Prompt Alternating-Time Epistemic Logics

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Abstract

In temporal logics, the operator $F$ expresses that at some time in the future something happens, e.g., a request is eventually granted. Unfortunately, there is no bound on the time until the eventuality is satisfied which in many cases does not correspond to the intuitive meaning system designers have, namely, that $F$ abstracts the idea that there is a bound on this time although its magnitude is not known. An elegant way to capture this meaning is through Prompt-LTL which extends LTL with the operator $F_P$ ("prompt eventually"). We extend this work by studying alternating-time epistemic temporal logics extended with $F_P$.

We study the model-checking problem of the logic Prompt-KATL∗, which is ATL∗ extended with epistemic operators and prompt eventually. We also obtain results for the model-checking problem of some of its fragments. Namely, of Prompt-KATL (ATL with epistemic operators and prompt eventually), Prompt-KCTL∗ (CTL∗ with epistemic operators and prompt eventually), and finally the existential fragments of Prompt-KATL∗ and Prompt-KATL.

Introduction

Alternating-time temporal logics are expressive tools for reasoning about multi-agent systems (Alur, Henzinger, and Kupferman 2002; van der Hoek and Wooldridge 2002; Chatterjee, Henzinger, and Piterman 2010; Mogavero et al. 2014). These powerful logics allow one to express individual or common goals of the agents throughout time, as well as specify the interactions among the agents (cooperation or adversarial). Model checking (Clarke and Emerson 1981; Queille and Sifakis 1981) specifications written in these logics allows one to verify the correct behavior of multi-agent systems using recently developed practical automatic tools (Lomuscio, Qu, and Raimondi 2009; Čermák et al. 2014; Čermák, Lomuscio, and Murano 2015).

A pioneering logic in this field is Alternating-Time Temporal Logic (ATL) and its fragments ATL (Alur, Henzinger, and Kupferman 2002), and CTL∗ (Emerson and Halpern 1986). ATL formulas are usually interpreted over concurrent game structures (CGS), which are labeled-state transition-systems with the ability of modeling the interaction among agents. For example, in a system with multiple agents and shared resources, the fact that a set of agents $A$ can ensure that, regardless of the actions of the other agents, every request to access a resource is eventually granted, can be expressed by the ATL∗ formula $\langle A \rangle \mathit{G}(\mathit{req} \rightarrow \mathit{grant})$.

A crucial shortcoming in real-life temporal-logic verification, deeply ingrained in the definition of linear-temporal logic (LTL), is that the satisfaction of a formula like $\mathit{grant}$ implies no a priori bound on when the grant occurs. I.e., the system may admit executions with longer and longer delays before the grant, and yet still satisfy the formula $\mathit{grant}$. Replacing the above formula with a formula specifying that the grant should occur within some fixed number of steps (say 3) is usually not an option, since one usually does not know the maximal delay that should be expected. This fact, that the $F$ operator of temporal-logics fails to capture the intuitive meaning of “within some bounded amount of time” has motivated the introduction of an extension of LTL, called “prompt-LTL”, in (Kupferman, Piterman, and Vardi 2009; Alur et al. 2001) that includes a new operator $F_P$ called “prompt eventually”. The semantics of $F_P \psi$ is such that it is satisfied only if there is some bound $k$, which is shared by all behaviours/computations of the system, such that whenever $F_P \psi$ should hold at some point along a computation then $\psi$ holds within at most $k$ steps. (Kupferman, Piterman, and Vardi 2009) goes on to show that prompt-LTL model checking is not more costly and slightly more complicated than LTL-model checking. It is important to note that prompt-LTL formulas are in positive normal form (i.e., with negations pushed all the way to the atoms), but it does not include the dual operator $G_P$ of the $F_P$ operator (however, the operator $G$ is included). As argued in (Kupferman, Piterman, and Vardi 2009), on the one hand $G_P$ is less useful than $F_P$ (its meaning is that there is some global bound $k$ such that whenever $G_P \psi$ should hold then $\psi$ holds for at least $k$ steps, and we do not care afterwards), and on the other hand adding it to the logic seriously complicates the decision procedures.

Since $ATL^*$ inherits its temporal operators from LTL it is natural to consider extending it with the $F_P$ operator, thus allowing one to specify that eventualities should not be delayed for an unbounded number of steps. We believe that, in a multi-agent setting, the need for the $F_P$ operator is ar-

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1It is an intriguing open question whether one can write in $ATL^*$ (or $CTL^*$) a formula that is equivalent to $F_P$. 

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guably even more natural than for closed single-agent systems (for which LTL is suited) as it allows one to specify that certain agents should not have the power to unboundedly delay other agents. Hence, we extend ATL* to include the \( F_p \) operator. We combine this with the extension of ATL* with epistemic operators that allow one to express what different agents know in a setting with imperfect information.

Reasoning about agents’ strategies in open system verification may require to act under partial information about the states of the system (Aminof, Murano, and Vardi 2007; Jamroga and Agotnes 2007; Aminof et al. 2013; Bulling and Jamroga 2014; Huang and van der Meyden 2014; Jamroga and Murano 2015). In many practical scenarios such as web-banking attacks, card games, market auctions, etc., agents have indeed a limited observability about the state content. One may think of states containing some private information that are visible only to a (possibly empty) subset of players (Reif 1984; Kupferman and Vardi 1997a). Each agent chooses his strategy based on what he can observe. To work with incomplete information systems, the syntax and the semantics of ATL* has been properly extended with epistemic operators (van der Hoek and Wooldridge 2002). The resulting logic is known as KATL*, sometimes simply called ATL* with imperfect information.

**Our contribution** We address the question of verifying prompt branching-time specifications in multi-agent systems for the first time. We present an extension of KATL* with the prompt eventually temporal operator, called Prompt-KATL*. We study the model-checking problem of this logic and some of its fragments. Namely, of Prompt-KATL (ATL with epistemic operators and prompt eventually), Prompt-KCTL* (CTL* with epistemic operators and prompt eventually), and finally the existential fragments of Prompt-KATL* and Prompt-KATL. We show that, for the case of perfect information, model-checking is decidable and not harder than for these logics without the prompt eventually operator. For the case of imperfect information, note that model checking of ATL*, and even ATL, over concurrent game structures with more than two agents and imperfect information is undecidable (Pnueli and Rosner 1989; Dima and Tiplea 2011)). Moreover, by (Vester 2013), this is already the case for the existential fragment of ATL. However, we show that the imperfect information case is always decidable for Prompt-KCTL*, and for Prompt-KATL* and Prompt-KATL it is decidable in the following two cases: (i) the players are constrained to memoryless strategies, or (ii) the players use memory-full but cooperative strategies and one restricts to the existential fragments of Prompt-KATL* and Prompt-KATL. Furthermore, in all cases, the complexity of our procedures is as good as for the non-prompt version of these logics.\(^2\)

**Related work** In (Alur et al. 2001), the authors introduce a parameterised extension of LTL in which the temporal operators are associated with variables in order to count the steps between successive occurrences of different events. A fragment of this logic is closely studied in (Kupferman, Piterman, and Vardi 2009), i.e., the extension of LTL by the prompt eventuality operator \( F_p \), called Prompt LTL. In (Almagor, Hirshfeld, and Kupferman 2010) the automata-theoretic counterpart of the \( F_p \) operator has been also introduced and studied.

Only recently has prompt LTL been studied outside the realm of closed systems. (Zimmermann 2013) studies two-player turn-based games of perfect information with respect to prompt LTL. (Chatterjee, Henzinger, and Horn 2009) lift the prompt semantics to \( \omega \)-regular games, under the parity winning condition, by introducing finitary parity games. They make use of the concept of “distance” between positions in a play that refers to the number of edges traversed in the game arena. The classical parity winning condition is then reformulated to take into consideration only those states occurring with a bounded distance. This idea has also been generalised to deal with more involved prompt parity conditions (Fijalkow and Zimmermann 2012; Mogavero, Murano, and Sorrentino 2013). Finally, from a practical point of view, one can see connections with bounded model checking of open systems. Indeed, a central question there is whether a model satisfies a given formula by looking at the model up to depth \( k \). We refer to (Huang, Luo, and Van Der Meyden 2011) for an overview.

Due space limitation, most of the proofs are just sketched.

**Definitions**

**Basic Notation** We denote the set of integers by \( \mathbb{Z} \), and write \( \mathbb{N}_0 := \mathbb{N} \cup \{0\} \). For a set \( \Gamma \), we write \( \Gamma^\omega \) (resp. \( \Gamma^* \)) for the set of infinite (resp. finite) sequences (also called words) of elements in \( \Gamma \), and \( \Gamma^+ \) for the non-empty finite sequences. We count positions in a sequence starting with 0, and write \( w_i \) for the \( i \)th element (called letter) of a word \( w \). The length (in \( \mathbb{N}_0 \cup \{\infty\} \)) of \( w \) is written \( |w| \). The suffix \( w_iw_{i+1} \cdots \) of \( w \) is written \( w_{\geq i} \) and \( w_{\leq i} \) is the prefix \( w_0 \cdots w_i \) of \( w \). We usually write \( a \) instead of the singleton set \( \{a\} \). Given \( n \in \mathbb{N} \), we consider a function \( f \) with domain \( \{1, \ldots, n\} \) as a vector with \( n \) coordinates. Thus we may write \( f = (f_1, \ldots, f_n) \), and use \( f(i) \) and \( f_i \) interchangeably.

**Game Structures** As for KATL*, models of Prompt-KATL* are Imperfect Information Concurrent Game Structures (iCGS), i.e., structures of the form \( S = \langle Ag, AP, Act, S, \lambda, \delta, \{\sim_a; a \in Ag\} \rangle \) where:

- \( Ag \) is a finite non-empty set of agents (also called players);
- \( AP \) is a finite non-empty set of atoms; \( Act \) is a finite non-empty set of actions for the agents;
- \( S \) is the set of states of the game structure;
- \( \lambda : S \to 2^{Ap} \) is a labeling function that assigns to a state \( s \) the set \( \lambda(s) \) of atoms that hold in that state;
- \( \delta : S \times Act^{Ag} \to S \) is a transition function that assigns to every state, and every choice of actions — one for each agent — a successor state;
and \( \sim_a \subseteq S \times S \) is an equivalence relation representing the imperfect information of agent \( a \), i.e., \( s \sim_a s' \) means that agent \( a \) cannot distinguish between \( s \) and \( s' \).

The equivalence-classes of \( \sim_a \) are called the observation sets of agent \( a \). The set \( Act^Ag \) is called the set of decisions. An iCGS is called finite if the set \( S \) is finite. An iCGS has perfect information if for every \( a \in Ag \) we have that \( \sim_a \) is the equality relation, i.e., if \( s \sim_a s' \) implies that \( s = s' \). In this case it may be written CGS.

Observe that we assume that all agents use the same set of actions \( Act \). However, it is sometimes convenient to assume that each agent \( a \) uses only some non-empty subset \( Act_a \subseteq Act \) of actions (one can simply define the transition relation to have all actions in \( Act \setminus Act_a \) duplicate the effect of some action in \( Act_a \)).

Computations and Strategies. A path in \( S \) is a finite or infinite sequence \( \pi_0 \pi_1 \ldots \in S^\omega \cup S^+ \) such that for all \( i \) there exists a decision \( d \in Act^Ag \) such that \( \pi_{i+1} = \delta(\pi_i, d) \).

We call finite paths histories, and infinite ones computations or plays. The set of computations in \( S \) is written \( cmp(S) \), and the set of computations in \( S \) that start with \( s \) is written \( cmp(S, s) \). We define \( hist(S) \) and \( hist(S, s) \), similarly.

A strategy (for a single agent) is a function \( \sigma : hist(S) \rightarrow Act \). For a non-empty set \( A \subseteq Ag \) of agents, and a strategy \( \sigma_A \) for each \( a \in A \), write \( \Sigma_A := \{ \sigma_a : a \in A \} \) for the set of strategies. A path \( h \) is consistent with \( \Sigma_A \) if it can be obtained by having the agents in \( A \) follow their strategies in \( \Sigma_A \), i.e., if for every position \( \pi_i \) of \( h \) there exists \( d \in Act^Ag \) such that: i) \( \pi_{i+1} = \delta(\pi_i, d) \); and ii) for every \( a \in A \), \( d(a) = \sigma_a(\pi_i) \). The set of computations starting with \( s \) that are consistent with \( \Sigma_A \), written \( out(s, \Sigma_A) \), is called the set of outcomes of \( \Sigma_A \) from \( s \).

For \( A \subseteq Ag \), we derive the following equivalence relations: \( \sim_a := \bigcap_{a \in A} \sim_a \), and \( \sim_A := (\bigcup_{a \in A} \sim_a)^* \), where \( ^* \) denotes the transitive closure (with respect to composition). Note that \( \sim_{\{a\}} = \sim_a \), and we use these interchangeably.

Extend \( \sim_A \) to histories point-wise: if \( h \) and \( h' \) are histories and \( h \sim_A h' \) and \( s \sim_A s' \) then \( hs \sim_A h's' \). A strategy \( \sigma \) is observational for agent \( a \) if for all \( h \sim_A h' \), we have \( \sigma(h) = \sigma(h') \). A set \( \Sigma_A \) of strategies for the agents in \( A \) is cooperatively observational if for all \( h \sim_A h' \) and \( a \in A \), we have \( \sigma_a(h) = \sigma_a(h') \).

2-Player Games. The proofs of Propositions 3 and 4 use the following notions. A 2-player concurrent game is a pair \( (S, \Upsilon) \) where \( S \) is an iCGS (called an arena) with two players (usually \( Ag = \{0, 1\} \), and \( \Upsilon \subseteq cmp(S) \) is a goal. A play \( \pi \in cmp(S) \) is won by Player 0 if \( \pi \in \Upsilon \), and is won by Player 1 otherwise. Given \( s \in S \), an observational strategy \( \sigma_i \) for Player \( i \in \{0, 1\} \) is called winning from \( s \) (and we say that Player \( i \) wins from \( s \)) if all plays starting in \( s \) that are consistent with \( \sigma_i \) are won by Player \( i \) (i.e., if \( out(s, \sigma_i) \subseteq \Upsilon \)). An LTL formula \( \psi \) induces a goal \( \Upsilon := \{ \pi \in cmp(S) \mid \psi \models \psi \} \), and we usually just say that the game has goal \( \psi \).

If Player 0 wins from \( s \) in the game with goal \( \psi \) we say that he can enforce \( \psi \) from \( s \).

Syntax of Prompt-KATL*.

Fix a finite set of atomic propositions (atoms) \( AP \), and a finite set of agents \( Ag \). The Prompt-KATL* state (\( \varphi \)) and path (\( \psi \)) formulas over \( AP \) and \( Ag \) are built using the following context-free grammar:

\[ \varphi ::= p \mid \neg \varphi \mid \varphi \lor \varphi \mid \varphi \land \varphi \mid ⟨⟨ A ⟩⟩ \psi \mid [[A]] \psi \mid \exists a \varphi \mid \nabla_A \varphi \mid C_A \varphi \mid \nabla_A \varphi \mid \nabla_A \varphi \mid \nabla_A \varphi \]

where \( p \) varies over \( AP \), \( A \) varies over subsets of \( Ag \), and \( a \) over elements of \( Ag \). The class of Prompt-KATL* formulas is the set of state formulas generated by the grammar.

The temporal operators are \( X \) (next), \( U \) (until), \( R \) (releases, the dual of until) and \( F_p \) (prompt eventually); the strategy quantifiers are \( \langle A \rangle \), \( [[A]] \), where \( \langle A \rangle \psi \) is read “the agents in \( A \) can enforce \( \psi \)”, and its dual \( [[A]] \psi \) is read “the agents in \( A \) cannot avoid \( \psi \)”; and the epistemic operators are \( \nabla_a \) (agent \( a \) knows that), \( D_A \) (the agents in \( A \) distributively know that), and \( C_A \) (amongst the agents in \( A \) it is common knowledge that), as well as the dual operators \( \nabla_a \), \( D_A \), \( C_A \).

Observe that, as discussed in the introduction, we do not include a dual operator to \( F_p \). Also note that if one removes the prompt eventually operator then the grammar above generates exactly all the formulas of KATL* in positive normal form (i.e., with negations pushed all the way to the atoms).

We have the usual syntactic sugar: we write \( F (\varphi) \) (eventually) instead of true \( U \varphi \); (globally) \( G \varphi \) instead of false \( R \varphi \); \( E \) (exists) instead of \( \langle A \rangle \psi \), and \( A \) (for all) instead of \( [[A]] \psi \); \( I \) (everybody knows) instead of \( \nabla_{a \in Ag} \nabla_a \psi \) (and its dual \( E \) for \( \forall a \in Ag \nabla_a \nabla_a \psi \)). Finally, we use a shorthand for repeated next: \( X^k \) (for \( k \in \mathbb{N} \)) is defined as: \( X^1 := X \), and \( X^{k+1} := XX^k \).

We now define some important syntactic fragments.

1. Prompt-ATL* formulas consists of the formulas of Prompt-KATL* in which no epistemic operator occurs.
2. Prompt-ATL formulas consists of the formulas of Prompt-ATL* in which every temporal operator is immediately preceded by a strategy quantifier.
3. Prompt-KCTL* is obtained from Prompt-KATL* by only allowing strategy quantifiers of the form \( E a \) and \( A \).
4. Prompt-LTL is the class of path formulas generated by the grammar above in which no strategy quantifier or epistemic operator appears.
5. The existential fragments of Prompt-KATL* and Prompt-KATL consist of those formulas in which the \( [[A]] \) quantifier does not occur.

Semantics of Prompt-KATL*.

We first define the semantics of KATL* (i.e., formulas that don’t mention the prompt operator \( F_p \)), and then define the semantics of Prompt-KATL*.

More formally, require that for every agent \( a \), there is some action \( \alpha \in Act_a \), such that for every \( d \in Act^Ag \) and \( s \in S \), we have: if \( d(a) \in Act \setminus Act_a \) then \( \delta(s, d) = \delta(s, d') \), where \( d' \) is obtained from \( d \) by letting \( d'(a) = \alpha \).

Agents using cooperative strategies are sometimes referred to as having distributive knowledge.

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Semantics of KATL*. The satisfaction relation $|=|$ is defined inductively, as usual. Formally, for all $s \in S, p \in AP$:
- $(s, s) |= p$ iff $p \in \lambda(s)$, and $(s, s) |= \neg p$ iff $p \notin \lambda(s)$.
- $(s, s) |= \varphi_1 \land \varphi_2$ iff $(s, s) |= \varphi_1$ and $(s, s) |= \varphi_2$.
- $(s, s) |= \varphi_1 \lor \varphi_2$ iff $(s, s) |= \varphi_1$ or $(s, s) |= \varphi_2$.
- $(s, s) |= \langle\langle A \rangle\rangle \psi$ iff there exists a set of observational strategies $\Sigma_A$, one for each agent $A$, s.t. $(s, \pi) |= \psi$ for all $\pi \in out(s, \Sigma_A)$.
- $(s, s) |= [\langle A \rangle] \psi$ iff for every set of observational strategies $\Sigma_A$, one for each agent $A$, there is a computation $\pi \in out(s, \Sigma_A)$ s.t. $(s, \pi) |= \psi$.
- $(s, s) |= \check{K}_a \psi$ (resp. $\check{\exists}_a \varphi$) iff $(s, s') |= \varphi$ for every $s'$ (resp. some $s'$) s.t. $s' \sim_A s$.
- $(s, s) |= \check{D}_A \varphi$ (resp. $\check{\forall}_A \varphi$) iff $(s, s') |= \varphi$ for every $s'$ (resp. some $s'$) s.t. $s' \sim_A s$.
- $(s, s) |= \check{C}_A \varphi$ (resp. $\check{\exists}_A \varphi$) iff $(s, s') |= \varphi$ for every $s'$ (resp. some $s'$) s.t. $s' \sim_A s$.

and for all $\pi \in cmp(S)$:
- $(s, \pi) |= \psi$, for $\psi$ a state formula, iff $(s, \pi_0) |= \psi$.
- $(s, \pi) |= \psi_1 \land \psi_2$ iff $(s, \pi) |= \psi_1$ and $(s, \pi) |= \psi_2$.
- $(s, \pi) |= \psi_1 \lor \psi_2$ iff $(s, \pi) |= \psi_1$ or $(s, \pi) |= \psi_2$.
- $(s, \pi) |= X \psi$ iff $(s, \pi_{\pi_2}) |\sim \psi$.
- $(s, \pi) |= \psi_1 U \psi_2$ iff there $i \in \mathbb{N}$ such that $(s, \pi_{\pi_2}) |= \psi_2$ and for all $j < i$, $(s, \pi_{\pi_2}) |= \psi_1$.
- $(s, \pi) |= \psi_1 R \psi_2$ iff for all $i$, either $(s, \pi_{\pi_2}) |= \psi_2$ or there exists $j < i$ such that $(s, \pi_{\pi_2}) |= \psi_1$.

We emphasise two points: strategies have perfect recall, and epistemic operators depend only on the current state and not on the history (the latter appears, e.g., in (Čermák et al. 2014)).

Semantics of Prompt-KATL*. For $k \in \mathbb{N}$, we use the notation $(S, s) \models_k \varphi$ and $(S, \pi) \models_k \psi$ to denote that the formula is satisfied in which every prompt eventuality is fulfilled within at most $k$ steps. Formally: define $(S, s) \models_k \varphi$ and $(S, \pi) \models_k \psi$ inductively, as above, with the following additional rule:
- $(S, \pi) \models_k F_p \psi$ iff there exists $j \leq k$ such that $(S, \pi_{\pi_2}) \models_k \psi$. Say that $\pi$ models $\psi$ with bound $k$.

Definition 1. For a Prompt-KATL* state-formula $\varphi$ and $s \in S$, define $(S, s) \models \varphi$ if there exists $k \in \mathbb{N}$ such that $(S, s) \models_k \varphi$. Say that $\varphi$ is satisfied at $s$.

Linearising branching-formulas and Prompt-LTL

Like CTL*, and ATL* after it, one can think of a Prompt-KATL* path formula $\psi$ over atoms AP as a Prompt-LTL formula $lin(\psi)$ over atoms which are the maximal state subformulas of $\psi$, as follows (Kupferman, Vardi, and Wolper 2000).

A formula $\varphi$ is a state subformula of $\psi$ if $\varphi$ is a state formula as well as a subformula of $\psi$. A formula $\varphi$ is a maximal state subformula of $\psi$ if $\varphi \neq \psi$, it is a state subformula of $\psi$, and it is not a proper subformula of any other state subformula of $\psi$. Let $max(\psi)$ be the set of maximal state subformulas of $\psi$.

Every Prompt-KATL* path formula $\psi$ can be viewed as a Prompt-LTL formula, call it $lin(\psi)$, whose atoms are elements of $max(\psi)$.

Definition 2. For a path formula $\psi$, define $lin(\psi)$ as follows — in each case note that $lin(\psi)$ is a formula over atoms $max(\psi)$:
- $lin(p) ::= p$ and $lin(\neg p) ::= \neg p$;
- For $\varphi \in \{\langle A \rangle, \neg A, K_a, D_A, \exists_A\}$, $lin(\varphi) ::= \varphi$;
- If $\psi = \varphi_1 \land \varphi_2$ for $\varphi \in \{\lor, \land\}$, then $lin(\varphi_1) \land lin(\varphi_2)$ is defined to be $lin(\varphi_1) \land lin(\varphi_2)$ (note this is well defined since at least one $\varphi_i$ is not a state formula);
- For $\varphi \in \{X, F_p\}$, $lin(\varphi) ::= olin(\varphi)$;
- For $\varphi \in \{U, R\}$, $lin(\varphi_1 \land \varphi_2) ::= lin(\varphi_1) \land lin(\varphi_2)$.

For example, if $\psi = (p U (\langle A \rangle F_p \varphi) \lor X \neg p)$, then its state subformulas are $\{\langle A \rangle F_p \varphi, q, \neg p\}$, and $max(\psi) = \{\langle A \rangle F_p \varphi, q, \neg p\}$, and thus $lin(\psi)$ is the Prompt-LTL formula $\langle\langle \langle A \rangle \rangle F_p \varphi \rangle \rangle F_p q \lor X \neg p$ over the atoms $max(\psi)$ (for illustration we box the subformulas that are treated as atoms).

For an iCGS $S := \langle Ag, AP, Act, S, \lambda, \delta, \{\ast \subseteq a \subseteq Ag\}\rangle$, and a Prompt-KATL* path formula $\psi$ over AP, define the iCGS $S_\psi := \langle Ag, max(\psi), Act, S, \lambda, \delta, \{\ast \subseteq a \subseteq Ag\}\rangle$ with atoms $max(\psi)$ and a labeling $\lambda_\psi : S \rightarrow max(\psi)$ defined by letting $\varphi \in \lambda_\psi(s)$ iff $(S, s) |= \varphi$. The next lemma says that a computation $\pi$ of $S$ satisfies $\psi$ iff, when viewed as a computation of $S_\psi$, it satisfies $lin(\psi)$:

Lemma 1. For every iCGS $S$, Prompt-KATL* path formula $\psi$ over atoms AP, and computation $\pi$ of $S$, we have that $(S, \pi) |= \psi$ if and only if $(S_\psi, \pi) |= lin(\psi)$.

Proof. The proof is by induction on the structure of $\psi$, using the following inductive hypothesis on the subformulas $\phi$ of $\psi$ (that are not proper subformulas of any formula in $max(\psi)$): for every position $i \in \mathbb{N}$ of $\pi$, we have that $(S, \pi_{\pi_2}) |= \phi$ iff $(S_\psi, \pi_{\pi_2}) |= lin(\phi)$.

Prompt linear-temporal logic.

We now consider Prompt-LTL in more detail.

Notation. For a path $\pi \in cmp(S)$ and a Prompt-LTL formula $\psi$, write $\pi |= \psi$ instead of $(S, \pi) |= \psi$. Also, if $\psi \in (2^AP)^{\omega}$ then we abuse notation and write $\psi \models \psi$.

Recall from the definitions of syntax that we define Prompt-LTL as the syntactic fragment of Prompt-KATL* in which $\langle\langle A \rangle\rangle$ and $[\langle A \rangle]$ do not occur. Thus, if $\psi$ is a Prompt-LTL formula and $S$ is a CGS, then $(S, s) |= A \psi$ expresses that there is a bound $k \in \mathbb{N}$ such that for every $\pi \in cmp(S)$ starting in $s$ we have that $(S, \pi) \models_k \psi$.

Remark: the syntax in (Kupferman, Piterman, and Vardi 2009) is different. There they write, e.g., $S \models \psi$ instead of $S \models A \psi$.  

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Proposition 1. (Kupferman, Piterman, and Vardi 2009) The following problem is decidable: given Prompt-LTL formula ψ, a finite CGS S, and a state s ∈ S, decide whether (S, s) |= Aψ. Moreover, the complexity is PSPACE in the size of ψ and NLOGSPACE in the size of S.

The previous proposition allows us to deal with universal quantifiers. We now deal with the existential quantifier.

Definition 3. If ψ is a Prompt-LTL formula, define live(ψ) as the LTL formula that results from ψ by replacing every F_p by F.

The following lemma says that (S, s) |= E ψ if and only if (S, s) |= E live(ψ). The reason, roughly, is as follows. For the forward direction, note that if ψ holds promptly on a computation, then in particular, it holds eventually (this is because our formulas are in positive-normal form and so the prompt eventualities cannot be negated). For the reverse direction, use the fact that if S has a computation satisfying live(ψ) then it has such a computation that is a lasso, i.e., of the form uw^ω (this holds for all LTL formulas, and thus live(ψ) in particular, (Vardi and Wolper 1994)). In this case, every subformula of ψ that holds in a suffix π_{2j} of π (with j ≥ |u| + |v|) also holds in the suffix π_{2j−|v|}. Thus all eventualities hold promptly (i.e., within |u| + |v| steps). The formal proof is in the full version of the paper.

Lemma 2. If π = uw^ω is a lasso, and ψ is a Prompt-LTL formula, then the following are equivalent:

a) π |= ψ,

b) π |= live(ψ),

c) π |= bψ where b = |u| + |v|.

Proof. Clearly c) → a). For a) → b) an easy induction on ψ shows that for every computation π, and every k ∈ N, if π |= bψ then π |= live(ψ). For b) → c) we prove, by induction on ψ, that for all i ∈ N, and all subformulas ψ' of ψ: if π_{≥i} |= live(ψ') then π_{≥i} |= bψ' (to complete the proof take i = 0 and ψ' = ψ). The only non-trivial case is ψ' = F_p ψ_1. Thus, suppose π_{≥i} |= live(F_p ψ_1). Then π_{≥i} |= F live(ψ_1), and so there exists j ≥ i such that π_{≥j} |= F live(ψ_1). There are two cases.

Suppose j < |u|. By induction, π_{≥j−1} |= b ψ_1, and so π_{≥i} |= max(j−i, b) F_p ψ_1. Since j < i < |u| − i < b, we have that π_{≥i} |= b F_p ψ_1, as required.

Suppose j ≥ |u|. Pick n_0 so that max{i, |u|} ≤ j − |v|n_0 ≤ max{i, |u|} + |v|. Since π = uw^ω, we have that π_{≥j−|v|n_0} |= live(ψ_1), and so by induction π_{≥j−|v|n_0} |= b ψ_1, and so π_{≥i} |= max(j−|v|n_0−i, b) F_p ψ_1. Since j − |v|n_0 − i ≤ max{i, |u|} + |v| − i ≤ |u| + |v| = b, we have π_{≥i} |= b F_p ψ_1, as required.

We now get:

Proposition 2. For every Prompt-LTL formula ψ, CGS S, and state s ∈ S, we have that (S, s) |= E ψ if and only if (S, s) |= E live(ψ). Moreover, the cost of checking this fact is PSPACE in the size of ψ and NLOGSPACE in the size of S.

Proof. Since live(ψ) is an LTL formula, we have that (S, s) |= E live(ψ) if and only if there is a lasso π = uw^ω starting in s such that (S, π) |= live(ψ) (Vardi and Wolper 1994). By Lemma 2 this is equivalent to (S, π) |= bψ where b = |u| + |v|. By definition of |= b, this is equivalent to (S, s) |= E ψ. The complexity follows from the model-checking complexity of LTL (Sistla and Clarke 1985; Kupferman, Piterman, and Vardi 2009).

The final lemma of this section will be used in the proof of Proposition 4 (used as part of Theorem 4 on co-operative strategies). The lemma relies on the alternating–color technique from (Kupferman, Piterman, and Vardi 2009), that we now describe. Given a computation π of an iCGS S, add a new atomic proposition red, and imagine that states in which red holds are colored red, and otherwise white. Given a Prompt-LTL formula ψ, derive from it an LTL formula col(ψ) by replacing every subformula of the form F_p ψ with a subformula that says that ψ holds before the color changes twice. Now, if we can come up with a coloring of π in which the colors alternate fast enough (say every k steps or less), such that the colored version of π models col(ψ), then we can deduce that π models ψ with bound 2k; conversely, if ψ models ψ with bound k, then by changing colors every k steps we can ensure that col(ψ) is satisfied. This allows us to replace reasoning about Prompt-LTL formulas with reasoning about colorings and LTL formulas. We now formally describe the required elements for applying this reasoning.

For w ∈ (2AP∪{red})^ω, a block is a maximal subword w_1 · · · w_i of w such that red ∈ w_i for all l ∈ [i, j) or red ∉ w_i for all l ∈ [i, j]. For k ∈ N, say that w is k-coloured if the size of every block of w has length at most k. Say that w ∈ (2AP∪{red})^ω is a colouring of v ∈ (2AP)^ω if and only if for every i, w_i ∩ AP = v_i. Say that w is a k-colouring of v if w is a colouring of v and is k-coloured.

For a Prompt-LTL formula ψ, let ψ' be the LTL formula obtained by replacing every subformula of ψ, of the form F_p ψ, by within(ψ) := (red U (¬red ω) ∨ (¬red U (red ω))), which states that ψ should hold at some point within this color block or the next. Let col(ψ) := ψ' ∧ ρ, where ρ := GF(red ∧ X¬red ω) states that the colors change infinitely often. It is important to note that while col(ψ) is exponentially larger than ψ, the number of subformulas it has is linear in the number of subformulas of ψ.

Lemma 3. For every Prompt-LTL formula ψ, word v ∈ (2AP)^ω and k ∈ N: (i) if there is a k-colouring w of ω such that w |= col(ψ) then v |= 2k ψ; (ii) if v |= k ψ then there is a k-colouring w of ω such that w |= col(ψ), moreover, w can be chosen with all blocks of size exactly k.

Proof. Although this lemma is easily extractable from (Kupferman, Piterman, and Vardi 2009), for completeness we provide the proof. For (i), given a k-colouring w of ψ, if w |= col(ψ) then the result (that w |= 2k ψ) follows by induction on ψ and the following observation: for every i ∈ N and every subformula F_p φ of ψ, we have w_{2i+1} |= within(φ) implies that v_{2i+1} |= 2k F_p φ.

For (ii), if v |= k ψ then colour v by blocks of size exactly k to get w. The result (that w |= col(ψ)) follows by induc-
tion on ψ and the following observation: for every i ∈ N and every subformula F_p ϕ of ψ, we have v_{≥i} ⊨^k F_p ϕ implies that w_{≥i} ⊨ within(ϕ).

Deciding the model-checking problems

The model-checking problem for a logic L is the following: given a formula ϕ from L and a finite iCGS S, decide whether S ⊨ ϕ.

Fact 1. Model checking ATL* is undecidable over concurrent game structures with |Ag| ≥ 3, and imperfect information, already for the existential fragment (Pnueli and Rosner 1989; Dima and Tiplea 2011).

Thus, also model checking Prompt-KATL* is undecidable for three or more agents with imperfect information. We show that one can regain decidability (and we provide the optimal complexity) in four ways: (i) restricting to iCGS with perfect information, or (ii) restricting to path quantifiers (instead of strategy quantifiers), or (iii) restricting to memoryless strategies, or (iv) restricting to cooperative strategies and the existential fragment.

Prompt-ATL* with perfect information

Proposition 3. The following problems are decidable: given a Prompt-LTL formula ψ, a finite (perfect-information) CGS S, a set of agents A ⊆ Ag, and state s ∈ S, decide whether (S, s) ⊨ ⟨⟨A⟩⟩ψ; and, similarly, decide whether (S, s) ⊨ ⋁_i<k [[A]]_iψ. Moreover, the complexity of these problems is 2EXPTIME in the number of subformulas of ψ, and polynomial in the size of S.

Proof. We will reduce each question to the problem of deciding if a given player has a winning strategy in Prompt-LTL turn-based games. The latter are solvable in 2EXPTIME (Zimmermann 2013).

We first consider the case of ⟨⟨A⟩⟩ψ. In the first step, build a two-player arena G such that (⋆): (S, s) ⊨ ⟨⟨A⟩⟩ψ if and only if there exists k ∈ N such that Player 0 can enforce (in G) the LTL goal ψ_s from s, where ψ_s is formed from ψ by replacing every subformula of the form F_p ϕ by ∨_{i≤k} X^i ϕ. The idea is that Player 0 corresponds to the coalition A and Player 1 to the coalition Ag \ A. In more detail: S = ⟨⟨Ag⟩⟩, AP, Act, S, λ, δ, define G to be the (perfect information) CGS with two agents, Player 0 and Player 1, ⟨⟨0, 1⟩⟩, AP, Act_0 ⊔ Act_1, S, λ, δ_G⟩ as follows:

- action sets Act_0 := Act^Ag and Act_1 := Act^Ag \ A;
- state set S, labeling λ;
- transition function δ_G : S × Act_0 × Act_1 → S that maps (s, (d_1, d_2)) → δ(s, d_1 ⊗ d_2) where d_1 ⊗ d_2 ∈ Act^Ag maps a to d_1(a) for a ∈ A and otherwise (for a ∈ Ag \ A) to d_2(a).

It is immediate from the definitions (of G and |=) that (⋆) holds. For the next step, recall the following folk fact (†): Player 0 has a winning strategy in a concurrent game G with goal T if and only if Player 0 has a winning strategy in the turn-based game G^{tb} which simulates G as follows: first, Player 0 moves (thus revealing his chosen action) and then Player 1 chooses his action and the simulated move is completed. To “skip” the intermediate nodes that G^{tb} introduces, we use the goal Υ^{tb} which is defined to be the set of all computations such that the subsequence consisting of only the even positioned nodes is in Υ. The intuitive reason that this lemma is true is that in the game G^{tb} Player 0 has no new information available to it, and thus it is exactly as hard (or easy) for him to win as in G.

Formally, we build the turn-based game G^{tb} of perfect information with goal Υ^{tb} as follows. Let G^{tb} be the 2-player game, over atoms AP, and such that:

- the state set S^{tb} = S ⊔ (S × Act_0);
- the labeling function λ^{tb} maps s → λ(s) and (s, d) → 0;
- the transition function δ^{tb} : S^{tb} × Act_0 × Act_1 → S^{tb} maps state s ∈ S and actions (d_0, d_1) to (s, d_0), and maps a state (s, d_0) ∈ S × Act_0 and actions (d'_0, d'_1) to δ(s, d_0 ⊗ d_1) for all s ∈ S, d_0, d'_0 ∈ Act_0 and d_1 ∈ Act_1.

Moreover, Υ^{tb} consists of those plays whose subsequence consisting of every other node satisfies ψ_k.

Thus, we have reduced our problem to deciding if there exists k ∈ N such that Player 0 has a winning strategy in the game G^{tb} with goal Υ^{tb}. The latter is the problem of solving a two-player turn-based Prompt-LTL game, which, by (Zimmermann 2013), can be done in 2EXPTIME. Moreover, that procedure is already able to deal with goals which only look at every second node of the play (so called “blinking semantics”). Thus, the procedure can be easily adapted to decide if there exists k ∈ N such that Player 0 has a winning strategy in the game G^{tb} with goal Υ^{tb}, which completes the ⟨⟨A⟩⟩ψ case.

We turn to the [[A]]_iψ case. Dually to the case above, build a two-player game H by associating the coalition A with Player 1 and associating the coalition Ag \ A with Player 0. Thus, (S, s) ⊨ [[A]]_iψ if and only if there exists k ∈ N such that Player 1 does not have a winning strategy in the game H with goal ψ_k. Dually again, build H^{tb} in which Player 1 moves first and use the fact (†) to deduce that, for every k ∈ N, Player 1 does not have a winning strategy in H with goal ψ_k if and only if Player 1 does not have a winning strategy in the game H^{tb} with goal Υ^{tb}. Since turn-based games of perfect information with LTL goals (the “blinking semantics” is easily accommodated) are determined (i.e., one of the players has a winning strategy), then this is equivalent to Player 0 having a winning strategy in the game G^{tb} with goal Υ^{tb}. But this is the same type of game we had already solved in the case of ⟨⟨A⟩⟩ψ. This completes the [[A]]_iψ case.

Theorem 1. The model checking problem for Prompt-ATL* (resp. Prompt-ATL) is 2EXPTIME-complete (resp. PTIME-complete).

Proof. By (Alur, Henzinger, and Kupferman 2002), the lower bound already holds for ATL* (resp. ATL). For the upper bound, we adapt the marking algorithm for model checking ATL* (Alur, Henzinger, and Kupferman 2002).

However, Player 1 can win in G^{tb} from states in which he can only prevent Player 0 from winning in G, but not ensure he himself wins.

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Let \( \varphi \) be a state Prompt-\( \text{ATL}^* \) formula, and mark every state \( s \in S \) by the state subformulas \( \varphi' \) of \( \varphi \) that are satisfied at \( s \). This is done inductively. The case that \( \varphi' \) is an atom, a negation of an atom, a disjunction or a conjunction is immediate (e.g., if \( \varphi' = \varphi_1 \lor \varphi_2 \) and \( s \) is marked by \( \varphi_1 \) then also mark \( s \) by \( \varphi_1 \lor \varphi_2 \)).

The case that \( \varphi' = \langle \langle A \rangle \rangle \psi \) is dealt with as follows. Recall that \( \text{lin}(\psi) \) is a Prompt-\( \text{LTL} \) formula over atoms \( \text{max}(\psi) \). By Lemma 1, deciding if \( (S, s) \models \langle \langle A \rangle \rangle \psi \) is equivalent to deciding if \( (S_0, s) \models \langle \langle A \rangle \rangle \text{lin}(\psi) \). By induction, the satisfaction of all state subformulas of \( \psi \) have already been determined. In particular, we have enough information to form \( A^* \).

By Proposition 3, deciding whether \( (S_0, s) \models \langle \langle A \rangle \rangle \text{lin}(\psi) \) can be done in \( \text{2EXPTIME} \). The case that \( \varphi' = \langle A \rangle \psi \) is similar.

The case of Prompt-\( \text{ATL} \) follows by noting that the formula \( \text{lin}(\psi) \) is always of constant size since Prompt-\( \text{ATL} \) does not allow temporal operators to be directly nested.

**Prompt-\( \text{KCTL}^* \)**

In the case of Prompt-\( \text{KCTL}^* \), the strategy quantifiers are restricted to \( \text{E}, \text{A} \). This inherent restriction on the strategy quantifiers results in a decidable model checking problem, also in the presence of imperfect information.

**Theorem 2.** Model checking Prompt-\( \text{KCTL}^* \) is \( \text{PSPACE-complete} \), and the structure complexity is \( \text{PTIME-complete} \).

**Proof.** The lower-bounds already hold for \( \text{CTL}^* \) (Kupferman, Vardi, and Wolper 2000). For the upper-bounds, let \( \varphi \) be a state Prompt-\( \text{KCTL}^* \) formula, and mark every state \( s \in S \) by the state subformulas \( \varphi' \) of \( \varphi \) that are satisfied at \( s \). This is done inductively as in Theorem 1. The cases we have to consider are the epistemic operators and the complexity of the path quantifiers. The case that \( \varphi' \) is of the form \( \mathbb{K}_a \phi \) is dealt with as follows: mark \( s \) by \( \mathbb{K}_a \phi \) if every state \( s' \sim_a s \) is already marked by \( \phi \); similarly, mark \( s \) by \( \mathbb{D}_a \phi \) if every state \( s' \sim_a s \) is already marked by \( \phi \) (the remaining epistemic operators, including the duals, are similar). Each of these steps can be performed in time polynomial in \( S \).

We now discuss the case that \( \varphi' \) is of the form \( \text{E} \psi \) or \( \text{A} \psi \). Recall that \( \text{lin}(\psi) \) is a Prompt-\( \text{LTL} \) formula over atoms \( \text{max}(\psi) \), and that \( \text{live}(\cdot) \) replaces every \( F \beta \) by \( F \) in a Prompt-\( \text{LTL} \) formula. Thus, \( \text{live}(\text{lin}(\psi)) \) is an LTL formula over atoms \( \text{max}(\psi) \). By induction, the satisfaction of all state subformulas of \( \psi \) has already been determined. In particular, we have enough information to form the CGS \( S_0 \) (note that since \( S \) is a CGS, so is \( S_0 \)). By Lemma 1 note that (1): for \( Q \in \{ \text{E}, \text{A} \} \), \( (S_0, s) \models \langle A \rangle \psi \) if and only if \( (S_0, s) \models \text{lin}(\psi) \).

For \( \varphi' = \langle A \rangle \psi \) mark \( s \) by \( \text{A} \psi \) if and only if \( (S_0, s) \models \text{lin}(\psi) \). To see this is correct use (1).

For \( \varphi' = \text{E} \psi \) mark \( s \) by \( \text{E} \psi \) if and only if \( (S_0, s) \models \text{lin}(\psi) \). To see that this is correct use (1). Proposition 2, and the fact that \( \text{lin}(\psi) \) is a Prompt-\( \text{LTL} \) formula over atoms \( \text{max}(\psi) \).

For the complexity, note that i) there are only a linear number of subformulas \( \varphi' \) of \( \varphi \), and ii) each case \( \varphi' \) of the algorithm can be done in \( \text{PSPACE} \) in the size of \( \varphi' \) and \( \text{NLOGSPACE} \) in the size of \( S \) (for the \( \varphi' = \text{A} \psi \) case use Proposition 1, and for the \( \varphi' = \text{E} \psi \) case use the fact that model checking LTL is in \( \text{PSPACE} \) (Sistla and Clarke 1985)).

**Prompt-\( \text{KATL}^* \) with memoryless strategies**

In this section we prove that model checking Prompt-\( \text{KATL}^* \) with memoryless strategies is \( \text{PSPACE-complete} \). We begin with the relevant definitions.

A strategy \( \sigma \) is called memoryless if for all histories \( h, h' \), and every state \( s \) we have \( \sigma(hs) = \sigma(h's) \). It is thus common to consider memoryless strategies as functions from states (not histories) to actions.

Define \( \models_{\text{mem}}^k \), like \( \models^k \) except replace the definition of the semantics of the strategy quantifiers to limit the agents to memoryless (observational) strategies as follows:

- \( (S, s) \models_{\text{mem}}^k \langle A \rangle \psi \) (resp. \( [\langle A \rangle \psi] \)) iff there exists a set \( \psi \) (resp. for all sets \( \Sigma_A \)) of memoryless observational strategies, one strategy for each agent in \( A \), such that for all (resp. for at least one) computation \( \pi \in \text{out}(s, \Sigma_A) \), we have \( (S, \pi) \models_{\text{mem}}^k \psi \).

Define \( (S, s) \models_{\text{mem}} \psi \) if there exists \( k \in \mathbb{N} \) such that \( (S, s) \models_{\text{mem}}^k \psi \).

**Theorem 3.** The following problem is \( \text{PSPACE-complete} \): given a Prompt-\( \text{KATL}^* \) formula \( \varphi \) and a finite iCGS \( S \), decide whether \( S \models_{\text{mem}} \varphi \).

**Proof.** The lower-bound already holds for \( \text{CTL}^* \) (Kupferman, Vardi, and Wolper 2000). For the upper bound, we use the marking algorithm as we did in Theorem 1. We show how to deal with the existential strategy quantifier (the universal quantifier is symmetric). Apply Lemma 1 and reduce \( (S, s) \models_{\text{mem}}^k \langle A \rangle \psi \) to \( (S_0, s) \models_{\text{mem}} \langle A \rangle \text{lin}(\psi) \). To solve the latter, observe the following: i) each agent has finitely many observational memoryless strategies (each of polynomial size) in \( S \), ii) instantiating a set \( \Sigma_A \) of such strategies, one for each agent in \( A \), results in a sub-area \( S' \) of \( S_0 \) in which we have to check whether \( (S', s) \models \text{A lin}(\psi) \). By Proposition 1, each step ii) can be done in \( \text{PSPACE} \). Thus one can, in \( \text{PSPACE} \), search for a set of observational memoryless strategies \( \Sigma_A \) such that ii) holds.

**Existential Fragment of Prompt-\( \text{KATL}^* \) with co-operative strategies**

In this section we prove that model checking the existential fragment of Prompt-\( \text{KATL}^* \) with co-operative strategies is \( \text{2EXPTIME-complete} \). We begin with some definitions.

Define \( \models_{\text{co}}^k \), like \( \models^k \) except replace the definition of the semantics of the strategy quantifiers \( \langle A \rangle, [\langle A \rangle] \) as follows:

- \( (S, s) \models_{\text{co}}^k \langle A \rangle \psi \) (resp. \( [\langle A \rangle \psi] \)) iff there exists a set (resp. for all sets of \( \Sigma_A \)) of strategies, one strategy for each agent in \( A \), that is co-operatively observational, such that for all computations (resp. for at least one computation) \( \pi \in \text{out}(s, \Sigma_A) \), we have \( (S, \pi) \models_{\text{co}}^k \psi \).

One can also define a “uniform” semantics in which also the agents not quantified over (i.e. the ones not in \( A \)) are limited to memoryless strategies. Our techniques can be easily adapted also to this uniform semantics.
Define \((S, s) \models_{co} \varphi\) iff there exists \(k \in \mathbb{N}\) such that \((S, s) \models_k \varphi\).

**Theorem 4.** The model-checking problem for the existential fragment of Prompt-KATL\(^*\) (resp. Prompt-KATL with cooperative strategies) is \(2\text{EXPTIME}\)-complete (resp. in \(\text{PTIME}\)).

The lower bound holds already for the existential fragment of ATL\(^*\) (see the proof of Theorem 5.6 in (Alur, Henzinger, and Kupferman 2002)). For the upper bound, the proof proceeds as in previous constructions, using the marking algorithm from the proof of Theorem 1. The interesting case is the strategy quantifier \(\langle A \rangle\), which is dealt with by the following proposition.

**Proposition 4.** The following problem is decidable: given a Prompt-LTL formula \(\psi\), a finite iCGS \(S\), a set of agents \(A \subseteq Ag\), and state \(s \in S\), decide whether \((S, s) \models_{co} \langle A \rangle \psi\). Moreover, this can be done in \(2\text{EXPTIME}\) in the number of subformulas of \(\psi\), and polynomial time in the size of \(S\).

**Proof.** The outline of the proof is as follows: we build a two-player arena \(G\) such that \((\ast)\): \((S, s) \models_{co} \langle A \rangle \psi\) if and only if there exists \(k \in \mathbb{N}\) such that Player 0 has a strategy in \(G\) that enforces the LTL goal \(\psi_k\) from \(s\) (as in Proposition 3, \(\psi_k\) is formed from \(\psi\) by replacing every subformula of the form \(F_P \phi\) by \(\bigvee_{i \in k} X_i^r \phi\)). The idea is that Player 0 corresponds to the coalition \(A\) and Player 1 to the coalition \(Ag \setminus A\). We are justified in treating all players in \(A\) as a single player by our assumption of cooperative strategies for them (the antagonists in \(Ag \setminus A\) are always treated as a single player since all of their strategies have to be defeated).

We then modify \(G\) to obtain a new arena \(G'\), which allows Player 0 to choose colours (red or \(\neg\text{red}\)) at every second step. We then prove \((\ast\ast)\) for every \(k \in \mathbb{N}\), if Player 0 has an observational strategy in \(G\) that enforces goal \(\psi_k\) from \(s\) then it has an observational strategy in \(G'\) that enforces, from \(s\), the following goal \((\dagger)\): \(\text{col}(\psi)\) holds and no colour consecutively repeats more than \(k\) times, and vice versa (but with \(\psi_{2k}\) substituted for \(\psi\), i.e., that if Player 0 has an observational strategy in \(G'\) that enforces \((\dagger)\) then it has an observational strategy in \(G\) that enforces \(\psi_{2k}\)). Finally, using the fact that LTL games of imperfect information admit finite-state strategies, we will show how to decide (in \(2\text{EXPTIME}\)) whether there exists \(k \in \mathbb{N}\) such that Agent 0 has a strategy in \(G'\) that enforces \((\dagger)\) from \(s\). Here are some more details.

If \(S = \langle Ag, AP, Act, S, \lambda, \delta, \{\sim_\alpha; a \in Ag\} \rangle\), define \(G\) to be the iCGS \(\langle\{0, 1\}, AP, Act_0 \cup Act_1, S, \lambda, \delta_G, \{\sim_{A0}, \sim_{A1}\}\rangle\) as follows:

- action sets \(Act_0 := Act^A\) and \(Act_1 := Act^{Ag \setminus A}\),
- state set \(S\), labeling \(\lambda\),
- transition function \(\delta_G : S \times Act_0 \cup Act_1 \rightarrow S\) that maps \((s, (d_1, d_2)) \mapsto \delta(s, d_1 \otimes d_2)\) where \(d_1 \otimes d_2 \in Act^{Ag}\) maps a to \(d_1(a)\) for \(a \in A\) and otherwise (for \(a \in Ag \setminus A\)) to \(d_2(a)\),
- relation \(\sim_{A0}, \sim_{A1}\) defined as follows: \(s \sim_0 r\) if \(s \sim_A r\) and \(s \sim_1 r\) if \(s \sim_{Ag \setminus A} r\).

It is immediate from the definitions (of \(G\) and \(\models_{co}\)) that \((\ast)\) holds. Define \(G'\) to be the iCGS obtained from \(G\) by adding a new atom \(\neg\text{red}\) and splitting every transition from \(s\) to \(s'\) using decision \(d\), into a diamond shape, where the first decision is either to go left from \(s\) (decision \(d\) with \(\text{red}\)) or go right (decision \(d\) with \(\neg\text{red}\)) to one of two intermediate nodes. The choice of colour is made entirely by Player 0. Then, from each of these intermediate nodes every decision leads to the node \(s'\). The observation sets are defined such that the intermediate nodes that are successors of indistinguishable nodes are themselves indistinguishable (and thus no new information leaks). Formally, \(G'\) is the iCGS\(^{10}\) \(\langle\{0, 1\}, AP \cup \{\text{red}\}, Act', S', \lambda', \delta', \{\sim_{0'}, \sim_{1'}\}\rangle\) where:

- \(Act'_0 := Act_0 \times \{\text{red}, \neg\text{red}\} \) and \(Act'_1 := Act_1\),
- \(S' := S \cup (S \times Act^{Ag}\times \{\text{red}, \neg\text{red}\})\),
- the labeling \(\lambda'\) maps \(s \mapsto \lambda(s)\) and \((s, d, x) \mapsto \{x\}\),
- transition function \(\delta' : S' \times Act'_0 \cup Act'_1 \rightarrow S'\) that, for actions \((d_0, x) \in Act'_0\) and \(d_1 \in Act_1\), maps state \(s \in S\) and actions \((d_0, x), d_1\) to \((s, d_0 \otimes d_1, x)\); and that maps \((s, d, x)\) and all actions of the players to \(\delta_G(s, d)\).

- observation sets \(\sim_{0'}, \sim_{1'}\) defined as follows: \(s \sim'_0 r\) if \(s \sim_0 r\) and \((s, d, x) \sim'_1 (r, d', x')\) if \(s \sim_1 r\).

We now describe, for every \(k \in \mathbb{N}\), the natural transformation of strategies for Player 0 between \(G\) and \(G'\). For \(h' \in \text{hist}(S')\) define \(\text{proj}(h') = \text{hist}(S)\) to be the string in which every second symbol of \(h'\) is removed. Fix an action \(a \in Act'_0\). An observational strategy \(\sigma_0 : \text{hist}(S) \rightarrow Act_0\) in \(G\) induces the strategy \(\sigma'_0 : \text{hist}(S') \rightarrow Act'_0\) in \(G'\) defined as follows. If \(h'\) ends in an element of \(S\) then \(\sigma'_0(h') := (\sigma_0(\text{proj}(h'), x), x\text{;} \text{where \(x = \text{red}\) if the integer part of \(\frac{\text{proj}(h')}{k}\) is odd, and otherwise (if it is even), \(x = \neg\text{red}\) (in words, use \(\sigma_0\) and swap the colour every \(k\) steps in \(G\)) \text{.}\) If \(h'\) does not end in an element of \(S\) then define \(\sigma'_0(h') := a\) (recall that all transitions from the intermediate nodes go to the same destination). It is easy to see that \(\sigma'_0\) is observational since \(\sigma_0\) is.

Before we prove the converse, we state a useful fact that follows from an induction on the length of histories: if \(\text{proj}(h'^0) \sim_0 \text{proj}(h'^2)\) then \(h'^0 \sim_{0'} h'^2\). Now, an observational strategy \(\sigma'_0 : \text{hist}(S') \rightarrow Act'_0\) in \(G'\) induces the strategy \(\sigma_0 : \text{hist}(S) \rightarrow Act_0\) in \(G\) defined as follows: \(\sigma_0(h) := \sigma'_0(h')\) where \(\text{proj}(h') = h\). This is well defined because, if \(\text{proj}(h'_1) = \text{proj}(h'_2) = h\) then \(h'_1 \sim_{0'} h'_2\) (by

\(^{10}\)We remind the reader that \(\text{col}(\psi)\) is defined before Lemma 3.

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the useful fact), and so $\sigma'_0(h'_i) = \sigma'_0(h'_2)$ (since $\sigma'_0$ is observational). To see that $\sigma_0$ is observational let $h_1 \sim_0 h_2$ be equivalent histories in $G$, and suppose $\text{proj}(h'_1) = h_i$. Then by the useful fact, also $h'_1 \sim_0 h'_2$, and thus, since $\sigma'_0$ is observational, we have $\sigma'_0(h'_1) = \sigma'_0(h'_2)$.

We now establish (**). Fix $k$. A strategy $\sigma_0$ in $G$ that ensures $\psi_k$ (by (Lemma 3) that every play consistent with $\sigma_0$ can be $k$-coloured by blocks of size exactly $k$ in a way that satisfies $\text{col}(\psi)$. Thus, the transformed strategy $\sigma'_0$ of $G'$ ensures (†). Conversely, suppose strategy $\sigma'_0$ in $G'$ ensures (‡). Then (by Lemma 3) the transformed strategy $\sigma_0$ of $G$ ensures $\psi_{2k}$.

Finally, we show that Player 0 can enforce (‡) from $s$ in $G'$ if and only if Player 0 can enforce $\text{col}(\psi)$ from $s$ in $G'$. The “only if” direction is immediate from the definitions. For the “if” direction, it is implicit in (Kupferman and Vardi 1997a; 1997b) that $\text{ATL}$ games of imperfect information admit finite-state strategies. We claim that if a strategy requires memory $k$ then it $((|S| \times k)$-colours the play. Indeed, let $h$ be a history, and take $i < j \leq |h|$ such that the infix between positions $i$ and $j$ is of length at least $|S| \times k$. Hence, there exists $i \leq i' < j' \leq j$ such that the last states of $x := h_{i'}$ and $y := h_{j'}$ are equal, and the strategy is in the same memory-state after processing the histories $x$ and $y$. In particular, the strategy gives the same action on $x$ as on $y$. Thus, the opponent can repeatedly play the infix between $i'$ and $j'$, and thus this infix must contain a colour change since we assumed that the strategy enforces $\text{col}(\psi)$ (which states, in particular, that the colours alternate infinitely often). This completes the “if” direction.

Conclusion

Reasoning about promptness has recently received attention for linear specifications (Alur et al. 2001; Kupferman, Piterman, and Vardi 2009; Zimmermann 2013). This is due to the fact that questions like “a specific state is eventually reached in a computation” have a clear meaning and application in the theory of formal verification, but are useless in practical scenarios if there is no bound on the time before the specific state is reached.

In this work, we initiated the study of prompt requirements on branching time specifications for multi-agent systems. We introduced the logic $\text{Prompt-KATL}^*$, an extension of the logic $\text{KATL}$, in which we added the prompt eventually temporal operator $\text{FP}$, and settled the model-checking complexity of many natural fragments, under various restrictions, i.e., perfect information, memoryless strategies, and the existential fragment with co-operative strategies. In particular, we prove that model-checking $\text{Prompt-KCTL}^*$ is $\text{PSpace}$-complete without any restrictions. Note that in the case of two players the restriction to co-operative strategies is not a real restriction as these correspond to ordinary strategies. Our results for $\text{Prompt-KATL}^*$ and $\text{Prompt-KATL}$ are summarised in the following tables:

<table>
<thead>
<tr>
<th>Perfect Info. or co-operative $\exists$ Fragment</th>
<th>Memoryless</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{2ExpTime}$-complete</td>
<td>$\text{PSpace}$-complete</td>
</tr>
</tbody>
</table>

Figure 1: Complexity of model checking $\text{Prompt-KATL}^*$

<table>
<thead>
<tr>
<th>Perfect Info. or co-operative $\exists$ Fragment</th>
<th>Memoryless</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{in PTime}$</td>
<td>$\text{in PSpace}$</td>
</tr>
</tbody>
</table>

Figure 2: Complexity of model checking $\text{Prompt-KATL}^*$

We remark that our upper-bounds for $\text{Prompt-KATL}^*$ obtained essentially as a side-effect of the results for $\text{Prompt-KATL}^*$, i.e., by analysing the complexity of the algorithms we provided for $\text{Prompt-KATL}^*$ in the restricted case of $\text{Prompt-KATL}$. However, one can directly reason on $\text{Prompt-KATL}$. For example, in the perfect-information case, one can show that (like for $\text{Prompt-CTL}$) $S \models \varphi$, for a $\text{Prompt-ATL}$ formula $\varphi$, iff $S \models \varphi$, where $\varphi$ is the $\text{ATL}$ formula obtained by replacing every prompt-eventuality $\text{FP}_p$ by a regular eventuality $\text{F}$. The underlying reason is that in $\text{Prompt-KATL}$ $\text{FP}_p$ only ranges over state sub-formulas and thus, reasoning about sub-formulas of the form $\langle A \rangle \text{FP}_p \varphi$ and $\langle A \rangle \text{FP}_p \varphi$ reduces to reasoning about two player games with reachability goals. In such games a (memoryless) winning strategy does not admit a lasso in which $\varphi$ does not hold on all its states (since then the opponent can pump the loop of the lasso and win). Thus, if the goal $\varphi$ is achieved, it is achieved within a number of steps that is at most the size of the model, and thus promptly. For the case of imperfect information, more complicated reasoning can be employed.

Our work opens up several avenues of research. First, an intriguing open question is whether model-checking $\text{Prompt-KATL}^*$ under cooperative strategies is decidable, even for two players (in this work we established the optimal complexity for its existential fragment). Second, the satisfiability problem has yet to be tackled, and we believe that the techniques introduces in this paper would yield useful for this investigation.

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References


memoryless strategies.

11Winning strategies in such games can be taken w.l.o.g. to be memoryless.

12Winning strategies in such games can be taken w.l.o.g. to be memoryless.


