is accepted then ψ is accepted as well". The resemblance of these informal interpretations is striking, but both approaches use fundamentally different semantics to formalise these interpretations. Here we ask the question of whether, and how we can interpret abstract dialectical frameworks in terms of conditional logic so that acceptance in the argumentative system is defined by a nonmonotonic inference relation based on conditionals. We continue work from (Kern-Isberner and Thimm 2018) by considering several translations of ADFs into conditional knowledge bases and applying the Z-inference relation (Goldszmidt and Pearl 1996) to these knowledge bases. We first show that there is a gap between argumentation and conditional semantics when applying several intuitive translations, but then define a class of translations that are *Z-adequate*—i. e. they preserve the semantics, see Section 3.1 for the formal definition—for the 2-valued model semantics of ADFs, and for other semantics under certain conditions on the ADFs. Furthermore, we show that none of the translations studied in this paper are Z-adequate for the grounded

semantics.

Outline of this Paper: After stating all the necessary preliminaries in Section 2, we investigate a family of translations from ADFs into conditional knowledge bases in Section 3, where we first define Z-adequacy for such translations (Section 3.1), after which we investigate the Z-adequacy of the suggested translations for 2-valued models (Section 3.2), preferred and stable semantics (Section 3.3) and grounded semantics (Section 3.4). We finally discuss related work in Section 4 before concluding (Section 5).

2 Preliminaries

In the following, we we briefly recall some general preliminaries on propositional logic, as well as technical details on conditional logic and ADFs (Brewka et al. 2013).

2.1 Propositional Logic

For a set At of atoms let $\mathcal{L}(\mathsf{At})$ be the corresponding propositional language constructed using the usual connectives $\wedge$ (*and*), $\vee$ (*or*), $\neg$ (*negation*) and $\supset$ (*material implication*). A (classical) *interpretation* (also called *possible world*) ω for a propositional language $\mathcal{L}(\mathsf{At})$ is a function $\omega : \mathsf{At} \to \{\mathsf{T}, \mathsf{F}\}$. Let $\Omega(\mathsf{At})$ denote the set of all interpretations for At. We simply write Ω if the set of atoms is implicitly given. An interpretation ω *satisfies* (or is a *model* of) an atom $a \in \mathsf{At}$, denoted by $\omega \models a$, if and only if $\omega(a) = \mathsf{T}$. The satisfaction relation $\models$ is extended to formulas as usual. As an abbreviation we sometimes identify an interpretation ω with its *complete conjunction*, i.e., if $a_1, \ldots, a_n \in \mathsf{At}$ are those atoms that are assigned T by ω and $a_{n+1}, \ldots, a_m \in \mathsf{At}$ are those propositions that are assigned F by ω we identify ω by $a_1 \ldots a_n \overline{a_{n+1}} \ldots \overline{a_m}$ (or any permutation of this). For example, the interpretation ω_1 on $\{a, b, c\}$ with $\omega(a) = \omega(c) = \mathsf{T}$ and $\omega(b) = \mathsf{F}$ is abbreviated by $a\bar{b}c$. For $\Phi \subseteq \mathcal{L}(\mathsf{At})$ we also define $\omega \models \Phi$ if and only if $\omega \models \phi$ for every $\phi \in \Phi$. Define the set of models $\mathsf{Mod}(X) = \{\omega \in \Omega(\mathsf{At}) \mid \omega \models X\}$ for every formula or set of formulas X. A formula or set of formulas X_1 *entails* another formula or set of formulas X_2, denoted by $X_1 \vdash X_2$, if $\mathsf{Mod}(X_1) \subseteq \mathsf{Mod}(X_2)$.

2.2 Reasoning with Nonmonotonic Conditionals

There are many different conditional logics (cf., e.g., (Kraus, Lehmann, and Magidor 1990; Nute 1984)), we will just use basic properties of conditionals that are common to many conditional logics and are especially important for nonmonotonic reasoning: Basically, we follow the approach of de Finetti (1974) who considered conditionals as *generalized indicator functions* for possible worlds resp. propositional interpretations ω:

$$((\psi|\phi))(\omega) = \begin{cases} 1 & : & \omega \models \phi \wedge \psi \\ 0 & : & \omega \models \phi \wedge \neg\psi \\ u & : & \omega \models \neg\phi \end{cases} \quad (1)$$

where u stands for *unknown* or *indeterminate*. In other words, a possible world ω *verifies* a conditional $(\psi|\phi)$ iff it satisfies both antecedent and conclusion $((\psi|\phi)(\omega) = 1)$; it *falsifies, or violates* it iff it satisfies the antecedence but not the conclusion $((\psi|\phi)(\omega) = 0)$; otherwise the conditional

is *not applicable*, i.e., the interpretation does not satisfy the antecedence $((\psi|\phi)(\omega) = u)$. We say that ω *satisfies* a conditional $(\psi|\phi)$ iff it does not falsify it, i.e., iff ω satisfies its *material counterpart* $\phi \supset \psi$. Hence, conditionals are three-valued logical entities and thus extend the binary setting of classical logics substantially in a way that is compatible with the probabilistic interpretation of conditionals as conditional probabilities. Such a conditional $(\psi|\phi)$ can be accepted as plausible if its verification $\phi \wedge \psi$ more plausible than its falsification $\phi \wedge \neg\psi$, where plausibility is often modelled by a total preorder on possible worlds. This is in full compliance with nonmonotonic inference relations $\phi \hspace{1pt}|\!\!\sim\hspace{1pt} \psi$ (Makinson 1988) expressing that from ϕ, ψ may be plausibly/defeasibly derived. An obvious implementation of total preorders are *ordinal conditional functions (OCFs)*, (also called *ranking functions*) $\kappa : \Omega \to \mathbb{N} \cup \{\infty\}$ (Spohn 1988). They express degrees of (im)plausibility of possible worlds and propositional formulas ϕ by setting $\kappa(\phi) := \min\{\kappa(\omega) \mid \omega \models \phi\}$. OCFs κ provide a particularly convenient formal environment for nonmonotonic and conditional reasoning, allowing for simply expressing the acceptance of conditionals and nonmonotonic inferences via stating that $(\psi|\phi)$ is accepted by κ iff $\phi \hspace{1pt}|\!\!\sim_\kappa\hspace{1pt} \psi$ iff $\kappa(\phi \wedge \psi) < \kappa(\phi \wedge \neg\psi)$, implementing formally the intuition of conditional acceptance based on plausibility mentioned above. For an OCF κ, $Bel(\kappa)$ denotes the propositional beliefs that are implied by all most plausible worlds, i.e. $Bel(\kappa) = \{\phi \mid \forall \omega \in \kappa^{-1}(0) : \omega \models \phi\}$. We denote with CL the framework of reasoning from conditional knowledge bases based on OCFs.

Specific examples of ranking models are system Z yielding the inference relation $|\!\!\sim_\mathsf{Z}$ (Goldszmidt and Pearl 1996) and c-representations (Kern-Isberner 2001). We focus on system Z defined as follows. A conditional $(\psi|\phi)$ is tolerated by a finite set of conditionals Δ if there is a possible world ω with $(\psi|\phi)(\omega) = 1$ and $(\psi'|\phi')(\omega) \neq 0$ for all $(\psi'|\phi') \in \Delta$, i.e. ω verifies $(\psi|\phi)$ and does not falsify any (other) conditional in Δ. The Z-partitioning $(\Delta_0, \ldots, \Delta_n)$ of Δ is defined as:

- $\Delta_0 = \{\delta \in \Delta \mid \Delta \text{ tolerates } \delta\}$;

- $\Delta_1, \ldots, \Delta_n$ is the Z-partitioning of $\Delta \setminus \Delta_0$.

For $\delta \in \Delta$ we define: $Z_\Delta(\delta) = i$ iff $\delta \in \Delta_i$ and $(\Delta_0, \ldots, \Delta_n)$ is the Z-partioning of Δ. Finally, the ranking function κ_Δ^Z is defined via: $\kappa_\Delta^Z(\omega) = \max\{Z(\delta) \mid \delta(\omega) = 0, \delta \in \Delta\}+1$, with $\max \emptyset = -1$. We can now define $\Delta \hspace{1pt}|\!\!\sim_\mathsf{Z}\hspace{1pt} \phi$ iff $\phi \in Bel(\kappa_\Delta^Z)$. Below the following Lemma about system Z will prove useful:

Lemma 1. $\omega \notin (\kappa_\Delta^Z)^{-1}(0)$ *iff* $\delta(\omega) = 0$ *for some* $\delta \in \Delta$.

Proof. This follows immediately in view of the fact that $\omega \in (\kappa_\Delta^Z)^{-1}(0)$ iff $\delta(\omega) \neq 0$ for every $\delta \in \Delta$. $\qquad \square$

Example 1. *Let* $\Delta = \{(b|\neg a), (a|\neg b), (c|\neg a \vee \neg b)\}$. *For this set of conditionals,* $\Delta = \Delta_0$ *and therefore we have:*

ω	κ_Δ^z	ω	κ_Δ^z	ω	κ_Δ^z	ω	κ_Δ^z
abc	0	$ab\bar{c}$	0	$a\bar{b}c$	0	$a\bar{b}\bar{c}$	1
$\bar{a}bc$	0	$\bar{a}b\bar{c}$	1	$\bar{a}\bar{b}c$	1	$\bar{a}\bar{b}\bar{c}$	1

Thus, $(\kappa_\Delta^Z)^{-1}(0) = \{abc, ab\bar{c}, a\bar{b}c, \bar{a}bc\}$. *This means that, for example,* $\Delta \hspace{1pt}|\!\!\sim_\mathsf{Z}\hspace{1pt} a \vee b$ *and* $\Delta \hspace{1pt}|\!\!\not\sim_\mathsf{Z}\hspace{1pt} c$.

2.3 Abstract Dialectical Frameworks

We briefly recall some technical details on ADFs following loosely the notation from (Brewka et al. 2013). An ADF D is a tuple $D = (S, L, C)$ where S is a set of statements, $L \subseteq S \times S$ is a set of links, and $C = \{C_s\}_{s \in S}$ is a set of total functions $C_s : 2^{par_D(s)} \to \{\top, \bot\}$ for each $s \in S$ with $par_D(s) = \{s' \in S \mid (s', s) \in L\}$ (also called acceptance functions). An acceptance function C_s defines the cases when the statement s can be accepted (truth value $\top$), depending on the acceptance status of its parents in D. By abuse of notation, we will often identify an acceptance function C_s by its equivalent *acceptance condition* which models the acceptable cases as a propositional formula.

Example 2. *We consider the following ADF $D_1 = (\{a, b, c\}, L, C)$ with $L = \{(a, b), (b, a), (a, c), (b, c)\}$ and:*
$$C_a = \neg b \quad C_b = \neg a \quad C_c = \neg a \vee \neg b$$
Informally, the acceptance conditions can be read as "a is accepted if b is not accepted", "b is accepted if a is not accepted" and "c is accepted if a is not accepted or b is not accepted".

An ADF $D = (S, L, C)$ is interpreted through 3-valued interpretations $v : S \to \{\top, \bot, u\}$, which assign to each statement in S either the value $\top$ (true, accepted), $\bot$ (false, rejected), or u (unknown). $\mathcal{V}$ consists of all three-valued interpretations whereas $\mathcal{V}^2$ consists of all the two-valued interpretations (i.e. interpretations such that for every $s \in S$, $v(s) \in \{\top, \bot\}$). Then v is a *model* of D if for all $s \in S$, if $v(s) \neq u$ then $v(s) = v(C_s)$. We define an order $\leq_i$ over $\{\top, \bot, u\}$ by making u the minimal element: $u <_i \top$ and $u <_i \bot$ and this order is lifted pointwise as follows (given two valuations v, w over S): $v \leq_i w$ iff $v(s) \leq_i w(s)$ for every $s \in S$. The set of two-valued interpretations extending a valuation v is defined as $[v]^2 = \{w \in \mathcal{V}^2 \mid v \leq_i w\}$. Given a set of valuations V, $\sqcap_i V(s) = v(s)$ if for every $v' \in V$, $v(s) = v'(s)$ and $\sqcap_i V(s) = u$ otherwise. $\Gamma_D(v) : S \to \{\top, \bot, u\}$ where $s \mapsto \sqcap_i\{w(C_s) \mid w \in [v]^2\}$.

For the definition of the stable model semantics, we need to define the reduct D^v of D given v, defined as: $D^v = (S^v, L^v, C^v)$ with:

- $S^v = \{s \in S \mid v(s) = \top\}$,
- $L^v = L \cap (S^v \times S^v)$, and
- $C^v = \{C_s[\{\phi \mid v(\phi) = \bot\}/\bot] \mid s \in S^v\}$.

where $C_s[\phi/\psi]$ is the formula obtained by substituting every occurence of ϕ in C_s by ψ.

Definition 1. Let $D = (S, L, C)$ be an ADF with $v : S \to \{\top, \bot, u\}$ an interpretation:

- v is a *2-valued model* iff $v \in \mathcal{V}^2$ and v is a model.
- v is *complete* for D iff $v = \Gamma_D(v)$.
- v is *preferred* for D iff v is $\leq_i$-maximally complete.
- v is *grounded* for D iff v is $\leq_i$-minimally complete.
- v is *stable* iff v is a model of D and $\{s \in S \mid v(s) = \top\} = \{s \in S \mid w(s) = \top\}$ where w is the grounded interpretation of D^v.

We denote by 2mod(D), complete(D), preferred(D), grounded(D) respectively stable(D) *the sets of 2-valued models and complete, preferred, grounded respectively stable interpretations of D.*

We recall the following relationships between the semantics defined above:

Theorem 1 ((Brewka et al. 2013)). *Given any ADF D, the following relationships hold:*

- stable(D) $\subseteq$ 2mod(D);
- 2mod(D) $\subseteq$ preferred(D);
- preferred(D) $\subseteq$ complete(D);
- grounded(D) $\subseteq$ complete(D).

Below we will make use of the following *ADF subclasses* which will prove useful in the generalisation of our results (in particular Theorem 2):

Definition 2 ((Diller et al. 2018)). *An ADF D is called:*

- weakly coherent *if* 2mod(D) $\subseteq$ stable(D);
- coherent *if* preferred(D) $\subseteq$ stable(D);
- semi-coherent *if* preferred(D) $\subseteq$ 2mod(D).

We finally define consequence relations for ADFs:

Definition 3. *Given an ADF $D = (S, L, C)$ and $s \in S$ and* sem $\in \{$2mod, preferred, grounded, stable$\}$*, we define:* $D \mid\!\sim^{\cap}_{\text{sem}} s[\neg s]$ *iff $v(s) = \top[\bot]$ for all $v \in$ sem(D).*[1]

Example 3 (Example 2 continued). *The ADF of Example 2 has three complete models v_1, v_2, v_3 with:*
$$v_1(a) = \top \quad v_1(b) = \bot \quad v_1(c) = \top$$
$$v_2(a) = \bot \quad v_2(b) = \top \quad v_2(c) = \top$$
$$v_3(a) = u \quad v_3(b) = u \quad v_3(c) = u$$

v_3 is the grounded interpretation whereas v_1 and v_2 are both preferred and 2-valued.

3 Interpreting ADFs in Conditional Logic

In this section, we systematically investigate translations from ADFs into conditional knowledge bases. The translations considered in this paper will be introduced in Section 3.1, where we will also formally define what it means for a translation to be adequate. We then investigate the adequacy for these translations with respect to the two-valued models (Section 3.2), the stable and the preferred semantics (Section 3.3) and the grounded semantics (Section 3.4).

3.1 Translations from ADFs to Conditional Logics

The general aim of this paper is to study translations of ADFs in CL. In more detail, let S be a set of propositions and $\mathfrak{D}_S$ be the set of all ADFs defined on the basis of S (i.e. all ADFs $D = (S, L, C)$), and $\mathfrak{C}_S$ be the set of all conditional knowledge bases over the propositional language generated by S. Then we investigate mappings $\Theta : \mathfrak{D}_S \to \mathfrak{C}_S$ (for arbitrary S).

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[1]Since (Brewka et al. 2013) showed the grounded extension to be unique for any ADF, we will omit $\cap$ from $\mid\!\sim_{\text{grounded}}$.

Once an ADF D has been translated into a conditional knowledge base $\Theta(D)$, we are able to use nonmonotonic inferences from this knowledge base by using, e.g., system Z:

Definition 4. *Let S be a set of atoms and $\theta : \mathfrak{D}_S \to \mathfrak{C}_S$ be a translation from* ADFs *to conditional knowledge bases. θ is Z-adequate with respect to semantics sem if: for every $D = (S, L, C)$ and every $s \in S$ it holds that $D \mathrel{\vdash\!\!\!\sim}_{\text{sem}}^{\cap} s$ iff $\Theta(D) \mathrel{\vdash\!\!\!\sim}_{\text{Z}} s$.*

There is a whole family of translations from ADFs to conditional logics which are *prima facie* apt to express the links between nodes s and their acceptance conditions C_s:

- $\Theta_1(D) = \{(s|C_s) \mid s \in S\}$
- $\Theta_2(D) = \{(C_s|s) \mid s \in S\}$
- $\Theta_3(D) = \Theta_1(D) \cup \Theta_2(D)$
- $\Theta_4(D) = \Theta_1(D) \cup \{(\neg s|\neg C_s) \mid s \in S\}$
- $\Theta_5(D) = \{((C_s \equiv s)|\top) \mid s \in S\}$

Θ_1 formalizes the intuition that whenever the condition of a node s is true, normally, s should be true as well. Likewise, Θ_2 formalizes the idea that if a node is true, its condition should be true as well. Θ_3 combines the two aforementioned intuitions. Θ_4 is a slight variation on this idea, combining Θ_1 with the constraint that whenever a condition of a node is false, the node itself should be false as well. Θ_5, finally, postulates that a node should be equivalent to its condition. Note that Θ_1 has already been investigated some small extent in (Kern-Isberner and Thimm 2018).

There are, of course, many more translations possible, for example one could suggest instead of $\Theta_1(D)$ the following $\Theta_1'(D) = \{(C_s \supset s|\top) \mid s \in S\}$. However, "shifting" the conditional to the right hand side does not impact the consequences of a translation:

Proposition 1. *Given a set of conditionals Δ, $\Delta \cup \{(\psi|\phi)\} \mathrel{\vdash\!\!\!\sim}_{\text{Z}} \theta$ iff $\Delta \cup \{(\phi \supset \psi|\top)\} \mathrel{\vdash\!\!\!\sim}_{\text{Z}} \theta$.*

Proof. Suppose Δ is a set of conditionals. In what follows, we will denote $\kappa^Z_{\Delta \cup \{(\psi|\phi)\}}$ by κ and $\kappa^Z_{\Delta \cup \{(\phi \supset \psi|\top)\}}$ by κ'. We show that $\kappa^{-1}(0) = \kappa'^{-1}(0)$, which implies the proposition. For this, suppose towards a contradiction that $\omega \in \kappa'^{-1}(0)$ yet $\omega \notin \kappa^{-1}(0)$. By Lemma 1 this means that there is some $(\lambda|\delta) \in \Delta \cup \{(\psi|\phi)\}$ s.t. $(\lambda|\delta)(\omega) = 0$. Since κ' accepts Δ, $(\lambda|\delta) = (\psi|\phi)$. Thus, $\omega \models \phi \wedge \neg \psi$. This means that $\omega \models \top \wedge \neg(\phi \supset \psi)$, i.e. $(\phi \supset \psi|\top)(\omega) = 0$. This contradicts $\kappa'(\omega) = 0$ and the assumption that κ' accepts $\Delta \cup \{(\phi \supset \psi|\top)\}$ and thus we have shown that $\kappa'^{-1}(0) \subseteq \kappa^{-1}(0)$. Analogously, we can show that $\kappa'^{-1}(0) \supseteq \kappa^{-1}(0)$ and thus $\kappa'^{-1}(0) = \kappa^{-1}(0)$. This implies $Bel(\kappa) = Bel(\kappa')$ $\square$

The above proposition thus establishes that within our perspective, it does not matter if we consider the conditional "ψ is plausible if ϕ is the case" or the conditional "$\phi \supset \psi$ is plausible". This does not imply that we can equivalently consider $\phi \supset \psi$ to be *true*.

3.2 Z-Adequacy w.r.t. Two-Valued Semantics

In this section we study Z-adequacy with respect to the 2mod-semantics for the translations suggested in the previous subsection. In particular, we will show that Θ_1 and Θ_2 are not Z-adequate whereas Θ_3, Θ_4 and Θ_5 are in fact Z-adequate for the 2mod-semantics.

We first observe that Θ_1 and Θ_2 are *not* Z-adequate.

Example 4 (Z-Inadequacy of Θ_1 w.r.t. 2mod). *We consider the following* ADF D_1 *from Example 2. Notice that $\Theta_1(D_1) = \{(b|\neg a), (a|\neg b), (c|\neg a \vee \neg b)\}$, which is the conditional knowledge base considered in Example 1. We therefore see that $\Theta_1(D_1) \not\mathrel{\vdash\!\!\!\sim}_{\text{Z}} c$ even though $D \mathrel{\vdash\!\!\!\sim}_{\text{2mod}}^{\cap} c$ and thus Θ_1 is not Z-adequate with respect to the* 2mod-*semantics.*

Example 5 (Z-Inadequacy of Θ_2 w.r.t. 2mod). *We consider the following* ADF $D_2 = (\{a, b, c\}, L, C)$ *where: $C_a = \neg b$ $C_b = \neg a$ $C_c = a \vee b$ D_2 has three complete models v_1, v_2, v_3 with: $v_1(a) = v_2(b) = v_1(c) = v_2(c) = \top$, $v_1(b) = v_2(a) = \bot$ and $v_3(a) = v_3(b) = v_3(c) = u$. Only v_1 and v_2 are 2-valued.*

Moving to $\Theta_2(D) = \{(\neg a|b), (\neg b|a), (a \vee b|c)\}$, we see that $(\kappa^Z_{\Theta_2(D)})^{-1}(0) = \{a\bar{b}c, a\bar{b}\bar{c}, \bar{a}bc, \bar{a}b\bar{c}, \bar{a}\bar{b}\bar{c}\}$. This means that $\Theta_2(D_2) \not\mathrel{\vdash\!\!\!\sim}_{\text{Z}} c$ even though $D \mathrel{\vdash\!\!\!\sim}_{\text{2mod}}^{\cap} c$, i.e. Θ_2 is not Z-adequate with respect to the 2mod-*semantics.*

We will now show that the translations Θ_3, Θ_4 and Θ_5 are Z-adequate for 2-valued models but not in general for the grounded semantics. For these results, the following conditions on translations will prove useful:

- **C1**: $\kappa^Z_{\Theta(D)}(C_s \wedge \neg s) > 0$ and $\kappa^Z_{\Theta(D)}(\neg C_s \wedge s) > 0$ for every $s \in S$.

- **C2**: $\{\bigwedge_{s \in S} C_s \equiv s\} \vdash \bigwedge_{(\psi|\phi) \in \Theta(D)} (\phi \supset \psi)$

  Θ_3, Θ_4 and Θ_5 satisfy both of the above conditions:

Proposition 2. *For any $i \in \{3, 4, 5\}$ and any* ADF D, $\Theta_i(D)$ *satisfies* **C1** *and* **C2**.

Proof. We show the claim for $i = 3$ and **C1**, the proofs for $i \in \{4, 5\}$ and **C2** are analogous. Suppose towards a contradiction that there is some ADF $D = (S, L, D)$ and some $s \in S$ s.t. $\kappa^Z_{\Theta_3(D)}(C_s \wedge \neg s) = 0$ or $\kappa^Z_{\Theta_3(D)}(\neg C_s \wedge s) = 0$. Suppose the former. Then $\kappa^Z_{\Theta_3(D)}(C_s \wedge \neg s) \geq \kappa^Z_{\Theta_3(D)}(C_s \wedge s)$, which contradicts $(s|C_s) \in \Theta_3(D)$. Likewise, $\kappa^Z_{\Theta_3(D)}(\neg C_s \wedge s) = 0$ contradicts $(C_s|s) \in \Theta_3(D)$. $\square$

Below we will use the notion of a world ω_v based on a 2-valued interpretation $v \in \mathcal{V}^2$ defined as:

$$\omega_v = \bigwedge_{v(s) = \top} s \wedge \bigwedge_{v(s) = \bot} \neg s$$

Likewise we define the valuation v_ω based on a world ω as $v_\omega(s) = \top$ if $\omega \models s$ and $v_\omega(s) = \bot$ otherwise.

Proposition 3. *For any Θ that satisfies* **C1** *for the* ADF D, $\omega \in (\kappa^Z_{\Theta(D)})^{-1}(0)$ *implies $v_\omega \in$ 2mod(D).*

Proof. Suppose that $\Theta(D)$ satisfies **C1** for the ADF $D = (S, L, C)$ and that $\omega \in (\kappa^Z_{\Theta(D)})^{-1}(0)$. We show that v_ω is a model of D. Indeed suppose towards a contradiction that $v_\omega(s) \neq v_\omega(C_s)$ for some $s \in S$. This means that $\omega \models s \wedge \neg C_s$ or $\omega \models \neg s \wedge C_s$. Since $\omega \in (\kappa^Z_{\Theta(D)})^{-1}(0)$, this contradicts $\Theta(D)$ satisfying **C1** for D. Thus, it has to be the case that v_ω is a model of D. That $v_\omega \in \mathcal{V}^2$ is clear from the fact that $\omega \models s \vee \neg s$ for every $s \in S$. $\square$

Proposition 4. *For any Θ that satisfies **C2** for the ADF D, $\omega_v \in (\kappa^Z_{\Theta(D)})^{-1}(0)$ if $v \in 2\mathrm{mod}(D)$.*

Proof. Suppose that $\Theta(D)$ satisfies **C2** for the ADF D, and suppose that v is a 2-valued model of D. Suppose towards a contradiction that $\omega_v \notin (\kappa^Z_{\Theta(D)})^{-1}(0)$. By Lemma 1 this means that $\omega_v \models \phi' \wedge \neg \psi'$ for some $(\psi'|\phi') \in \Theta(D)$. But then since $\{\bigwedge_{s \in S} C_s \equiv s\} \vdash \bigwedge_{(\psi|\phi) \in \Theta(D)}(\phi \supset \psi)$, by contraposition, and since $\{\phi' \wedge \neg \psi'\} \vdash \neg(\bigwedge_{(\psi|\phi) \in \Theta(D)}(\phi \supset \psi))$, $\omega_v \models \neg \bigwedge_{s \in S} C_s \equiv s$. But then there is some $s \in S$ s. t. $\omega_v \models s \wedge \neg C_s$ or $\omega_v \models \neg s \wedge C_s$. But then $v(s) \neq v(C_s)$, contradiction to v being a 2-valued model of D. $\square$

We can now derive the Z-adequacy with respect to the 2-valued model semantics for the translations Θ_3, Θ_4 and Θ_5:

Theorem 2. *For any ADF D, and $i \in \{3, 4, 5\}$: $D \mid\!\sim^\cap_{2\mathrm{mod}} s[\neg s]$ iff $\Theta_i(D) \mid\!\sim_Z s[\neg s]$ for any $s \in S$.*

Proof. Let $i \in \{1, 2, 3\}$ and D be an ADF. By definition, $D \mid\!\sim^\cap_{2\mathrm{mod}} s[\neg s]$ iff for every model $v \in \mathcal{V}^2$, $v(s) = \top[\bot]$. By Propositions 2, 3 and 4, $(\kappa^Z_{\Theta_i(D)})^{-1}(0) = \{\omega_v \mid v \in \mathcal{V}^2, v \in 2\mathrm{mod}(D)\}$. Thus, $D \mid\!\sim^\cap_{2\mathrm{mod}} s[\neg s]$ iff for every $\omega \in (\kappa^Z_{\Theta_i(D)})^{-1}(0)$, $\omega \models s[\neg s]$, which implies: $D \mid\!\sim^\cap_{2\mathrm{mod}} s[\neg s]$ iff $\Theta_i(D) \mid\!\sim_Z s[\neg s]$. $\square$

Remark 1. *In (Kern-Isberner and Thimm 2018), where Θ_1 was first proposed, the authors noted that there might be cases where an ADF D has a grounded interpretation but no ranking exists since there might be a conditional $(s|C_s) \in \Theta_1(D)$ s.t. $C_s \wedge s$ is not satisfiable, which implies that there does not exist any world ω for which $((s|C_s))(\omega) = 1$. This would mean that there is no ranking κ that accepts $(s|C_s)$. For Theorem 2, such a situation is unproblematic, since if $C_s \wedge s$ (or $\neg C_s \wedge \neg s$ for that matter) is not satisfiable, there will be no 2-valued model for D.*

3.3 Z-Adequacy w.r.t. Stable and Preferred Semantics

We can strengthen Theorem 2 to obtain Z-adequacy with respect to the stable and preferred semantics for specific subclasses of ADFs (the proof is an easy consequence of Theorems 1 and 2):

Theorem 3. *For any ADF D, and $i \in \{3, 4, 5\}$ the following results hold:*

1. *$D \mid\!\sim^\cap_{\mathrm{stable}} s[\neg s]$ iff $\Theta_i(D) \mid\!\sim_Z s[\neg s]$ for any $s \in S$, if D is weakly coherent.*
2. *$D \mid\!\sim^\cap_{\mathrm{preferred}} s[\neg s]$ iff $\Theta_i(D) \mid\!\sim_Z s[\neg s]$ for any $s \in S$, if D is semi-coherent.*

3.4 Z-Inadequacy w.r.t. the Grounded Semantics

Theorem 2 can also be used to derive the Z-inadequacy of Θ_3, Θ_4 and Θ_5 with respect to the grounded semantics:

Proposition 5. *For any $\Theta(D)$ that satisfies **C1** and **C2**, Θ is not Z-adequate with respect to* grounded.

Proof. We consider the following ADF $D = (\{a, b, c, d\}, L, C)$ where:
$$C_a = \neg b; \quad C_b = \neg a; \quad C_c = \neg a \wedge \neg b; \quad C_d = \neg c$$
This ADF has the following 2-valued models: v_1 which assigns $v_1(a) = v_1(d) = \top$ and $v_1(b) = v_1(c) = \bot$ and v_2 with $v_2(b) = v_2(d) = \top$ and $v_2(a) = v_2(c) = \bot$. Since $v_1(d) = v_2(d) = \top$, by Proposition 3 and Proposition 4, $\Theta(D) \mid\!\sim_Z d$. However, the grounded assignment v_G sets $v_G(a) = v_G(b) = v_G(c) = v_G(d) = u$. $\square$

4 Related Works

Our aim in this paper is to lay foundations of integrative techniques for argumentative and conditional reasoning. There are previous works, which have similar aims or are otherwise related to this endeavour. We will discuss those in the following.

First, there is huge body of work on *structured argumentation* (see e. g. (Besnard et al. 2014)). In these approaches, arguments are constructed on the basis of a knowledge base possibly consisting of conditionals. An attack relation between these arguments is constructed based on some syntactic criteria. Acceptable arguments are then identified by applying argumentation semantics to the resulting argumentation frameworks. Thus, even though structured argumentation syntactically uses conditional knowledge bases, it relies semantically on formal argumentation.

There have been some attempts to bridge the gap between specific structured argumentation formalisms and conditional reasoning. For example, in (Kern-Isberner and Simari 2011) conditional reasoning based on System Z (Goldszmidt and Pearl 1996) and DeLP (García and Simari 2004) are combined in a novel way. Roughly, the paper provides a novel semantics for DeLP by borrowing concepts from System Z that allows using *plausibility* as a criterion for comparing the strength of arguments and counterarguments. Our approach differs both in goal (we investigate the correspondence between argumentation and conditional logics instead of integrating insights from the latter into the former) and generality (DeLP is specific and arguably rather peculiar argumentation formalism whereas ADFs are the most general formalism around).

Several works investigate postulates for nonmonotonic reasoning known from conditional logics (Kraus, Lehmann, and Magidor 1990) for specific structured argumentation formalisms, such as assumption-based argumentation (Čyras and Toni 2015; Heyninck and Straßer 2018) and ASPIC$^+$ (Li, Oren, and Parsons 2017). These works revealed gaps between nonmonotonic reasoning and argumentation which we try to bridge in this paper.

Besnard et al. (Besnard, Grégoire, and Raddaoui 2013) develop a structured argumentation approach where general

conditional logic is used as the base knowledge representation formalism. Their framework is constructed in a similar fashion as the deductive argumentation approach (Besnard and Hunter 2008) but they also provide with *conditional contrariety* a new conflict relation for arguments, based on conditional logical terms. Even though insights from conditional logics are used in that paper, this approach stays well within the paradigm of structured argumentation. In (Weydert 2013) a new semantics for abstract argumentation is presented, which is also rooted in conditional logical terms. In more detail, a ranking interpretation is provided for extensions of arguments instantiated by strict and defeasible rules by using conditional ranking semantics. Thus, Weydert presupposes a conditional knowledge base that is used to contruct an argumentation framework whereas we investigate what are sensible translations of ADFs into conditional knowledge bases. In (Strass 2015) Strass presents a translation from an ASPIC-style defeasible logic theory to ADFs. While actually Strass embeds one argumentative formalism (the ASPIC-style theory) into another argumentative formalism (ADFs) and shows how the latter can simulate the former, the process of embedding is similar to our approach.

5 Conclusion

In this paper we systematically investigated translations from ADFs into conditional knowledge bases based on the syntactic similarities between the two frameworks. We have shown that there is a class of translations that is semantically adequate with respect to the 2-valued model semantics and, under certain assumptions on the ADF, also for the stable and preferred semantics. Finally, we have shown that, at least for the translations under consideration, Z-adequacy with respect to the grounded semantics is not guaranteed. In future work we want to both generalize these results by investigating translations that are Z-adequate for grounded semantics and looking at other ways to obtain a ranking κ on the basis of a translation Θ (from e. g. (Kern-Isberner 2001; Kraus, Lehmann, and Magidor 1990)). Furthermore, we plan to take advantage of the results from this paper to transfer features and results from conditional logics to ADFs.

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